CHAPTER
3

STEADY HEAT CONDUCTION

I
n heat transfer analysis, we are often interested in the rate of heat transfer
through a medium under steady conditions and surface temperatures. Such
problems can be solved easily without involving any differential equations
by the introduction of *thermal resistance concepts* in an analogous manner to
electrical circuit problems. In this case, the thermal resistance corresponds
to electrical resistance, temperature difference corresponds to voltage, and the
heat transfer rate corresponds to electric current.

We start this chapter with *one-dimensional steady heat conduction* in
a plane wall, a cylinder, and a sphere, and develop relations for *thermal resis-
tances* in these geometries. We also develop thermal resistance relations for
convection and radiation conditions at the boundaries. We apply this concept
to heat conduction problems in *multilayer* plane walls, cylinders, and spheres
and generalize it to systems that involve heat transfer in two or three dimen-
sions. We also discuss the *thermal contact resistance* and the *overall heat
transfer coefficient* and develop relations for the critical radius of insulation
for a cylinder and a sphere. Finally, we discuss steady heat transfer from
finned surfaces and some complex geometrics commonly encountered in
practice through the use of *conduction shape factors.*
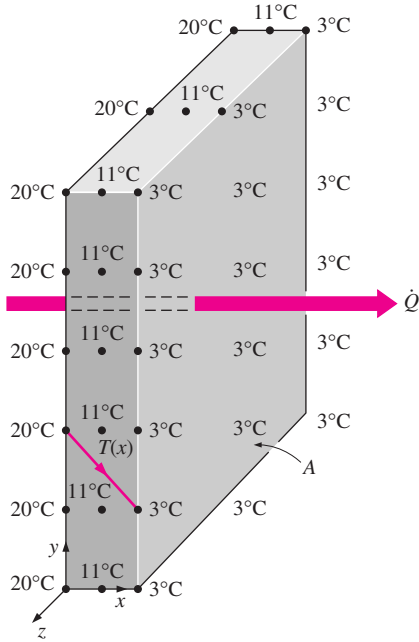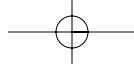
CONTENTS

FIGURE 3–1
Heat flow through a wall is one-dimensional when the temperature of the wall varies in one direction only.

3–1 ▪ STEADY HEAT CONDUCTION IN PLANE WALLS

Consider steady heat conduction through the walls of a house during a winter day. We know that heat is continuously lost to the outdoors through the wall. We intuitively feel that heat transfer through the wall is in the *normal direction* to the wall surface, and no significant heat transfer takes place in the wall in other directions (Fig. 3–1).

Recall that heat transfer in a certain direction is driven by the *temperature gradient* in that direction. There will be no heat transfer in a direction in which there is no change in temperature. Temperature measurements at several locations on the inner or outer wall surface will confirm that a wall surface is nearly *isothermal.* That is, the temperatures at the top and bottom of a wall surface as well as at the right or left ends are almost the same. Therefore, there will be no heat transfer through the wall from the top to the bottom, or from left to right, but there will be considerable temperature difference between the inner and the outer surfaces of the wall, and thus significant heat transfer in the direction from the inner surface to the outer one.

The small thickness of the wall causes the temperature gradient in that direction to be large. Further, if the air temperatures in and outside the house remain constant, then heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional.* The temperature of the wall in this case will depend on one direction only (say the *x*-direction) and can be expressed as $T(x)$.

Noting that heat transfer is the only energy interaction involved in this case and there is no heat generation, the *energy balance* for the wall can be expressed as

$$\begin{pmatrix} \text{Rate of} \\ \text{heat transfer} \\ \text{into the wall} \end{pmatrix} - \begin{pmatrix} \text{Rate of} \\ \text{heat transfer} \\ \text{out of the wall} \end{pmatrix} = \begin{pmatrix} \text{Rate of change} \\ \text{of the energy} \\ \text{of the wall} \end{pmatrix}$$
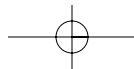
or

$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \frac{dE_{\text{wall}}}{dt} \qquad \text{(3-1)}$$

But $dE_{\text{wall}}/dt = 0$ for *steady* operation, since there is no change in the temperature of the wall with time at any point. Therefore, the rate of heat transfer into the wall must be equal to the rate of heat transfer out of it. In other words, *the rate of heat transfer through the wall must be constant,* $\dot{Q}_{\text{cond, wall}} = $ constant.

Consider a plane wall of thickness L and average thermal conductivity k. The two surfaces of the wall are maintained at constant temperatures of T_1 and T_2. For one-dimensional steady heat conduction through the wall, we have $T(x)$. Then Fourier's law of heat conduction for the wall can be expressed as

$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx} \qquad \text{(W)} \qquad \text{(3-2)}$$

where the rate of conduction heat transfer $\dot{Q}_{\text{cond wall}}$ and the wall area A are constant. Thus we have $dT/dx = $ constant, which means that *the temperature*
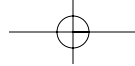
through the wall varies linearly with x. That is, the temperature distribution in the wall under steady conditions is a *straight line* (Fig. 3–2).

Separating the variables in the above equation and integrating from $x = 0$, where $T(0) = T_1$, to $x = L$, where $T(L) = T_2$, we get

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}} \, dx = -\int_{T=T_1}^{T_2} kA \, dT$$

Performing the integrations and rearranging gives

$$\dot{Q}_{\text{cond, wall}} = kA \frac{T_1 - T_2}{L} \qquad \text{(W)} \qquad \text{(3-3)}$$

which is identical to Eq. 3–1. Again, *the rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness.* Also, once the rate of heat conduction is available, the temperature $T(x)$ at any location x can be determined by replacing T_2 in Eq. 3–3 by T, and L by x.

The Thermal Resistance Concept

Equation 3–3 for heat conduction through a plane wall can be rearranged as

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad \text{(W)} \qquad \text{(3-4)}$$

where

$$R_{\text{wall}} = \frac{L}{kA} \qquad \text{(°C/W)} \qquad \text{(3-5)}$$

is the *thermal resistance* of the wall against heat conduction or simply the **conduction resistance** of the wall. Note that the thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium.

The equation above for heat flow is analogous to the relation for *electric current flow I,* expressed as

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e} \qquad \qquad \text{(3-6)}$$

where $R_e = L/\sigma_e A$ is the *electric resistance* and $\mathbf{V}_1 - \mathbf{V}_2$ is the *voltage difference* across the resistance (σ_e is the electrical conductivity). Thus, the *rate of heat transfer* through a layer corresponds to the *electric current,* the *thermal resistance* corresponds to *electrical resistance,* and the *temperature difference* corresponds to *voltage difference* across the layer (Fig. 3–3).

Consider convection heat transfer from a solid surface of area A_s and temperature T_s to a fluid whose temperature sufficiently far from the surface is T_∞, with a convection heat transfer coefficient h. Newton's law of cooling for convection heat transfer rate $\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$ can be rearranged as

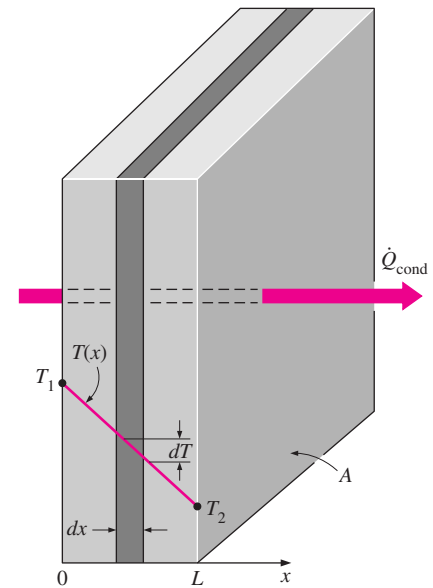$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}} \qquad \text{(W)} \qquad \text{(3-7)}$$

FIGURE 3–2
Under steady conditions, the temperature distribution in a plane wall is a straight line.

$$\dot{Q} = \frac{T_1 - T_2}{R}$$

(*a*) Heat flow

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e}$$

(*b*) Electric current flow

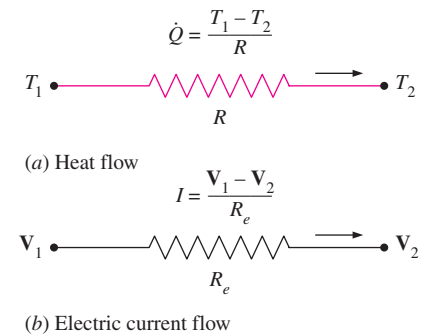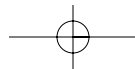**FIGURE 3–3**
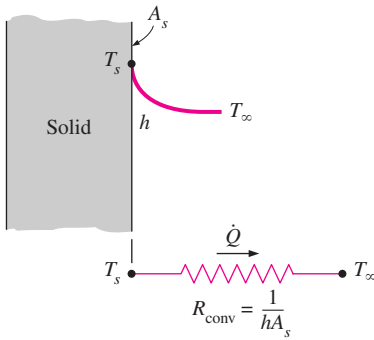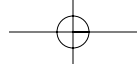Analogy between thermal and electrical resistance concepts.

FIGURE 3–4
Schematic for convection
resistance at a surface.

where

$$R_{conv} = \frac{1}{hA_s} \qquad (°C/W) \qquad \text{(3-8)}$$

is the *thermal resistance* of the surface against heat convection, or simply the **convection resistance** of the surface (Fig. 3–4). Note that when the convection heat transfer coefficient is very large ($h \to \infty$), the convection resistance becomes *zero* and $T_s \approx T_\infty$. That is, the surface offers *no resistance to convection,* and thus it does not slow down the heat transfer process. This situation is approached in practice at surfaces where boiling and condensation occur. Also note that the surface does not have to be a plane surface. Equation 3–8 for convection resistance is valid for surfaces of any shape, provided that the assumption of $h =$ constant and uniform is reasonable.

When the wall is surrounded by a gas, the *radiation effects,* which we have ignored so far, can be significant and may need to be considered. The rate of radiation heat transfer between a surface of emissivity ε and area A_s at temperature T_s and the surrounding surfaces at some average temperature T_{surr} can be expressed as

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) = h_{rad}A_s(T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}} \qquad (W) \qquad \text{(3-9)}$$

where

$$R_{rad} = \frac{1}{h_{rad}A_s} \qquad (K/W) \qquad \text{(3-10)}$$

is the *thermal resistance* of a surface against radiation, or the *radiation resistance,* and

$$h_{rad} = \frac{\dot{Q}_{rad}}{A_s(T_s - T_{surr})} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr}) \qquad (W/m^2 \cdot K) \qquad \text{(3-11)}$$

is the **radiation heat transfer coefficient.** Note that both T_s and T_{surr} *must* be in K in the evaluation of h_{rad}. The definition of the radiation heat transfer coefficient enables us to express radiation conveniently in an analogous manner to convection in terms of a temperature difference. But h_{rad} depends strongly on temperature while h_{conv} usually does not.

A surface exposed to the surrounding air involves convection and radiation simultaneously, and the total heat transfer at the surface is determined by adding (or subtracting, if in the opposite direction) the radiation and convection components. The convection and radiation resistances are parallel to each other, as shown in Fig. 3–5, and may cause some complication in the thermal resistance network. When $T_{surr} \approx T_\infty$, the radiation effect can properly be accounted for by replacing h in the convection resistance relation by

$$h_{combined} = h_{conv} + h_{rad} \qquad (W/m^2 \cdot K) \qquad \text{(3-12)}$$

FIGURE 3–5
Schematic for convection and
radiation resistances at a surface.

where $h_{combined}$ is the **combined heat transfer coefficient.** This way all the complications associated with radiation are avoided.
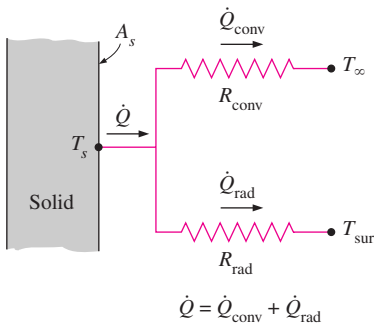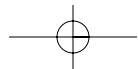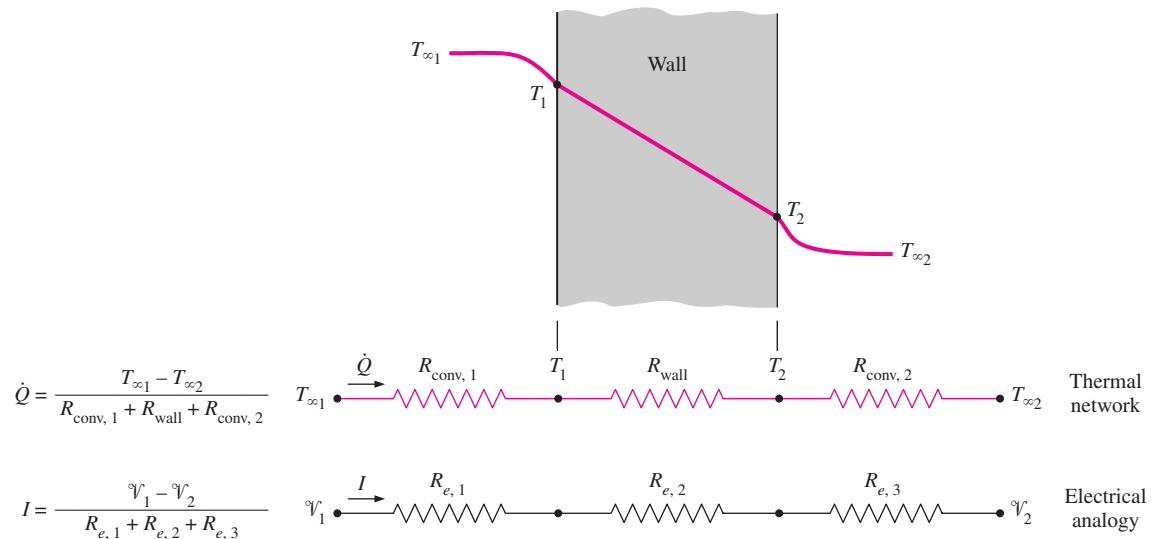
FIGURE 3–6
The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

Thermal Resistance Network

Now consider steady one-dimensional heat flow through a plane wall of thickness L, area A, and thermal conductivity k that is exposed to convection on both sides to fluids at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with heat transfer coefficients h_1 and h_2, respectively, as shown in Fig. 3–6. Assuming $T_{\infty 2} < T_{\infty 1}$, the variation of temperature will be as shown in the figure. Note that the temperature varies linearly in the wall, and asymptotically approaches $T_{\infty 1}$ and $T_{\infty 2}$ in the fluids as we move away from the wall.

Under steady conditions we have

$$\begin{pmatrix} \text{Rate of} \\ heat\ convection \\ \text{into the wall} \end{pmatrix} = \begin{pmatrix} \text{Rate of} \\ heat\ conduction \\ \text{through the wall} \end{pmatrix} = \begin{pmatrix} \text{Rate of} \\ heat\ convection \\ \text{from the wall} \end{pmatrix}$$

or

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA \frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2}) \qquad \textbf{(3-13)}$$

which can be rearranged as

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}} \qquad \textbf{(3-14)}$$

Adding the numerators and denominators yields (Fig. 3–7)

$$\dot{Q} = \frac{T_\infty - T_{\infty 2}}{R_{\text{total}}} \qquad \text{(W)} \qquad \textbf{(3-15)}$$

If

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \frac{a_n}{b_n} = c$$

then

$$\frac{a_1 + a_2 + \cdots + a_n}{b_1 + b_2 + \cdots + b_n} = c$$

For example,

$$\frac{1}{4} = \frac{2}{8} = \frac{5}{20} = 0.25$$

and

$$\frac{1 + 2 + 5}{4 + 8 + 20} = 0.25$$

FIGURE 3–7
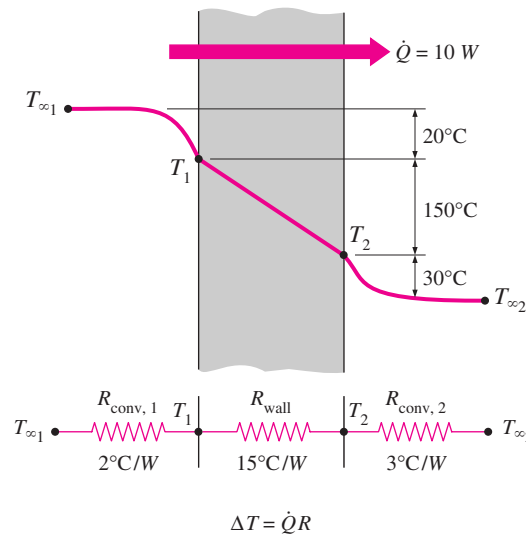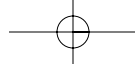A useful mathematical identity.

FIGURE 3–8
The temperature drop across a layer is proportional to its thermal resistance.

where

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (°C/W) \tag{3-16}$$

Note that the heat transfer area A is constant for a plane wall, and the rate of heat transfer through a wall separating two mediums is equal to the temperature difference divided by the total thermal resistance between the mediums. Also note that the thermal resistances are in *series,* and the equivalent thermal resistance is determined by simply *adding* the individual resistances, just like the electrical resistances connected in series. Thus, the electrical analogy still applies. We summarize this as *the rate of steady heat transfer between two surfaces is equal to the temperature difference divided by the total thermal resistance between those two surfaces.*

Another observation that can be made from Eq. 3–15 is that the ratio of the temperature drop to the thermal resistance across any layer is constant, and thus the temperature drop across any layer is proportional to the thermal resistance of the layer. The larger the resistance, the larger the temperature drop. In fact, the equation $\dot{Q} = \Delta T/R$ can be rearranged as

$$\Delta T = \dot{Q} R \qquad (°C) \tag{3-17}$$

which indicates that the *temperature drop* across any layer is equal to the *rate of heat transfer* times the *thermal resistance* across that layer (Fig. 3–8). You may recall that this is also true for voltage drop across an electrical resistance when the electric current is constant.

It is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA \, \Delta T \qquad (W) \tag{3-18}$$

where U is the **overall heat transfer coefficient.** A comparison of Eqs. 3–15 and 3–18 reveals that
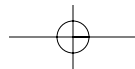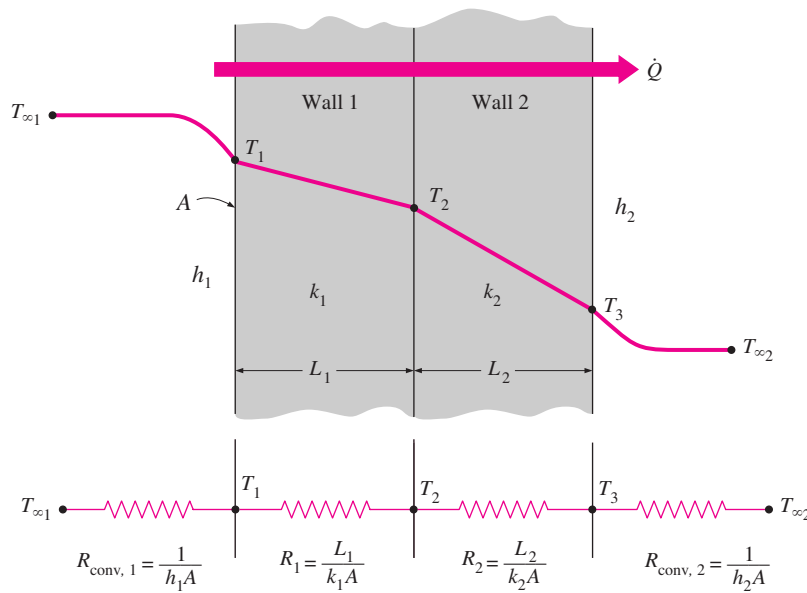
The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

$$UA = \frac{1}{R_{\text{total}}} \quad \text{(3-19)}$$

Therefore, for a unit area, the overall heat transfer coefficient is equal to the inverse of the total thermal resistance.

Note that we do not need to know the surface temperatures of the wall in order to evaluate the rate of steady heat transfer through it. All we need to know is the convection heat transfer coefficients and the fluid temperatures on both sides of the wall. The *surface temperature* of the wall can be determined as described above using the thermal resistance concept, but by taking the surface at which the temperature is to be determined as one of the terminal surfaces. For example, once $\dot{Q}$ is evaluated, the surface temperature T_1 can be determined from
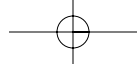
$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_{\infty 1} - T_1}{1/h_1 A} \quad \text{(3-20)}$$

Multilayer Plane Walls

In practice we often encounter plane walls that consist of several layers of different materials. The thermal resistance concept can still be used to determine the rate of steady heat transfer through such *composite* walls. As you may have already guessed, this is done by simply noting that the conduction resistance of each wall is *L/kA* connected in series, and using the electrical analogy. That is, by dividing the *temperature difference* between two surfaces at known temperatures by the *total thermal resistance* between them.

Consider a plane wall that consists of two layers (such as a brick wall with a layer of insulation). The rate of steady heat transfer through this two-layer composite wall can be expressed as (Fig. 3–9)

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \quad \text{(3-21)}$$
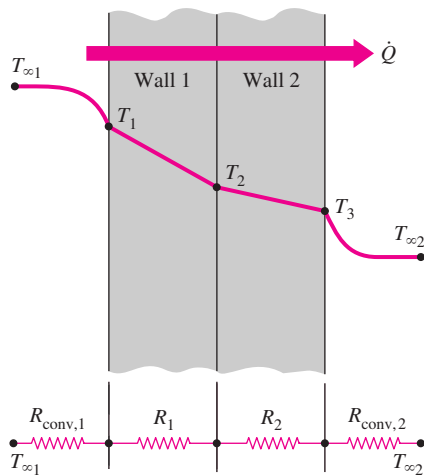
To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{\text{conv},2}}$

FIGURE 3–10
The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.

FIGURE 3–11
Schematic for Example 3–1.

where R_{total} is the *total thermal resistance,* expressed as

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall, 1}} + R_{\text{wall, 2}} + R_{\text{conv, 2}}$$

$$= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A} \qquad \text{(3-22)}$$

The subscripts 1 and 2 in the R_{wall} relations above indicate the first and the second layers, respectively. We could also obtain this result by following the approach used above for the single-layer case by noting that the rate of steady heat transfer $\dot{Q}$ through a multilayer medium is constant, and thus it must be the same through each layer. Note from the thermal resistance network that the resistances are *in series,* and thus the *total thermal resistance* is simply the *arithmetic sum* of the individual thermal resistances in the path of heat flow.

This result for the *two-layer* case is analogous to the *single-layer* case, except that an *additional resistance* is added for the *additional layer.* This result can be extended to plane walls that consist of *three* or *more layers* by adding an *additional resistance* for each *additional layer.*

Once $\dot{Q}$ is *known,* an unknown surface temperature T_j at any surface or interface j can be determined from

$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total}, i-j}} \qquad \text{(3-23)}$$

where T_i is a *known* temperature at location i and $R_{\text{total}, i-j}$ is the total thermal resistance between locations i and j. For example, when the fluid temperatures $T_{\infty 1}$ and $T_{\infty 2}$ for the two-layer case shown in Fig. 3–9 are available and $\dot{Q}$ is calculated from Eq. 3–21, the interface temperature T_2 between the two walls can be determined from (Fig. 3–10)

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv, 1}} + R_{\text{wall, 1}}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}} \qquad \text{(3-24)}$$

The temperature drop across a layer is easily determined from Eq. 3–17 by multiplying $\dot{Q}$ by the thermal resistance of that layer.

The thermal resistance concept is widely used in practice because it is intuitively easy to understand and it has proven to be a powerful tool in the solution of a wide range of heat transfer problems. But its use is limited to systems through which the rate of heat transfer $\dot{Q}$ remains *constant*; that is, to systems involving *steady* heat transfer with *no heat generation* (such as resistance heating or chemical reactions) within the medium.
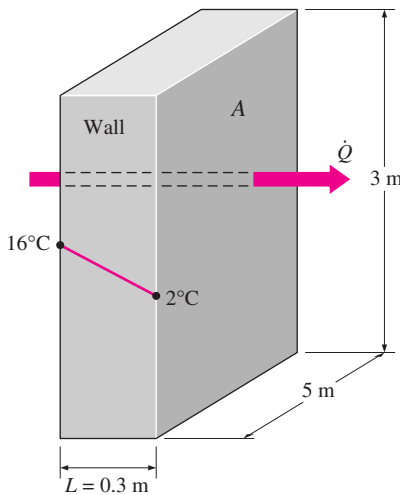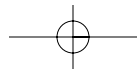
EXAMPLE 3–1   **Heat Loss through a Wall**

Consider a 3-m-high, 5-m-wide, and 0.3-m-thick wall whose thermal conductivity is $k = 0.9$ W/m · °C (Fig. 3–11). On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 16°C and 2°C, respectively. Determine the rate of heat loss through the wall on that day.

SOLUTION   The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

Assumptions   **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties   The thermal conductivity is given to be $k = 0.9$ W/m · °C.

Analysis   Noting that the heat transfer through the wall is by conduction and the area of the wall is $A = 3$ m $\times$ 5 m $= 15$ m^2, the steady rate of heat transfer through the wall can be determined from Eq. 3–3 to be

$$\dot{Q} = kA\,\frac{T_1 - T_2}{L} = (0.9\ \text{W/m} \cdot \text{°C})(15\ \text{m}^2)\,\frac{(16 - 2)\text{°C}}{0.3\ \text{m}} = \mathbf{630\ W}$$

We could also determine the steady rate of heat transfer through the wall by making use of the thermal resistance concept from

$$\dot{Q} = \frac{\Delta T_{\text{wall}}}{R_{\text{wall}}}$$

where

$$R_{\text{wall}} = \frac{L}{kA} = \frac{0.3\ \text{m}}{(0.9\ \text{W/m} \cdot \text{°C})(15\ \text{m}^2)} = 0.02222\text{°C/W}$$

Substituting, we get

$$\dot{Q} = \frac{(16 - 2)\text{°C}}{0.02222\text{°C/W}} = 630\ \text{W}$$

Discussion   This is the same result obtained earlier. Note that heat conduction through a plane wall with specified surface temperatures can be determined directly and easily without utilizing the thermal resistance concept. However, the thermal resistance concept serves as a valuable tool in more complex heat transfer problems, as you will see in the following examples.

EXAMPLE 3–2   **Heat Loss through a Single-Pane Window**

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m · °C. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is -10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2 · °C and $h_2 = 40$ W/m^2 · °C, which includes the effects of radiation.

SOLUTION   Heat loss through a window glass is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.
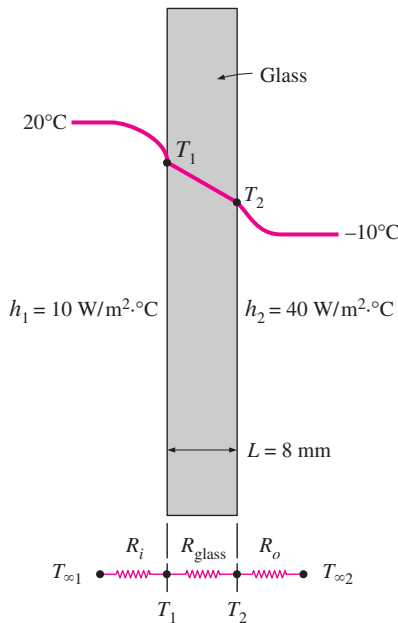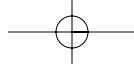
FIGURE 3–12
Schematic for Example 3–2.

Assumptions   **1** Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer through the wall is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties   The thermal conductivity is given to be $k = 0.78$ W/m · °C.

Analysis   This problem involves conduction through the glass window and convection at its surfaces, and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 3–12. Noting that the area of the window is $A = 0.8$ m $\times$ 1.5 m = 1.2 m^2, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot °\text{C})(1.2 \text{ m}^2)} = 0.08333°\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008 \text{ m}}{(0.78 \text{ W/m} \cdot °\text{C})(1.2 \text{ m}^2)} = 0.00855°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2 \cdot °\text{C})(1.2 \text{ m}^2)} = 0.02083°\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{glass}} + R_{\text{conv, 2}} = 0.08333 + 0.00855 + 0.02083$$
$$= 0.1127°\text{C/W}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°\text{C}}{0.1127°\text{C/W}} = \textbf{266 W}$$
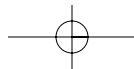
Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

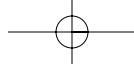$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} \longrightarrow \quad T_1 = T_{\infty 1} - \dot{Q} R_{\text{conv, 1}}$$
$$= 20°\text{C} - (266 \text{ W})(0.08333°\text{C/W})$$
$$= \textbf{-2.2°C}$$

Discussion   Note that the inner surface temperature of the window glass will be $-2.2°$C even though the temperature of the air in the room is maintained at 20°C. Such low surface temperatures are highly undesirable since they cause the formation of fog or even frost on the inner surfaces of the glass when the humidity in the room is high.

EXAMPLE 3–3   **Heat Loss through Double-Pane Windows**

Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass ($k = 0.78$ W/m · °C) separated by a 10-mm-wide stagnant air space ($k = 0.026$ W/m · °C). Determine the steady rate of heat

transfer through this double-pane window and the temperature of its inner sur-
face for a day during which the room is maintained at 20°C while the tempera-
ture of the outdoors is −10°C. Take the convection heat transfer coefficients on
the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2 · °C and $h_2 = 40$ W/m^2 · °C, which includes the effects of radiation.

SOLUTION   A double-pane window is considered. The rate of heat transfer
through the window and the inner surface temperature are to be determined.
Analysis   This example problem is identical to the previous one except that
the single 8-mm-thick window glass is replaced by two 4-mm-thick glasses that
enclose a 10-mm-wide stagnant air space. Therefore, the thermal resistance
network of this problem will involve two additional conduction resistances cor-
responding to the two additional layers, as shown in Fig. 3–13. Noting that the
area of the window is again $A = 0.8$ m $\times$ 1.5 m $= 1.2$ m^2, the individual re-
sistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot °C)(1.2 \text{ m}^2)} = 0.08333 °C/W$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.004 \text{ m}}{(0.78 \text{ W/m} \cdot °C)(1.2 \text{ m}^2)} = 0.00427 °C/W$$

$$R_2 = R_{\text{air}} = \frac{L_2}{k_2 A} = \frac{0.01 \text{ m}}{(0.026 \text{ W/m} \cdot °C)(1.2 \text{ m}^2)} = 0.3205 °C/W$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2 \cdot °C)(1.2 \text{ m}^2)} = 0.02083 °C/W$$

Noting that all three resistances are in series, the total resistance is

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{glass, 1}} + R_{\text{air}} + R_{\text{glass, 2}} + R_{\text{conv, 2}}$$
$$= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083$$
$$= 0.4332 °C/W$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°C}{0.4332 °C/W} = \textbf{69.2 W}$$

which is about one-fourth of the result obtained in the previous example. This
explains the popularity of the double- and even triple-pane windows in cold
climates. The drastic reduction in the heat transfer rate in this case is due to
the large thermal resistance of the air layer between the glasses.
   The inner surface temperature of the window in this case will be

$$T_1 = T_{\infty 1} - \dot{Q} R_{\text{conv, 1}} = 20°C - (69.2 \text{ W})(0.08333 °C/W) = \textbf{14.2°C}$$

which is considerably higher than the −2.2°C obtained in the previous ex-
ample. Therefore, a double-pane window will rarely get fogged. A double-pane
window will also reduce the heat gain in summer, and thus reduce the air-
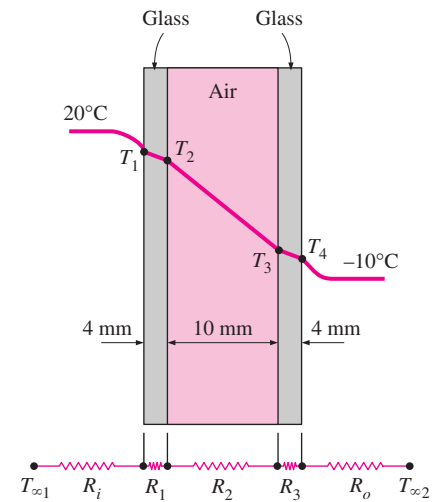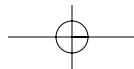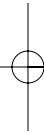conditioning costs.

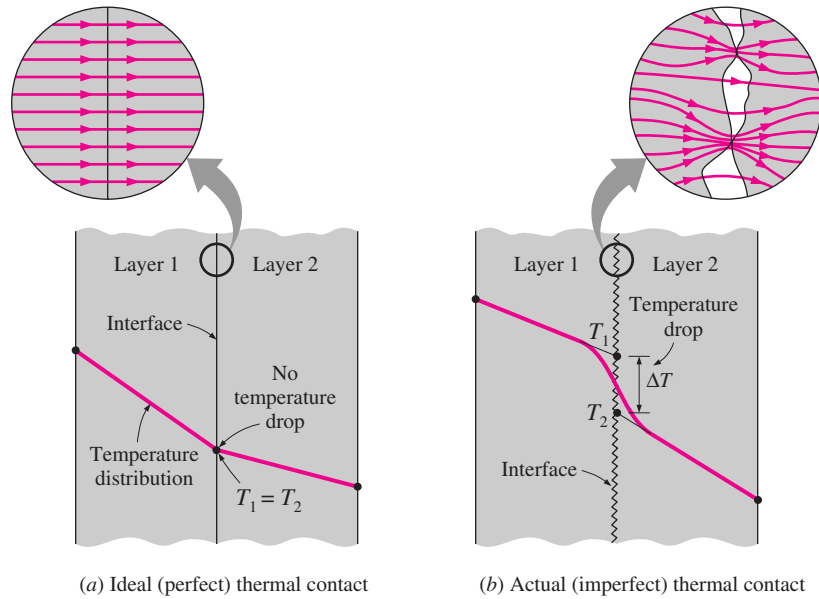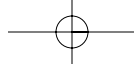**FIGURE 3–13**
Schematic for Example 3–3.

FIGURE 3–14
Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

(*a*) Ideal (perfect) thermal contact   (*b*) Actual (imperfect) thermal contact

3–2 ▪ THERMAL CONTACT RESISTANCE

In the analysis of heat conduction through multilayer solids, we assumed "perfect contact" at the interface of two layers, and thus no temperature drop at the interface. This would be the case when the surfaces are perfectly smooth and they produce a perfect contact at each point. In reality, however, even flat surfaces that appear smooth to the eye turn out to be rather rough when examined under a microscope, as shown in Fig. 3–14, with numerous peaks and valleys. That is, a surface is *microscopically rough* no matter how smooth it appears to be.

When two such surfaces are pressed against each other, the peaks will form good material contact but the valleys will form voids filled with air. As a result, an interface will contain numerous *air gaps* of varying sizes that act as *insulation* because of the low thermal conductivity of air. Thus, an interface offers some resistance to heat transfer, and this resistance per unit interface area is called the **thermal contact resistance,** R_c. The value of R_c is determined experimentally using a setup like the one shown in Fig. 3–15, and as expected, there is considerable scatter of data because of the difficulty in characterizing the surfaces.

Consider heat transfer through two metal rods of cross-sectional area A that are pressed against each other. Heat transfer through the interface of these two rods is the sum of the heat transfers through the *solid contact spots* and the *gaps* in the noncontact areas and can be expressed as

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}} \qquad \text{(3-25)}$$

It can also be expressed in an analogous manner to Newton's law of cooling as

$$\dot{Q} = h_c A\, \Delta T_{\text{interface}} \qquad \text{(3-26)}$$
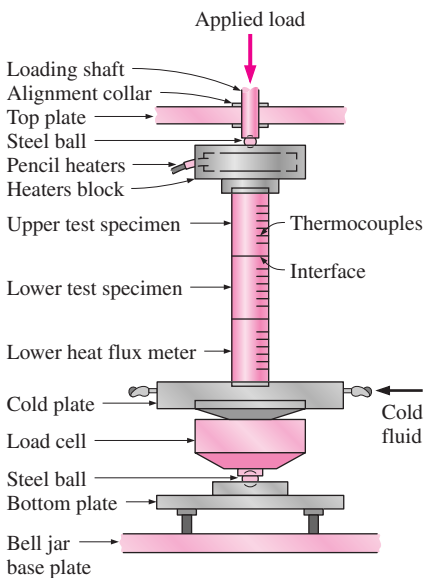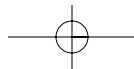

FIGURE 3–15
A typical experimental setup for the determination of thermal contact resistance (from Song et al., Ref. 11).

where A is the apparent interface area (which is the same as the cross-sectional area of the rods) and $\Delta T_{\text{interface}}$ is the effective temperature difference at the interface. The quantity h_c, which corresponds to the convection heat transfer coefficient, is called the **thermal contact conductance** and is expressed as

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C}) \qquad\qquad \textbf{(3-27)}$$

It is related to thermal contact resistance by

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \qquad (\text{m}^2 \cdot {}^\circ\text{C/W}) \qquad\qquad \textbf{(3-28)}$$

That is, thermal contact resistance is the inverse of thermal contact conductance. Usually, thermal contact conductance is reported in the literature, but the concept of thermal contact resistance serves as a better vehicle for explaining the effect of interface on heat transfer. Note that R_c represents thermal contact resistance *per unit area.* The thermal resistance for the entire interface is obtained by dividing R_c by the apparent interface area A.

The thermal contact resistance can be determined from Eq. 3–28 by measuring the temperature drop at the interface and dividing it by the heat flux under steady conditions. The value of thermal contact resistance depends on the *surface roughness* and the *material properties* as well as the *temperature* and *pressure* at the interface and the *type of fluid* trapped at the interface. The situation becomes more complex when plates are fastened by bolts, screws, or rivets since the interface pressure in this case is nonuniform. The thermal contact resistance in that case also depends on the plate thickness, the bolt radius, and the size of the contact zone. Thermal contact resistance is observed to *decrease* with *decreasing surface roughness* and *increasing interface pressure,* as expected. Most experimentally determined values of the thermal contact resistance fall between 0.000005 and 0.0005 m^2 · °C/W (the corresponding range of thermal contact conductance is 2000 to 200,000 W/m^2 · °C).

When we analyze heat transfer in a medium consisting of two or more layers, the first thing we need to know is whether the thermal contact resistance is *significant* or not. We can answer this question by comparing the magnitudes of the thermal resistances of the layers with typical values of thermal contact resistance. For example, the thermal resistance of a 1-cm-thick layer of an insulating material per unit surface area is

$$R_{c,\,\text{insulation}} = \frac{L}{k} = \frac{0.01 \text{ m}}{0.04 \text{ W/m} \cdot {}^\circ\text{C}} = 0.25 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

whereas for a 1-cm-thick layer of copper, it is

$$R_{c,\,\text{copper}} = \frac{L}{k} = \frac{0.01 \text{ m}}{386 \text{ W/m} \cdot {}^\circ\text{C}} = 0.000026 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

Comparing the values above with typical values of thermal contact resistance, we conclude that thermal contact resistance is significant and can even dominate the heat transfer for good heat conductors such as metals, but can be

TABLE 3–1

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of 10 μm and interface pressure of 1 atm (from Fried, Ref. 5)

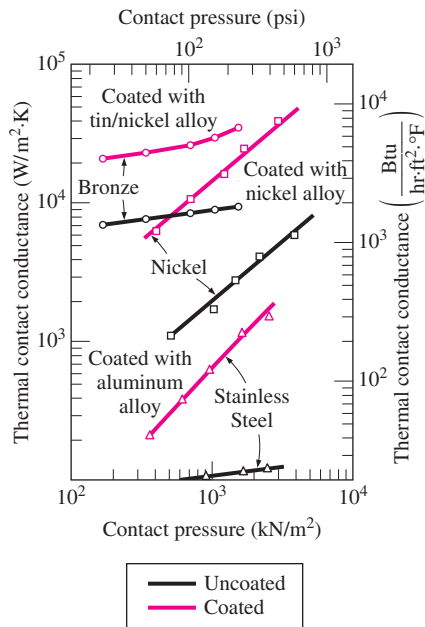| Fluid at the Interface	Contact Conductance, h_c, W/m^2 · °C
Air	3640
Helium	9520
Hydrogen	13,900
Silicone oil	19,000
Glycerin	37,700

FIGURE 3–16
Effect of metallic coatings on thermal contact conductance (from Peterson, Ref. 10).

disregarded for poor heat conductors such as insulations. This is not surprising since insulating materials consist mostly of air space just like the interface itself.

The thermal contact resistance can be minimized by applying a thermally conducting liquid called a *thermal grease* such as silicon oil on the surfaces before they are pressed against each other. This is commonly done when attaching electronic components such as power transistors to heat sinks. The thermal contact resistance can also be reduced by replacing the air at the interface by a *better conducting gas* such as helium or hydrogen, as shown in Table 3–1.

Another way to minimize the contact resistance is to insert a *soft metallic foil* such as tin, silver, copper, nickel, or aluminum between the two surfaces. Experimental studies show that the thermal contact resistance can be reduced by a factor of up to 7 by a metallic foil at the interface. For maximum effectiveness, the foils must be very thin. The effect of metallic coatings on thermal contact conductance is shown in Fig. 3–16 for various metal surfaces.

There is considerable uncertainty in the contact conductance data reported in the literature, and care should be exercised when using them. In Table 3–2 some experimental results are given for the contact conductance between similar and dissimilar metal surfaces for use in preliminary design calculations. Note that the *thermal contact conductance* is *highest* (and thus the contact resistance is lowest) for *soft metals* with *smooth surfaces* at *high pressure*.

EXAMPLE 3–4   Equivalent Thickness for Contact Resistance

The thermal contact conductance at the interface of two 1-cm-thick aluminum plates is measured to be 11,000 W/m^2 · °C. Determine the thickness of the aluminum plate whose thermal resistance is equal to the thermal resistance of the interface between the plates (Fig. 3–17).

SOLUTION The thickness of the aluminum plate whose thermal resistance is equal to the thermal contact resistance is to be determined.
Properties The thermal conductivity of aluminum at room temperature is $k = 237$ W/m · °C (Table A-3).
Analysis Noting that thermal contact resistance is the inverse of thermal contact conductance, the thermal contact resistance is

$$R_c = \frac{1}{h_c} = \frac{1}{11,000 \text{ W/m}^2 \cdot °C} = 0.909 \times 10^{-4} \text{ m}^2 \cdot °C/W$$

For a unit surface area, the thermal resistance of a flat plate is defined as

$$R = \frac{L}{k}$$

where L is the thickness of the plate and k is the thermal conductivity. Setting $R = R_c$, the equivalent thickness is determined from the relation above to be

$$L = kR_c = (237 \text{ W/m} \cdot °C)(0.909 \times 10^{-4} \text{ m}^2 \cdot °C/W) = 0.0215 \text{ m} = \textbf{2.15 cm}$$

TABLE 3–2

Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface Condition	Rough-ness, μm	Tempera-ture, °C	Pressure, MPa	h_c,* W/m^2 · °C
Identical Metal Pairs					
416 Stainless steel	Ground	2.54	90–200	0.3–2.5	3800
304 Stainless steel	Ground	1.14	20	4–7	1900
Aluminum	Ground	2.54	150	1.2–2.5	11,400
Copper	Ground	1.27	20	1.2–20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.7–7	11,400
Dissimilar Metal Pairs					
Stainless steel–				10	2900
Aluminum		20–30	20	20	3600
Stainless steel–				10	16,400
Aluminum		1.0–2.0	20	20	20,800
Steel Ct-30–				10	50,000
Aluminum	Ground	1.4–2.0	20	15–35	59,000
Steel Ct-30–				10	4800
Aluminum	Milled	4.5–7.2	20	30	8300
				5	42,000
Aluminum-Copper	Ground	1.3–1.4	20	15	56,000
				10	12,000
Aluminum-Copper	Milled	4.4–4.5	20	20–35	22,000

*Divide the given values by 5.678 to convert to Btu/h · ft^2 · °F.

Discussion   Note that the interface between the two plates offers as much re-sistance to heat transfer as a 2.3–cm-thick aluminum plate. It is interesting that the thermal contact resistance in this case is greater than the sum of the thermal resistances of both plates.

FIGURE 3–17
Schematic for Example 3–4.

EXAMPLE 3–5   Contact Resistance of Transistors

Four identical power transistors with aluminum casing are attached on one side of a 1-cm-thick 20-cm × 20-cm square copper plate (k = 386 W/m · °C) by screws that exert an average pressure of 6 MPa (Fig. 3–18). The base area of each transistor is 8 cm^2, and each transistor is placed at the center of a 10-cm × 10-cm quarter section of the plate. The interface roughness is estimated to be about 1.5 μm. All transistors are covered by a thick Plexiglas layer, which is a poor conductor of heat, and thus all the heat generated at the junction of the transistor must be dissipated to the ambient at 20°C through the back surface of the copper plate. The combined convection/radiation heat transfer coefficient at the back surface can be taken to be 25 W/m^2 · °C. If the case temperature of

FIGURE 3–18
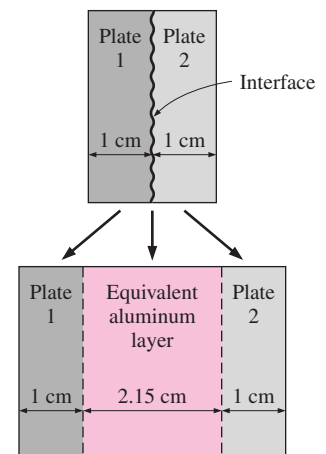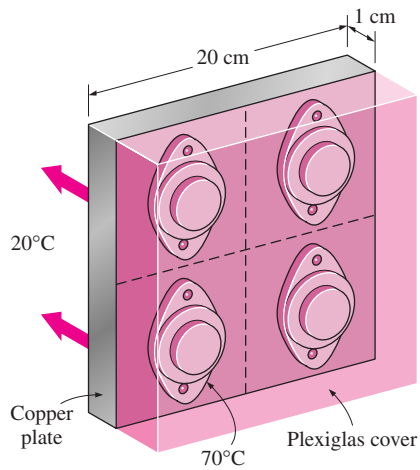Schematic for Example 3–5.

the transistor is not to exceed 70°C, determine the maximum power each transistor can dissipate safely, and the temperature jump at the case-plate interface.

SOLUTION   Four identical power transistors are attached on a copper plate. For a maximum case temperature of 70°C, the maximum power dissipation and the temperature jump at the interface are to be determined.

Assumptions   **1** Steady operating conditions exist. **2** Heat transfer can be approximated as being one-dimensional, although it is recognized that heat conduction in some parts of the plate will be two-dimensional since the plate area is much larger than the base area of the transistor. But the large thermal conductivity of copper will minimize this effect. **3** All the heat generated at the junction is dissipated through the back surface of the plate since the transistors are covered by a thick Plexiglas layer. **4** Thermal conductivities are constant.

Properties   The thermal conductivity of copper is given to be $k = 386$ W/m · °C. The contact conductance is obtained from Table 3–2 to be $h_c = 42,000$ W/m^2 · °C, which corresponds to copper-aluminum interface for the case of 1.3–1.4 μm roughness and 5 MPa pressure, which is sufficiently close to what we have.

Analysis   The contact area between the case and the plate is given to be 8 cm^2, and the plate area for each transistor is 100 cm^2. The thermal resistance network of this problem consists of three resistances in series (interface, plate, and convection), which are determined to be

$$R_{\text{interface}} = \frac{1}{h_c A_c} = \frac{1}{(42,000 \text{ W/m}^2 \cdot \text{°C})(8 \times 10^{-4} \text{ m}^2)} = 0.030\text{°C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.01 \text{ m}}{(386 \text{ W/m} \cdot \text{°C})(0.01 \text{ m}^2)} = 0.0026\text{°C/W}$$

$$R_{\text{conv}} = \frac{1}{h_o A} = \frac{1}{(25 \text{ W/m}^2 \cdot \text{°C})(0.01 \text{ m}^2)} = 4.0\text{°C/W}$$

The total thermal resistance is then

$$R_{\text{total}} = R_{\text{interface}} + R_{\text{plate}} + R_{\text{ambient}} = 0.030 + 0.0026 + 4.0 = 4.0326\text{°C/W}$$

Note that the thermal resistance of a copper plate is very small and can be ignored altogether. Then the rate of heat transfer is determined to be
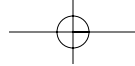
$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{(70 - 20)\text{°C}}{4.0326\text{°C/W}} = \textbf{12.4 W}$$

Therefore, the power transistor should not be operated at power levels greater than 12.4 W if the case temperature is not to exceed 70°C.
   The temperature jump at the interface is determined from

$$\Delta T_{\text{interface}} = \dot{Q} R_{\text{interface}} = (12.4 \text{ W})(0.030\text{°C/W}) = \textbf{0.37°C}$$

which is not very large. Therefore, even if we eliminate the thermal contact resistance at the interface completely, we will lower the operating temperature of the transistor in this case by less than 0.4°C.

3–3 · GENERALIZED THERMAL RESISTANCE NETWORKS

The *thermal resistance* concept or the *electrical analogy* can also be used to solve steady heat transfer problems that involve parallel layers or combined series-parallel arrangements. Although such problems are often two- or even three-dimensional, approximate solutions can be obtained by assuming one-dimensional heat transfer and using the thermal resistance network.

Consider the composite wall shown in Fig. 3–19, which consists of two parallel layers. The thermal resistance network, which consists of two parallel resistances, can be represented as shown in the figure. Noting that the total heat transfer is the sum of the heat transfers through each layer, we have

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{3-29}$$

Utilizing electrical analogy, we get

$$\dot{Q} = \frac{T_1 - T_2}{R_{total}} \tag{3-30}$$

where

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{total} = \frac{R_1 R_2}{R_1 + R_2} \tag{3-31}$$

since the resistances are in parallel.

Now consider the combined series-parallel arrangement shown in Fig. 3–20. The total rate of heat transfer through this composite system can again be expressed as

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \tag{3-32}$$

where

$$R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv} \tag{3-33}$$

and

$$R_1 = \frac{L_1}{k_1 A_1}, \qquad R_2 = \frac{L_2}{k_2 A_2}, \qquad R_3 = \frac{L_3}{k_3 A_3}, \qquad R_{conv} = \frac{1}{h A_3} \tag{3-34}$$

Once the individual thermal resistances are evaluated, the total resistance and the total rate of heat transfer can easily be determined from the relations above.

The result obtained will be somewhat approximate, since the surfaces of the third layer will probably not be isothermal, and heat transfer between the first two layers is likely to occur.

Two assumptions commonly used in solving complex multidimensional heat transfer problems by treating them as one-dimensional (say, in the
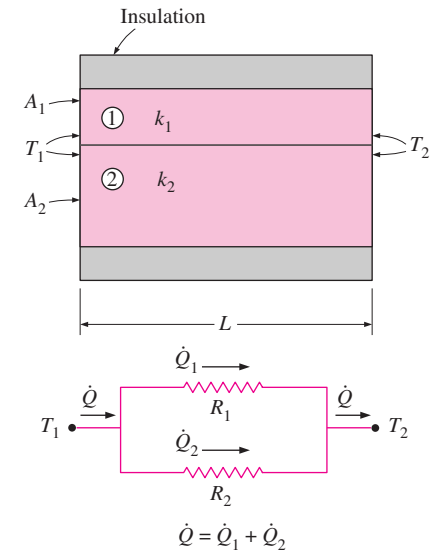

FIGURE 3–19
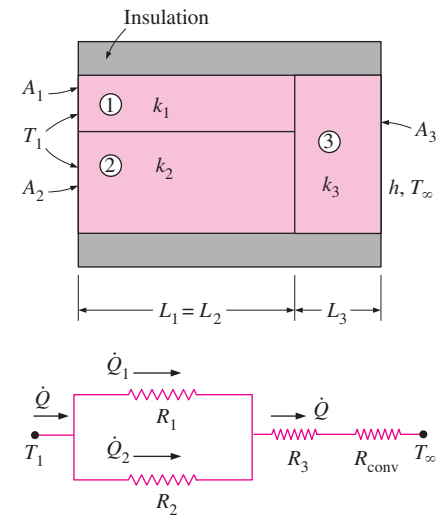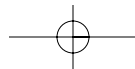Thermal resistance network for two parallel layers.

FIGURE 3–20
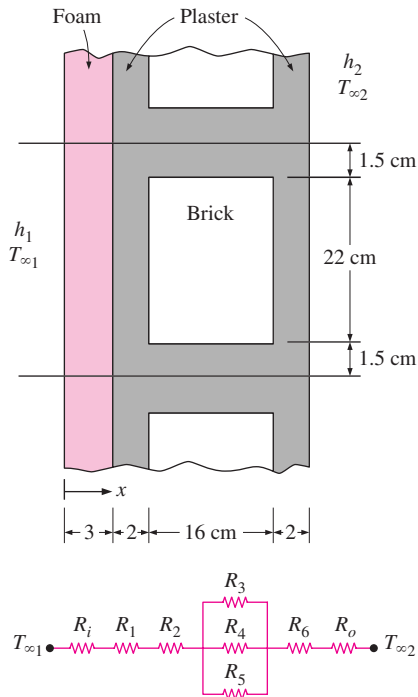Thermal resistance network for combined series-parallel arrangement.

x-direction) using the thermal resistance network are (1) any plane wall normal to the *x*-axis is *isothermal* (i.e., to assume the temperature to vary in the *x*-direction only) and (2) any plane parallel to the *x*-axis is *adiabatic* (i.e., to assume heat transfer to occur in the *x*-direction only). These two assumptions result in different resistance networks, and thus different (but usually close) values for the total thermal resistance and thus heat transfer. The actual result lies between these two values. In geometries in which heat transfer occurs predominantly in one direction, either approach gives satisfactory results.

EXAMPLE 3–6   Heat Loss through a Composite Wall

A 3-m-high and 5-m-wide wall consists of long 16-cm $\times$ 22-cm cross section horizontal bricks ($k = 0.72$ W/m $\cdot$ °C) separated by 3-cm-thick plaster layers ($k = 0.22$ W/m $\cdot$ °C). There are also 2-cm-thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ($k = 0.026$ W/m $\cdot$ °C) on the inner side of the wall, as shown in Fig. 3–21. The indoor and the outdoor temperatures are 20°C and -10°C, and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10$ W/m$^2 \cdot$ °C and $h_2 = 25$ W/m$^2 \cdot$ °C, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

SOLUTION The composition of a composite wall is given. The rate of heat transfer through the wall is to be determined.
Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer can be approximated as being one-dimensional since it is predominantly in the *x*-direction. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is negligible.
Properties The thermal conductivities are given to be $k = 0.72$ W/m $\cdot$ °C for bricks, $k = 0.22$ W/m $\cdot$ °C for plaster layers, and $k = 0.026$ W/m $\cdot$ °C for the rigid foam.
Analysis There is a pattern in the construction of this wall that repeats itself every 25-cm distance in the vertical direction. There is no variation in the horizontal direction. Therefore, we consider a 1-m-deep and 0.25-m-high portion of the wall, since it is representative of the entire wall.
  Assuming any cross section of the wall normal to the *x*-direction to be *isothermal,* the thermal resistance network for the representative section of the wall becomes as shown in Fig. 3–21. The individual resistances are evaluated as:

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot °\text{C})(0.25 \times 1 \text{ m}^2)} = 0.4°\text{C/W}$$

$$R_1 = R_{\text{foam}} = \frac{L}{kA} = \frac{0.03 \text{ m}}{(0.026 \text{ W/m} \cdot °\text{C})(0.25 \times 1 \text{ m}^2)} = 4.6°\text{C/W}$$

$$R_2 = R_6 = R_{\text{plaster, side}} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.22 \text{ W/m} \cdot °\text{C})(0.25 \times 1 \text{ m}^2)}$$
$$= 0.36°\text{C/W}$$

$$R_3 = R_5 = R_{\text{plaster, center}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.22 \text{ W/m} \cdot °\text{C})(0.015 \times 1 \text{ m}^2)}$$
$$= 48.48°\text{C/W}$$

FIGURE 3–21
Schematic for Example 3–6.

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.72 \text{ W/m} \cdot \text{°C})(0.22 \times 1 \text{ m}^2)} = 1.01 \text{°C/W}$$

$$R_O = R_{conv, 2} = \frac{1}{h_2 A} = \frac{1}{(25 \text{ W/m}^2 \cdot \text{°C})(0.25 \times 1 \text{ m}^2)} = 0.16 \text{°C/W}$$

The three resistances R_3, R_4, and R_5 in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{mid}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03 \text{ W/°C}$$

which gives

$$R_{mid} = 0.97 \text{°C/W}$$

Now all the resistances are in series, and the total resistance is

$$\begin{aligned} R_{total} &= R_i + R_1 + R_2 + R_{mid} + R_6 + R_o \\ &= 0.4 + 4.6 + 0.36 + 0.97 + 0.36 + 0.16 \\ &= 6.85 \text{°C/W} \end{aligned}$$

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20 - (-10)] \text{°C}}{6.85 \text{°C/W}} = 4.38 \text{ W} \qquad (\text{per } 0.25 \text{ m}^2 \text{ surface area})$$

or $4.38/0.25 = 17.5$ W per m^2 area. The total area of the wall is $A = 3$ m $\times$ 5 m $= 15$ m^2. Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{total} = (17.5 \text{ W/m}^2)(15 \text{ m}^2) = \textbf{263 W}$$

Of course, this result is approximate, since we assumed the temperature within the wall to vary in one direction only and ignored any temperature change (and thus heat transfer) in the other two directions.

Discussion   In the above solution, we assumed the temperature at any cross section of the wall normal to the *x*-direction to be *isothermal*. We could also solve this problem by going to the other extreme and assuming the surfaces parallel to the *x*-direction to be *adiabatic*. The thermal resistance network in this case will be as shown in Fig. 3–22. By following the approach outlined above, the total thermal resistance in this case is determined to be $R_{total} = 6.97 \text{°C/W}$, which is very close to the value 6.85°C/W obtained before. Thus either approach would give roughly the same result in this case. This example demonstrates that either approach can be used in practice to obtain satisfactory results.
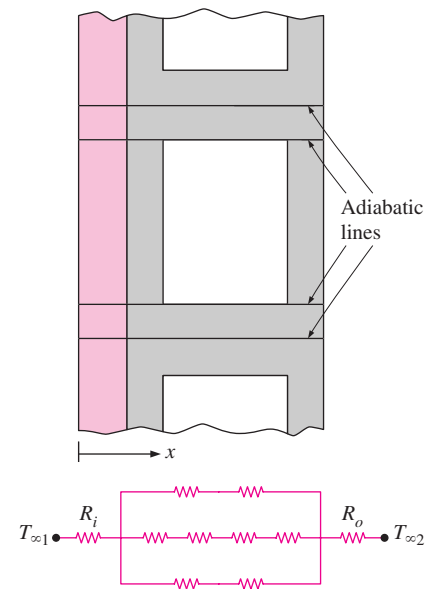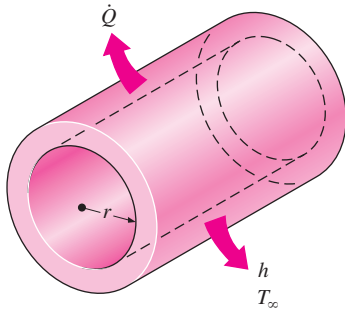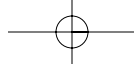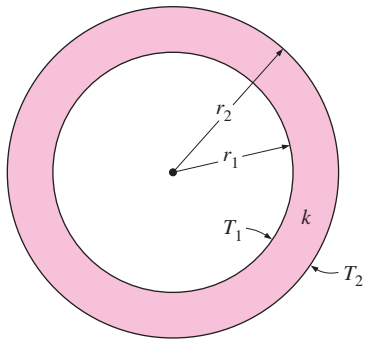

FIGURE 3–22
Alternative thermal resistance network for Example 3–6 for the case of surfaces parallel to the primary direction of heat transfer being adiabatic.

FIGURE 3–23
Heat is lost from a hot water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.

FIGURE 3–24
A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

3–4 ▪ HEAT CONDUCTION IN CYLINDERS AND SPHERES

Consider steady heat conduction through a hot water pipe. Heat is continuously lost to the outdoors through the wall of the pipe, and we intuitively feel that heat transfer through the pipe is in the normal direction to the pipe surface and no significant heat transfer takes place in the pipe in other directions (Fig. 3–23). The wall of the pipe, whose thickness is rather small, separates two fluids at different temperatures, and thus the temperature gradient in the radial direction will be relatively large. Further, if the fluid temperatures inside and outside the pipe remain constant, then heat transfer through the pipe is *steady*. Thus heat transfer through the pipe can be modeled as *steady* and *one-dimensional*. The temperature of the pipe in this case will depend on one direction only (the radial *r*-direction) and can be expressed as $T = T(r)$. The temperature is independent of the azimuthal angle or the axial distance. This situation is approximated in practice in long cylindrical pipes and spherical containers.

In *steady* operation, there is no change in the temperature of the pipe with time at any point. Therefore, the rate of heat transfer into the pipe must be equal to the rate of heat transfer out of it. In other words, heat transfer through the pipe must be constant, $\dot{Q}_{cond, cyl} = $ constant.

Consider a long cylindrical layer (such as a circular pipe) of inner radius r_1, outer radius r_2, length *L,* and average thermal conductivity *k* (Fig. 3–24). The two surfaces of the cylindrical layer are maintained at constant temperatures T_1 and T_2. There is no heat generation in the layer and the thermal conductivity is constant. For one-dimensional heat conduction through the cylindrical layer, we have $T(r)$. Then Fourier's law of heat conduction for heat transfer through the cylindrical layer can be expressed as

$$\dot{Q}_{cond, cyl} = -kA\frac{dT}{dr} \qquad \text{(W)} \tag{3-35}$$

where $A = 2\pi rL$ is the heat transfer area at location *r.* Note that *A* depends on *r*, and thus it *varies* in the direction of heat transfer. Separating the variables in the above equation and integrating from $r = r_1$, where $T(r_1) = T_1$, to $r = r_2$, where $T(r_2) = T_2$, gives
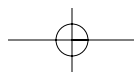
$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{cond, cyl}}{A} dr = -\int_{T=T_1}^{T_2} k\, dT \tag{3-36}$$

Substituting $A = 2\pi rL$ and performing the integrations give

$$\dot{Q}_{cond, cyl} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \text{(W)} \tag{3-37}$$

since $\dot{Q}_{cond, cyl} = $ constant. This equation can be rearranged as

$$\dot{Q}_{cond, cyl} = \frac{T_1 - T_2}{R_{cyl}} \qquad \text{(W)} \tag{3-38}$$

where

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times (\text{Length}) \times (\text{Thermal conductivity})} \tag{3-39}$$

is the *thermal resistance* of the cylindrical layer against heat conduction, or simply the **conduction resistance** of the cylinder layer.

We can repeat the analysis above for a *spherical layer* by taking $A = 4\pi r^2$ and performing the integrations in Eq. 3–36. The result can be expressed as

$$\dot{Q}_{\text{cond, sph}} = \frac{T_1 - T_2}{R_{\text{sph}}} \tag{3-40}$$
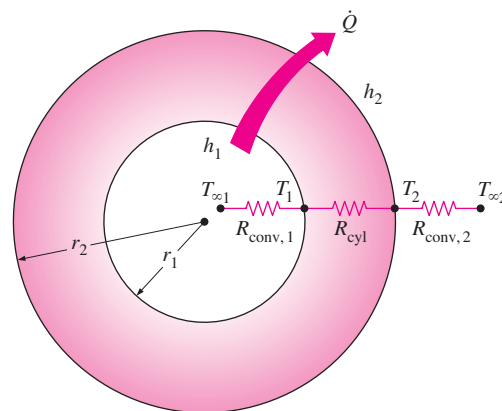
where

$$R_{\text{sph}} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})} \tag{3-41}$$

is the *thermal resistance* of the spherical layer against heat conduction, or simply the **conduction resistance** of the spherical layer.

Now consider steady one-dimensional heat flow through a cylindrical or spherical layer that is exposed to convection on both sides to fluids at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with heat transfer coefficients h_1 and h_2, respectively, as shown in Fig. 3–25. The thermal resistance network in this case consists of one conduction and two convection resistances in series, just like the one for the plane wall, and the rate of heat transfer under steady conditions can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \tag{3-42}$$

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{cyl}} + R_{\text{conv},2}$$

FIGURE 3–25
The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

where

$$R_{total} = R_{conv, 1} + R_{cyl} + R_{conv, 2}$$

$$= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2} \qquad \textbf{(3-43)}$$

for a *cylindrical* layer, and

$$R_{total} = R_{conv, 1} + R_{sph} + R_{conv, 2}$$

$$= \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2} \qquad \textbf{(3-44)}$$

for a spherical layer. Note that A in the convection resistance relation $R_{conv} = 1/hA$ is the *surface area at which convection occurs.* It is equal to $A = 2\pi rL$ for a cylindrical surface and $A = 4\pi r^2$ for a spherical surface of radius r. Also note that the thermal resistances are in series, and thus the total thermal resistance is determined by simply adding the individual resistances, just like the electrical resistances connected in series.

Multilayered Cylinders and Spheres

Steady heat transfer through multilayered cylindrical or spherical shells can be handled just like multilayered plane walls discussed earlier by simply adding an *additional resistance* in series for each *additional layer.* For example, the steady heat transfer rate through the three-layered composite cylinder of length L shown in Fig. 3–26 with convection on both sides can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} \qquad \textbf{(3-45)}$$
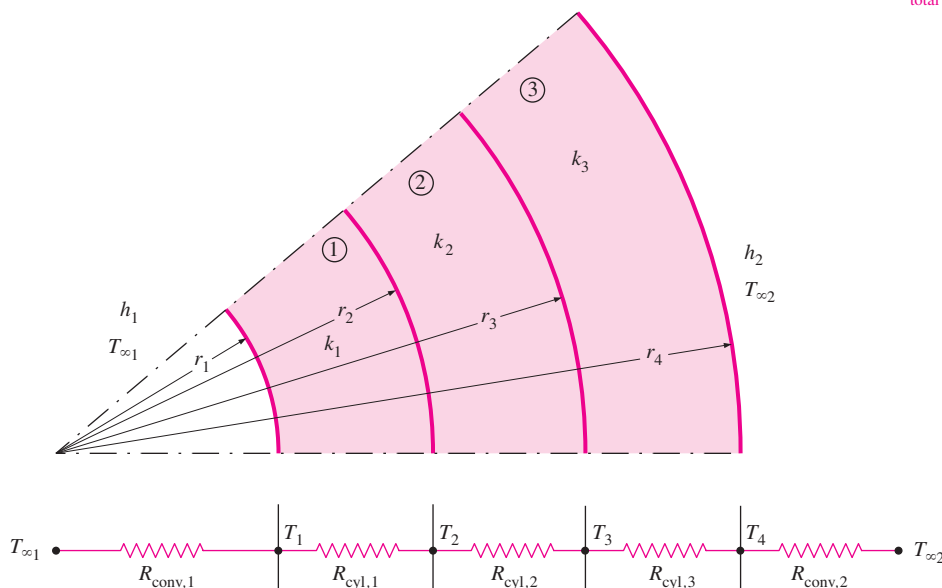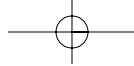

FIGURE 3–26
The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.

where R_{total} is the *total thermal resistance,* expressed as

$$R_{total} = R_{conv, 1} + R_{cyl, 1} + R_{cyl, 2} + R_{cyl, 3} + R_{conv, 2}$$

$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4} \qquad \text{(3-46)}$$

where $A_1 = 2\pi r_1 L$ and $A_4 = 2\pi r_4 L$. Equation 3–46 can also be used for a three-layered spherical shell by replacing the thermal resistances of cylindrical layers by the corresponding spherical ones. Again, note from the thermal resistance network that the resistances are in series, and thus the total thermal resistance is simply the *arithmetic sum* of the individual thermal resistances in the path of heat flow.

Once $\dot{Q}$ is known, we can determine any intermediate temperature T_j by applying the relation $\dot{Q} = (T_i - T_j)/R_{total, i-j}$ across any layer or layers such that T_i is a *known* temperature at location i and $R_{total, i-j}$ is the total thermal resistance between locations i and j (Fig. 3–27). For example, once $\dot{Q}$ has been calculated, the interface temperature T_2 between the first and second cylindrical layers can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{conv, 1} + R_{cyl, 1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}} \qquad \text{(3-47)}$$

We could also calculate T_2 from

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{conv, 2}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}} \qquad \text{(3-48)}$$
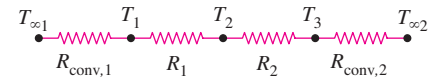
Although both relations will give the same result, we prefer the first one since it involves fewer terms and thus less work.

The thermal resistance concept can also be used for *other geometries,* provided that the proper conduction resistances and the proper surface areas in convection resistances are used.

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}}$$

$$= \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_1}$$

$$= \frac{T_1 - T_3}{R_1 + R_2}$$

$$= \frac{T_2 - T_3}{R_2}$$

$$= \frac{T_2 - T_{\infty 2}}{R_2 + R_{conv,2}}$$

$$= \cdots$$

FIGURE 3–27
The ratio $\Delta T/R$ across any layer is equal to $\dot{Q}$, which remains constant in one-dimensional steady conduction.

EXAMPLE 3–7   Heat Transfer to a Spherical Container

A 3-m internal diameter spherical tank made of 2-cm-thick stainless steel ($k = 15$ W/m · °C) is used to store iced water at $T_{\infty 1} = 0$°C. The tank is located in a room whose temperature is $T_{\infty 2} = 22$°C. The walls of the room are also at 22°C. The outer surface of the tank is black and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are $h_1 = 80$ W/m² · °C and $h_2 = 10$ W/m² · °C, respectively. Determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period.
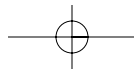
SOLUTION   A spherical container filled with iced water is subjected to convection and radiation heat transfer at its outer surface. The rate of heat transfer and the amount of ice that melts per day are to be determined.
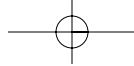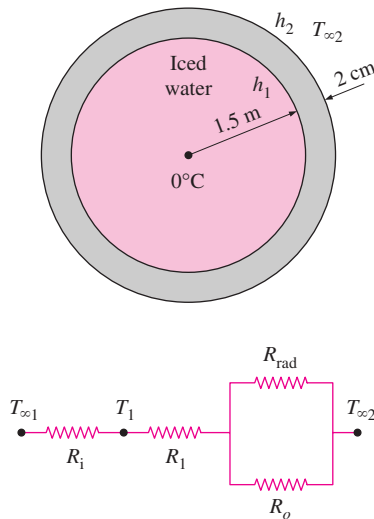
FIGURE 3–28
Schematic for Example 3–7.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant.

Properties The thermal conductivity of steel is given to be $k = 15$ W/m · °C. The heat of fusion of water at atmospheric pressure is $h_{if} = 333.7$ kJ/kg. The outer surface of the tank is black and thus its emissivity is $\varepsilon = 1$.

Analysis (*a*) The thermal resistance network for this problem is given in Fig. 3–28. Noting that the inner diameter of the tank is $D_1 = 3$ m and the outer diameter is $D_2 = 3.04$ m, the inner and the outer surface areas of the tank are

$$A_1 = \pi D_1^2 = \pi(3 \text{ m})^2 = 28.3 \text{ m}^2$$
$$A_2 = \pi D_2^2 = \pi(3.04 \text{ m})^2 = 29.0 \text{ m}^2$$

Also, the radiation heat transfer coefficient is given by

$$h_{\text{rad}} = \varepsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$$

But we do not know the outer surface temperature T_2 of the tank, and thus we cannot calculate h_{rad}. Therefore, we need to assume a T_2 value now and check the accuracy of this assumption later. We will repeat the calculations if necessary using a revised value for T_2.

We note that T_2 must be between 0°C and 22°C, but it must be closer to 0°C, since the heat transfer coefficient inside the tank is much larger. Taking $T_2 = 5°C = 278$ K, the radiation heat transfer coefficient is determined to be

$$h_{\text{rad}} = (1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(295 \text{ K})^2 + (278 \text{ K})^2][(295 + 278) \text{ K}]$$
$$= 5.34 \text{ W/m}^2 \cdot \text{K} = 5.34 \text{ W/m}^2 \cdot °\text{C}$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A_1} = \frac{1}{(80 \text{ W/m}^2 \cdot °\text{C})(28.3 \text{ m}^2)} = 0.000442°\text{C/W}$$

$$R_1 = R_{\text{sphere}} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52 - 1.50) \text{ m}}{4\pi (15 \text{ W/m} \cdot °\text{C})(1.52 \text{ m})(1.50 \text{ m})}$$
$$= 0.000047°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_2} = \frac{1}{(10 \text{ W/m}^2 \cdot °\text{C})(29.0 \text{ m}^2)} = 0.00345°\text{C/W}$$
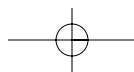
$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_2} = \frac{1}{(5.34 \text{ W/m}^2 \cdot °\text{C})(29.0 \text{ m}^2)} = 0.00646°\text{C/W}$$

The two parallel resistances R_o and R_{rad} can be replaced by an equivalent resistance R_{equiv} determined from

$$\frac{1}{R_{\text{equiv}}} = \frac{1}{R_o} + \frac{1}{R_{\text{rad}}} = \frac{1}{0.00345} + \frac{1}{0.00646} = 444.7 \text{ W/°C}$$

which gives

$$R_{\text{equiv}} = 0.00225°\text{C/W}$$

Now all the resistances are in series, and the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_{\text{equiv}} = 0.000442 + 0.000047 + 0.00225 = 0.00274°\text{C/W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{\text{total}}} = \frac{(22 - 0)°\text{C}}{0.00274°\text{C/W}} = \textbf{8029 W} \qquad (\text{or } \dot{Q} = 8.027 \text{ kJ/s})$$

To check the validity of our original assumption, we now determine the outer surface temperature from

$$\dot{Q} = \frac{T_{\infty 2} - T_2}{R_{\text{equiv}}} \longrightarrow \quad T_2 = T_{\infty 2} - \dot{Q}R_{\text{equiv}}$$
$$= 22°\text{C} - (8029 \text{ W})(0.00225°\text{C/W}) = 4°\text{C}$$

which is sufficiently close to the 5°C assumed in the determination of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations using 4°C for T_2.
(b) The total amount of heat transfer during a 24-h period is

$$Q = \dot{Q}\,\Delta t = (8.029 \text{ kJ/s})(24 \times 3600 \text{ s}) = 673,700 \text{ kJ}$$

Noting that it takes 333.7 kJ of energy to melt 1 kg of ice at 0°C, the amount of ice that will melt during a 24-h period is

$$m_{\text{ice}} = \frac{Q}{h_{if}} = \frac{673,700 \text{ kJ}}{333.7 \text{ kJ/kg}} = \textbf{2079 kg}$$

Therefore, about 2 metric tons of ice will melt in the tank every day.

Discussion An easier way to deal with combined convection and radiation at a surface when the surrounding medium and surfaces are at the same temperature is to add the radiation and convection heat transfer coefficients and to treat the result as the convection heat transfer coefficient. That is, to take $h = 10 + 5.34 = 15.34$ W/m² · °C in this case. This way, we can ignore radiation since its contribution is accounted for in the convection heat transfer coefficient. The convection resistance of the outer surface in this case would be

$$R_{\text{combined}} = \frac{1}{h_{\text{combined}} A_2} = \frac{1}{(15.34 \text{ W/m}^2 \cdot °\text{C})(29.0 \text{ m}^2)} = 0.00225°\text{C/W}$$

which is identical to the value obtained for the equivalent resistance for the parallel convection and the radiation resistances.

EXAMPLE 3–8   Heat Loss through an Insulated Steam Pipe

Steam at $T_{\infty 1} = 320°\text{C}$ flows in a cast iron pipe ($k = 80$ W/m · °C) whose inner and outer diameters are $D_1 = 5$ cm and $D_2 = 5.5$ cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation with $k = 0.05$ W/m · °C. Heat is lost to the surroundings at $T_{\infty 2} = 5°\text{C}$ by natural convection and radiation, with
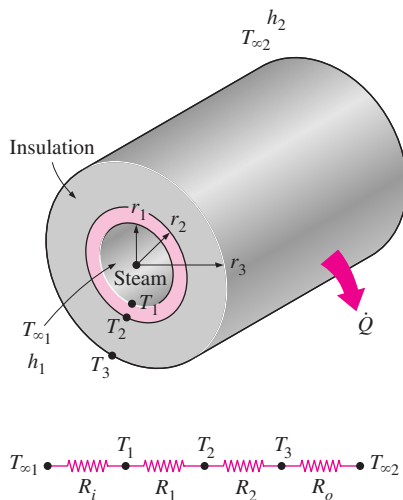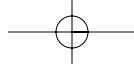
FIGURE 3–29
Schematic for Example 3–8.

a combined heat transfer coefficient of $h_2 = 18$ W/m^2 · °C. Taking the heat transfer coefficient inside the pipe to be $h_1 = 60$ W/m^2 · °C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

SOLUTION   A steam pipe covered with glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions   **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties   The thermal conductivities are given to be $k = 80$ W/m · °C for cast iron and $k = 0.05$ W/m · °C for glass wool insulation.

Analysis   The thermal resistance network for this problem involves four resistances in series and is given in Fig. 3–29. Taking $L = 1$ m, the areas of the surfaces exposed to convection are determined to be

$$A_1 = 2\pi r_1 L = 2\pi(0.025 \text{ m})(1 \text{ m}) = 0.157 \text{ m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575 \text{ m})(1 \text{ m}) = 0.361 \text{ m}^2$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(60 \text{ W/m}^2 \cdot °\text{C})(0.157 \text{ m}^2)} = 0.106°\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80 \text{ W/m} \cdot °\text{C})(1 \text{ m})} = 0.0002°\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05 \text{ W/m} \cdot °\text{C})(1 \text{ m})} = 2.35°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_3} = \frac{1}{(18 \text{ W/m}^2 \cdot °\text{C})(0.361 \text{ m}^2)} = 0.154°\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61°\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(320 - 5)°\text{C}}{2.61°\text{C/W}} = \textbf{121 W} \qquad \text{(per m pipe length)}$$

The heat loss for a given pipe length can be determined by multiplying the above quantity by the pipe length L.

The temperature drops across the pipe and the insulation are determined from Eq. 3–17 to be

$$\Delta T_{\text{pipe}} = \dot{Q} R_{\text{pipe}} = (121 \text{ W})(0.0002°\text{C/W}) = \textbf{0.02°C}$$
$$\Delta T_{\text{insulation}} = \dot{Q} R_{\text{insulation}} = (121 \text{ W})(2.35°\text{C/W}) = \textbf{284°C}$$

That is, the temperatures between the inner and the outer surfaces of the pipe differ by 0.02°C, whereas the temperatures between the inner and the outer surfaces of the insulation differ by 284°C.
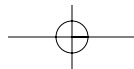
Discussion Note that the thermal resistance of the pipe is too small relative to the other resistances and can be neglected without causing any significant error. Also note that the temperature drop across the pipe is practically zero, and thus the pipe can be assumed to be isothermal. The resistance to heat flow in insulated pipes is primarily due to insulation.

3–5 ▪ CRITICAL RADIUS OF INSULATION

We know that adding more insulation to a wall or to the attic always decreases heat transfer. The thicker the insulation, the lower the heat transfer rate. This is expected, since the heat transfer area A is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

Adding insulation to a cylindrical pipe or a spherical shell, however, is a different matter. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection. The heat transfer from the pipe may increase or decrease, depending on which effect dominates.

Consider a cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is maintained constant (Fig. 3–30). The pipe is now insulated with a material whose thermal conductivity is k and outer radius is r_2. Heat is lost from the pipe to the surrounding medium at temperature T_∞, with a convection heat transfer coefficient h. The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as (Fig. 3–31)

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}}$$ **(3-49)**

The variation of $\dot{Q}$ with the outer radius of the insulation r_2 is plotted in Fig. 3–31. The value of r_2 at which $\dot{Q}$ reaches a maximum is determined from the requirement that $d\dot{Q}/dr_2 = 0$ (zero slope). Performing the differentiation and solving for r_2 yields the **critical radius of insulation** for a cylindrical body to be

$$r_{\text{cr, cylinder}} = \frac{k}{h} \qquad \text{(m)}$$ **(3-50)**

Note that the critical radius of insulation depends on the thermal conductivity of the insulation k and the external convection heat transfer coefficient h. The rate of heat transfer from the cylinder increases with the addition of insulation for $r_2 < r_{\text{cr}}$, reaches a maximum when $r_2 = r_{\text{cr}}$, and starts to decrease for $r_2 > r_{\text{cr}}$. Thus, insulating the pipe may actually increase the rate of heat transfer from the pipe instead of decreasing it when $r_2 < r_{\text{cr}}$.

The important question to answer at this point is whether we need to be concerned about the critical radius of insulation when insulating hot water pipes or even hot water tanks. Should we always check and make sure that the outer
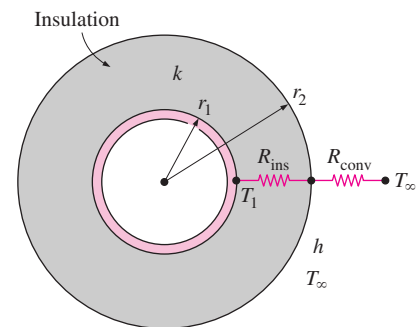

FIGURE 3–30
An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.
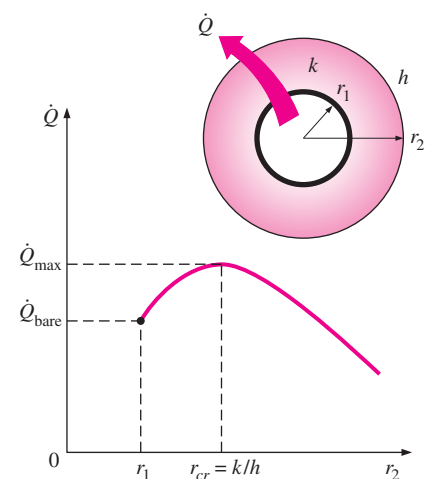

FIGURE 3–31

radius of insulation exceeds the critical radius before we install any insulation? Probably not, as explained here.

The value of the critical radius r_{cr} will be the largest when k is large and h is small. Noting that the lowest value of h encountered in practice is about 5 W/m$^2 \cdot$ °C for the case of natural convection of gases, and that the thermal conductivity of common insulating materials is about 0.05 W/m$^2 \cdot$ °C, the largest value of the critical radius we are likely to encounter is

$$r_{cr, max} = \frac{k_{max, insulation}}{h_{min}} \approx \frac{0.05 \text{ W/m} \cdot \text{°C}}{5 \text{ W/m}^2 \cdot \text{°C}} = 0.01 \text{ m} = 1 \text{ cm}$$

This value would be even smaller when the radiation effects are considered. The critical radius would be much less in forced convection, often less than 1 mm, because of much larger h values associated with forced convection. Therefore, we can insulate hot water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.

The radius of electric wires may be smaller than the critical radius. Therefore, the plastic electrical insulation may actually *enhance* the heat transfer from electric wires and thus keep their steady operating temperatures at lower and thus safer levels.

The discussions above can be repeated for a sphere, and it can be shown in a similar manner that the critical radius of insulation for a spherical shell is

$$r_{cr, sphere} = \frac{2k}{h} \tag{3-51}$$

where k is the thermal conductivity of the insulation and h is the convection heat transfer coefficient on the outer surface.

EXAMPLE 3–9 **Heat Loss from an Insulated Electric Wire**

A 3-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is $k = 0.15$ W/m $\cdot$ °C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30$°C with a heat transfer coefficient of $h = 12$ W/m$^2 \cdot$ °C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

SOLUTION An electric wire is tightly wrapped with a plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient incorporates the radiation effects, if any.

Properties The thermal conductivity of plastic is given to be $k = 0.15$ W/m $\cdot$ °C.

Analysis Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = \dot{W}_e = VI = (8 \text{ V})(10 \text{ A}) = 80 \text{ W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. 3–32. The values of these two resistances are determined to be

$$A_2 = (2\pi r_2)L = 2\pi(0.0035 \text{ m})(5 \text{ m}) = 0.110 \text{ m}^2$$

$$R_{conv} = \frac{1}{hA_2} = \frac{1}{(12 \text{ W/m}^2 \cdot °\text{C})(0.110 \text{ m}^2)} = 0.76°\text{C/W}$$

$$R_{plastic} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15 \text{ W/m} \cdot °\text{C})(5 \text{ m})} = 0.18°\text{C/W}$$

and therefore

$$R_{total} = R_{plastic} + R_{conv} = 0.76 + 0.18 = 0.94°\text{C/W}$$

Then the interface temperature can be determined from

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \quad \longrightarrow \quad \begin{aligned} T_1 &= T_\infty + \dot{Q}R_{total} \\ &= 30°\text{C} + (80 \text{ W})(0.94°\text{C/W}) = \textbf{105°C} \end{aligned}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 3–50 to be

$$r_{cr} = \frac{k}{h} = \frac{0.15 \text{ W/m} \cdot °\text{C}}{12 \text{ W/m}^2 \cdot °\text{C}} = 0.0125 \text{ m} = 12.5 \text{ mm}$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will *enhance* heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer $\dot{Q}$ will *increase* when the interface temperature T_1 is held constant, or T_1 will *decrease* when $\dot{Q}$ is held constant, which is the case here.

Discussion It can be shown by repeating the calculations above for a 4-mm-thick plastic cover that the interface temperature drops to 90.6°C when the thickness of the plastic cover is doubled. It can also be shown in a similar manner that the interface reaches a minimum temperature of 83°C when the outer radius of the plastic cover equals the critical radius.
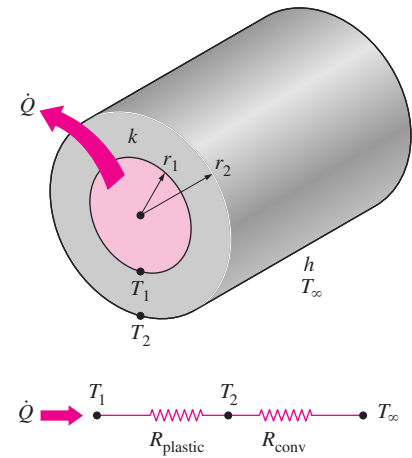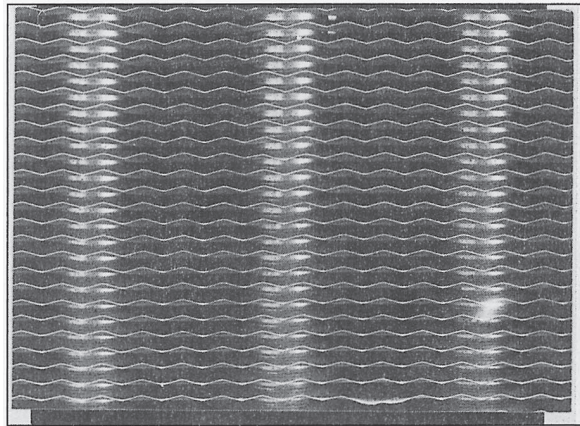

FIGURE 3–32
Schematic for Example 3–9.

FIGURE 3–33
The thin plate fins of a car radiator greatly increase the rate of heat transfer to the air (photo by Yunus Çengel and James Kleiser).

3–6 ▪ HEAT TRANSFER FROM FINNED SURFACES

The rate of heat transfer from a surface at a temperature T_s to the surrounding medium at T_∞ is given by Newton's law of cooling as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

where A_s is the heat transfer surface area and h is the convection heat transfer coefficient. When the temperatures T_s and T_∞ are fixed by design considerations, as is often the case, there are *two ways* to increase the rate of heat transfer: to increase the *convection heat transfer coefficient h* or to increase the *surface area A_s.* Increasing h may require the installation of a pump or fan, or replacing the existing one with a larger one, but this approach may or may not be practical. Besides, it may not be adequate. The alternative is to increase the surface area by attaching to the surface *extended surfaces* called *fins* made of highly conductive materials such as aluminum. Finned surfaces are manufactured by extruding, welding, or wrapping a thin metal sheet on a surface. Fins enhance heat transfer from a surface by exposing a larger surface area to convection and radiation.

Finned surfaces are commonly used in practice to enhance heat transfer, and they often increase the rate of heat transfer from a surface severalfold. The car radiator shown in Fig. 3–33 is an example of a finned surface. The closely packed thin metal sheets attached to the hot water tubes increase the surface area for convection and thus the rate of convection heat transfer from the tubes to the air many times. There are a variety of innovative fin designs available in the market, and they seem to be limited only by imagination (Fig. 3–34).

In the analysis of fins, we consider *steady* operation with *no heat generation* in the fin, and we assume the thermal conductivity k of the material to remain constant. We also assume the convection heat transfer coefficient h to be *constant* and *uniform* over the entire surface of the fin for convenience in the analysis. We recognize that the convection heat transfer coefficient h, in general, varies along the fin as well as its circumference, and its value at a point is a strong function of the *fluid motion* at that point. The value of h is usually much lower at the *fin base* than it is at the *fin tip* because the fluid is surrounded by solid surfaces near the base, which seriously disrupt its motion to
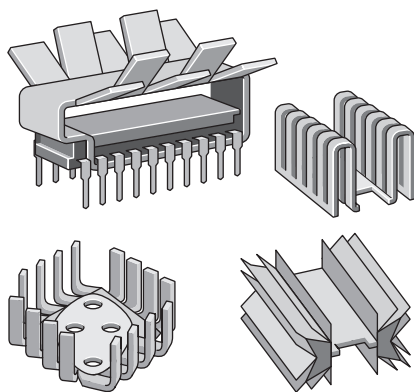
FIGURE 3–34
Some innovative fin designs.

the point of "suffocating" it, while the fluid near the fin tip has little contact with a solid surface and thus encounters little resistance to flow. Therefore, adding too many fins on a surface may actually decrease the overall heat transfer when the decrease in h offsets any gain resulting from the increase in the surface area.

Fin Equation

Consider a volume element of a fin at location x having a length of Δx, cross-sectional area of A_c, and a perimeter of p, as shown in Fig. 3–35. Under steady conditions, the energy balance on this volume element can be expressed as

$$\begin{pmatrix} \text{Rate of } heat \\ conduction \text{ into} \\ \text{the element at } x \end{pmatrix} = \begin{pmatrix} \text{Rate of } heat \\ conduction \text{ from the} \\ \text{element at } x + \Delta x \end{pmatrix} + \begin{pmatrix} \text{Rate of } heat \\ convection \text{ from} \\ \text{the element} \end{pmatrix}$$

or

$$\dot{Q}_{\text{cond}, x} = \dot{Q}_{\text{cond}, x + \Delta x} + \dot{Q}_{\text{conv}}$$

where

$$\dot{Q}_{\text{conv}} = h(p\,\Delta x)(T - T_\infty)$$

Substituting and dividing by Δx, we obtain

$$\frac{\dot{Q}_{\text{cond}, x + \Delta x} - \dot{Q}_{\text{cond}, x}}{\Delta x} + hp(T - T_\infty) = 0 \tag{3-52}$$

Taking the limit as $\Delta x \to 0$ gives

$$\frac{d\dot{Q}_{\text{cond}}}{dx} + hp(T - T_\infty) = 0 \tag{3-53}$$

From Fourier's law of heat conduction we have

$$\dot{Q}_{\text{cond}} = -kA_c \frac{dT}{dx} \tag{3-54}$$

where A_c is the cross-sectional area of the fin at location x. Substitution of this relation into Eq. 3–53 gives the differential equation governing heat transfer in fins,

$$\frac{d}{dx}\left(kA_c \frac{dT}{dx}\right) - hp(T - T_\infty) = 0 \tag{3-55}$$

In general, the cross-sectional area A_c and the perimeter p of a fin vary with x, which makes this differential equation difficult to solve. In the special case of *constant cross section* and *constant thermal conductivity,* the differential equation 3–55 reduces to
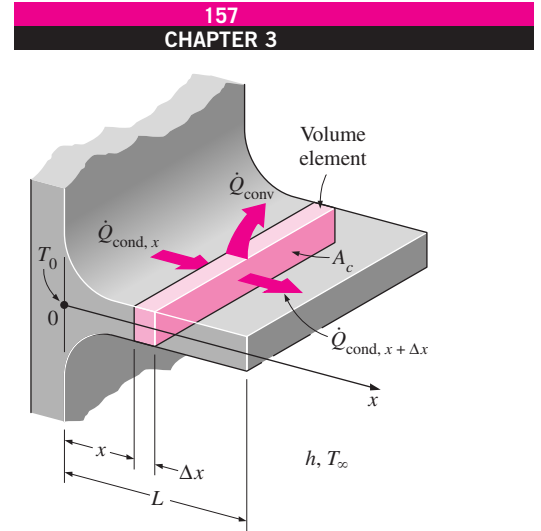
$$\frac{d^2\theta}{dx^2} - a^2\theta = 0 \tag{3-56}$$

FIGURE 3–35
Volume element of a fin at location x having a length of Δx, cross-sectional area of A_c, and perimeter of p.

where

$$a^2 = \frac{hp}{kA_c} \tag{3-57}$$

and $\theta = T - T_\infty$ is the *temperature excess.* At the fin base we have $\theta_b = T_b - T_\infty$.

Equation 3–56 is a linear, homogeneous, second-order differential equation with constant coefficients. A fundamental theory of differential equations states that such an equation has two linearly independent solution functions, and its general solution is the linear combination of those two solution functions. A careful examination of the differential equation reveals that subtracting a constant multiple of the solution function θ from its second derivative yields zero. Thus we conclude that the function θ and its second derivative must be *constant multiples* of each other. The only functions whose derivatives are constant multiples of the functions themselves are the *exponential functions* (or a linear combination of exponential functions such as sine and cosine hyperbolic functions). Therefore, the solution functions of the differential equation above are the exponential functions e^{-ax} or e^{ax} or constant multiples of them. This can be verified by direct substitution. For example, the second derivative of e^{-ax} is $a^2 e^{-ax}$, and its substitution into Eq. 3–56 yields zero. Therefore, the general solution of the differential equation Eq. 3–56 is

$$\theta(x) = C_1 e^{ax} + C_2 e^{-ax} \tag{3-58}$$

where C_1 and C_2 are arbitrary constants whose values are to be determined from the boundary conditions at the base and at the tip of the fin. Note that we need only two conditions to determine C_1 and C_2 uniquely.

The temperature of the plate to which the fins are attached is normally known in advance. Therefore, at the fin base we have a *specified temperature* boundary condition, expressed as

Boundary condition at fin base: $\qquad \theta(0) = \theta_b = T_b - T_\infty \qquad$ **(3-59)**

At the fin tip we have several possibilities, including specified temperature, negligible heat loss (idealized as an insulated tip), convection, and combined convection and radiation (Fig. 3–36). Next, we consider each case separately.

1   Infinitely Long Fin ($T_{\text{fin tip}} = T_\infty$)

For a sufficiently long fin of *uniform* cross section (A_c = constant), the temperature of the fin at the fin tip will approach the environment temperature T_∞ and thus θ will approach zero. That is,

Boundary condition at fin tip: $\quad \theta(L) = T(L) - T_\infty = 0 \qquad$ as $\qquad L \rightarrow \infty$

This condition will be satisfied by the function e^{-ax}, but not by the other prospective solution function e^{ax} since it tends to infinity as x gets larger. Therefore, the general solution in this case will consist of a constant multiple of e^{-ax}. The value of the constant multiple is determined from the requirement that at the fin base where $x = 0$ the value of θ will be θ_b. Noting that

T_b T_∞

L

x

0

Specified
temperature

(*a*) Specified temperature
(*b*) Negligible heat loss
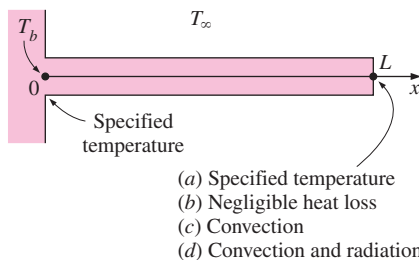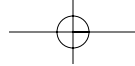(*c*) Convection
(*d*) Convection and radiation

FIGURE 3–36
Boundary conditions at the
fin base and the fin tip.

$e^{-ax} = e^0 = 1$, the proper value of the constant is θ_b, and the solution function we are looking for is $\theta(x) = \theta_b e^{-ax}$. This function satisfies the differential equation as well as the requirements that the solution reduce to θ_b at the fin base and approach zero at the fin tip for large x. Noting that $\theta = T - T_\infty$ and $a = \sqrt{hp/kA_c}$, the variation of temperature along the fin in this case can be expressed as

Very long fin:
$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-ax} = e^{-x\sqrt{hp/kA_c}} \qquad \text{(3-60)}$$

Note that the temperature along the fin in this case decreases *exponentially* from T_b to T_∞, as shown in Fig. 3–37. The steady rate of *heat transfer* from the entire fin can be determined from Fourier's law of heat conduction

Very long fin:
$$\dot{Q}_{\text{long fin}} = -kA_c \frac{dT}{dx}\bigg|_{x=0} = \sqrt{hpkA_c} \, (T_b - T_\infty) \qquad \text{(3-61)}$$

where p is the perimeter, A_c is the cross-sectional area of the fin, and x is the distance from the fin base. Alternatively, the rate of heat transfer from the fin could also be determined by considering heat transfer from a differential volume element of the fin and integrating it over the entire surface of the fin. That is,

$$\dot{Q}_{\text{fin}} = \int_{A_{\text{fin}}} h[T(x) - T_\infty] \, dA_{\text{fin}} = \int_{A_{\text{fin}}} h\theta(x) \, dA_{\text{fin}} \qquad \text{(3-62)}$$

The two approaches described are equivalent and give the same result since, under steady conditions, the heat transfer from the exposed surfaces of the fin is equal to the heat transfer to the fin at the base (Fig. 3–38).

2 Negligible Heat Loss from the Fin Tip (Insulated fin tip, $\dot{Q}_{\text{fin tip}} = 0$)

Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic situation is for heat transfer from the fin tip to be negligible since the heat transfer from the fin is proportional to its surface area, and the surface area of the fin tip is usually a negligible fraction of the total fin area. Then the fin tip can be assumed to be insulated, and the condition at the fin tip can be expressed as

Boundary condition at fin tip:
$$\frac{d\theta}{dx}\bigg|_{x=L} = 0 \qquad \text{(3-63)}$$

The condition at the fin base remains the same as expressed in Eq. 3–59. The application of these two conditions on the general solution (Eq. 3–58) yields, after some manipulations, this relation for the temperature distribution:

Adiabatic fin tip:
$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh a(L - x)}{\cosh aL} \qquad \text{(3-64)}$$
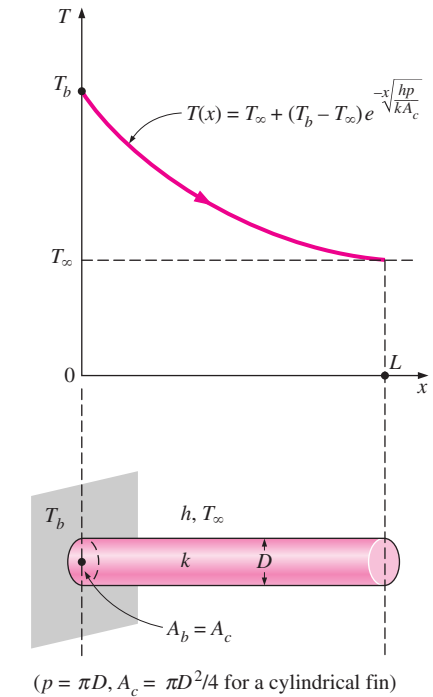

FIGURE 3–37
A long circular fin of uniform cross section and the variation of temperature along it.
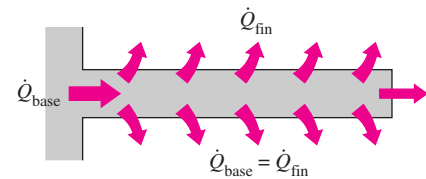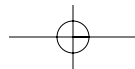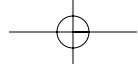

FIGURE 3–38
Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.

The rate of heat transfer from the fin can be determined again from Fourier's law of heat conduction:

Adiabatic fin tip:

$$\dot{Q}_{\text{insulated tip}} = -kA_c \frac{dT}{dx}\bigg|_{x=0}$$

$$= \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL \qquad \textbf{(3-65)}$$

Note that the heat transfer relations for the very long fin and the fin with negligible heat loss at the tip differ by the factor tanh aL, which approaches 1 as L becomes very large.

3 Convection (or Combined Convection and Radiation) from Fin Tip

The fin tips, in practice, are exposed to the surroundings, and thus the proper boundary condition for the fin tip is convection that also includes the effects of radiation. The fin equation can still be solved in this case using the convection at the fin tip as the second boundary condition, but the analysis becomes more involved, and it results in rather lengthy expressions for the temperature distribution and the heat transfer. Yet, in general, the fin tip area is a small fraction of the total fin surface area, and thus the complexities involved can hardly justify the improvement in accuracy.

A practical way of accounting for the heat loss from the fin tip is to replace the *fin length L* in the relation for the *insulated tip* case by a **corrected length** defined as (Fig. 3–39)

Corrected fin length:

$$L_c = L + \frac{A_c}{p} \qquad \textbf{(3-66)}$$

where A_c is the cross-sectional area and p is the perimeter of the fin at the tip. Multiplying the relation above by the perimeter gives $A_{\text{corrected}} = A_{\text{fin (lateral)}} + A_{\text{tip}}$, which indicates that the fin area determined using the corrected length is equivalent to the sum of the lateral fin area plus the fin tip area.

The corrected length approximation gives very good results when the variation of temperature near the fin tip is small (which is the case when $aL \geq 1$) and the heat transfer coefficient at the fin tip is about the same as that at the lateral surface of the fin. Therefore, *fins subjected to convection at their tips can be treated as fins with insulated tips by replacing the actual fin length by the corrected length in Eqs. 3–64 and 3–65*.

Using the proper relations for A_c and p, the corrected lengths for rectangular and cylindrical fins are easily determined to be

$$L_{c,\text{ rectangular fin}} = L + \frac{t}{2} \qquad \text{and} \qquad L_{c,\text{ cylindrical fin}} = L + \frac{D}{4}$$

where t is the thickness of the rectangular fins and D is the diameter of the cylindrical fins.

Fin Efficiency

Consider the surface of a *plane wall* at temperature T_b exposed to a medium at temperature T_∞. Heat is lost from the surface to the surrounding medium by

$\dot{Q}_{\text{fin}}$

Convection

L

(*a*) Actual fin with convection at the tip

$\dot{Q}_{\text{fin}}$

$\dfrac{A_c}{p}$
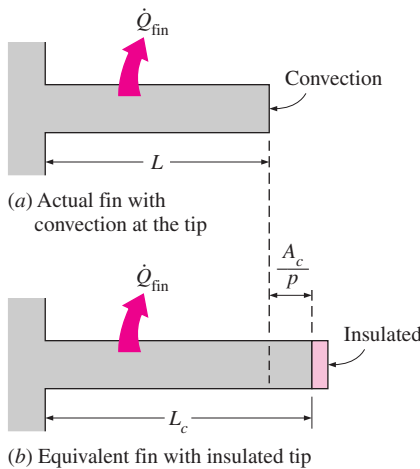
Insulated

L_c

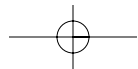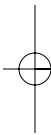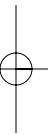(*b*) Equivalent fin with insulated tip

FIGURE 3–39

Corrected fin length L_c is defined such that heat transfer from a fin of length L_c with insulated tip is equal to heat transfer from the actual fin of length L with convection at the fin tip.

convection with a heat transfer coefficient of h. Disregarding radiation or accounting for its contribution in the convection coefficient h, heat transfer from a surface area A_s is expressed as $\dot{Q} = hA_s(T_s - T_\infty)$.

Now let us consider a fin of constant cross-sectional area $A_c = A_b$ and length L that is attached to the surface with a perfect contact (Fig. 3–40). This time heat will flow from the surface to the fin *by conduction* and from the fin to the surrounding medium *by convection* with the same heat transfer coefficient h. The temperature of the fin will be T_b at the fin base and gradually decrease toward the fin tip. Convection from the fin surface causes the temperature at any cross section to drop somewhat from the midsection toward the outer surfaces. However, the cross-sectional area of the fins is usually very small, and thus the temperature at any cross section can be considered to be uniform. Also, the fin tip can be assumed for convenience and simplicity to be insulated by using the corrected length for the fin instead of the actual length.

In the limiting case of *zero thermal resistance* or *infinite thermal conductivity* ($k \rightarrow \infty$), the temperature of the fin will be uniform at the base value of T_b. The heat transfer from the fin will be *maximum* in this case and can be expressed as

$$\dot{Q}_{\text{fin, max}} = hA_{\text{fin}}(T_b - T_\infty) \tag{3-67}$$

In reality, however, the temperature of the fin will drop along the fin, and thus the heat transfer from the fin will be less because of the decreasing temperature difference $T(x) - T_\infty$ toward the fin tip, as shown in Fig. 3–41. To account for the effect of this decrease in temperature on heat transfer, we define a **fin efficiency** as

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\begin{array}{c}\text{Ideal heat transfer rate from the fin}\\ \text{if the entire fin were at base temperature}\end{array}} \tag{3-68}$$

or

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} \dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) \tag{3-69}$$

where A_{fin} is the total surface area of the fin. This relation enables us to determine the heat transfer from a fin when its efficiency is known. For the cases of constant cross section of *very long fins* and *fins with insulated tips,* the fin efficiency can be expressed as

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{aL} \tag{3-70}$$

and

$$\eta_{\text{insulated tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{\tanh aL}{aL} \tag{3-71}$$

since $A_{\text{fin}} = pL$ for fins with constant cross section. Equation 3–71 can also be used for fins subjected to convection provided that the fin length L is replaced by the corrected length L_c.
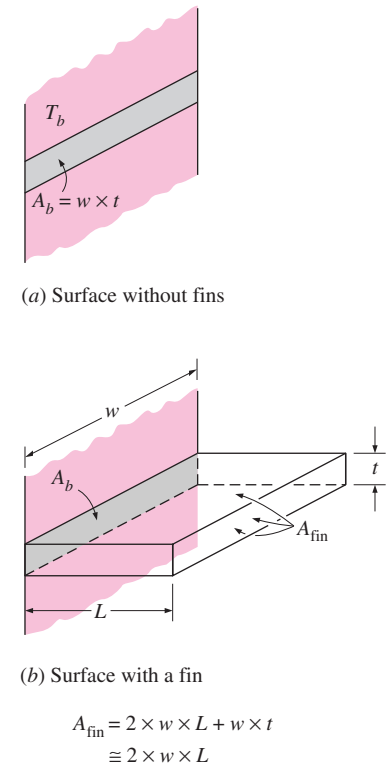

(a) Surface without fins

(b) Surface with a fin

$$A_{\text{fin}} = 2 \times w \times L + w \times t$$
$$\cong 2 \times w \times L$$

FIGURE 3–40
Fins enhance heat transfer from a surface by enhancing surface area.
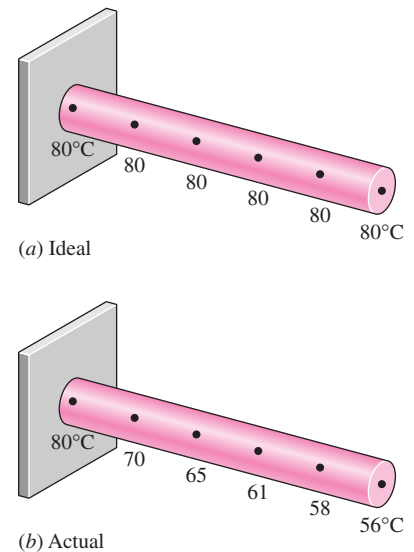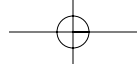

(a) Ideal

(b) Actual

FIGURE 3–41
Ideal and actual temperature distribution in a fin.

Fin efficiency relations are developed for fins of various profiles and are plotted in Fig. 3–42 for fins on a *plain surface* and in Fig. 3–43 for *circular fins* of constant thickness. The fin surface area associated with each profile is also given on each figure. For most fins of constant thickness encountered in practice, the fin thickness t is too small relative to the fin length L, and thus the fin tip area is negligible.
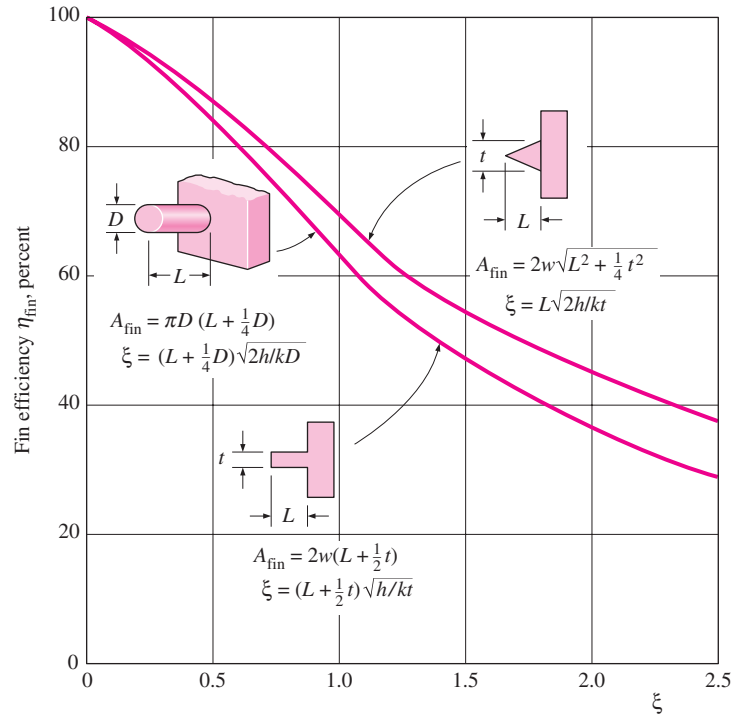

FIGURE 3–42

Efficiency of circular, rectangular, and triangular fins on a plain surface of width w (from Gardner, Ref. 6).

$$A_{\text{fin}} = \pi D\left(L + \tfrac{1}{4}D\right)$$
$$\xi = \left(L + \tfrac{1}{4}D\right)\sqrt{2h/kD}$$

$$A_{\text{fin}} = 2w\sqrt{L^2 + \tfrac{1}{4}t^2}$$
$$\xi = L\sqrt{2h/kt}$$

$$A_{\text{fin}} = 2w\left(L + \tfrac{1}{2}t\right)$$
$$\xi = \left(L + \tfrac{1}{2}t\right)\sqrt{h/kt}$$
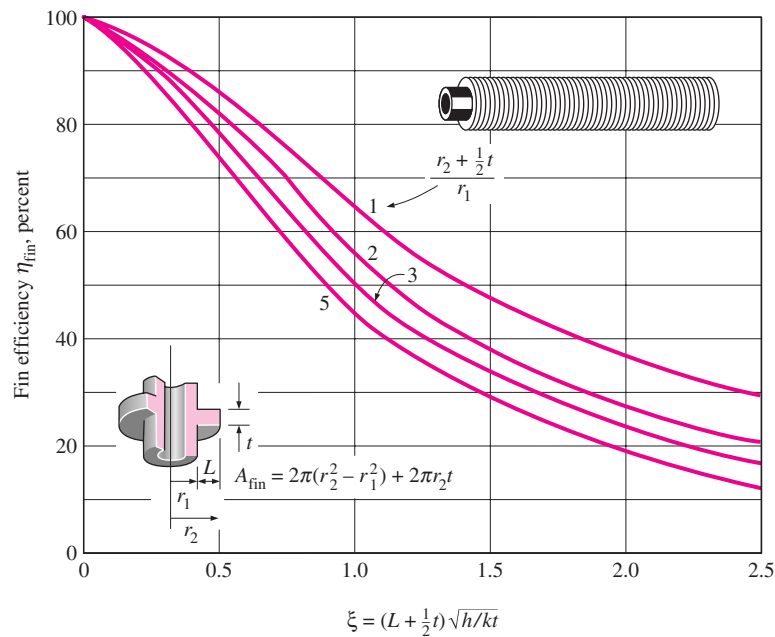
FIGURE 3–43

Efficiency of circular fins of length L and constant thickness t (from Gardner, Ref. 6).

$$\frac{r_2 + \tfrac{1}{2}t}{r_1}$$

$$A_{\text{fin}} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t$$

$$\xi = \left(L + \tfrac{1}{2}t\right)\sqrt{h/kt}$$
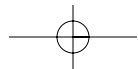
Note that fins with triangular and parabolic profiles contain less material and are more efficient than the ones with rectangular profiles, and thus are more suitable for applications requiring minimum weight such as space applications.

An important consideration in the design of finned surfaces is the selection of the proper *fin length L.* Normally the *longer* the fin, the *larger* the heat transfer area and thus the *higher* the rate of heat transfer from the fin. But also the larger the fin, the bigger the mass, the higher the price, and the larger the fluid friction. Therefore, increasing the length of the fin beyond a certain value cannot be justified unless the added benefits outweigh the added cost. Also, the fin efficiency decreases with increasing fin length because of the decrease in fin temperature with length. Fin lengths that cause the fin efficiency to drop below 60 percent usually cannot be justified economically and should be avoided. The efficiency of most fins used in practice is above 90 percent.

Fin Effectiveness

Fins are used to *enhance* heat transfer, and the use of fins on a surface cannot be recommended unless the enhancement in heat transfer justifies the added cost and complexity associated with the fins. In fact, there is no assurance that adding fins on a surface will *enhance* heat transfer. The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case. The performance of fins expressed in terms of the *fin effectiveness* ε_{fin} is defined as (Fig. 3–44)

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } base\ area\ A_b}{\text{Heat transfer rate from the surface of } area\ A_b} \tag{3-72}$$

Here, A_b is the cross-sectional area of the fin at the base and $\dot{Q}_{\text{no fin}}$ represents the rate of heat transfer from this area if no fins are attached to the surface. An effectiveness of $\varepsilon_{\text{fin}} = 1$ indicates that the addition of fins to the surface does not affect heat transfer at all. That is, heat conducted to the fin through the base area A_b is equal to the heat transferred from the same area A_b to the surrounding medium. An effectiveness of $\varepsilon_{\text{fin}} < 1$ indicates that the fin actually acts as *insulation,* slowing down the heat transfer from the surface. This situation can occur when fins made of low thermal conductivity materials are used. An effectiveness of $\varepsilon_{\text{fin}} > 1$ indicates that fins are *enhancing* heat transfer from the surface, as they should. However, the use of fins cannot be justified unless ε_{fin} is sufficiently larger than 1. Finned surfaces are designed on the basis of *maximizing* effectiveness for a specified cost or *minimizing* cost for a desired effectiveness.

Note that both the fin efficiency and fin effectiveness are related to the performance of the fin, but they are different quantities. However, they are related to each other by

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\eta_{\text{fin}}\,hA_{\text{fin}}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\,\eta_{\text{fin}} \tag{3-73}$$

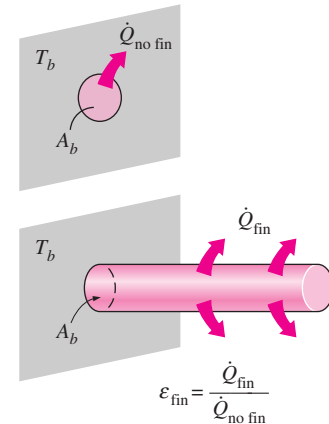$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}}$$

FIGURE 3–44
The effectiveness of a fin.

Therefore, the fin effectiveness can be determined easily when the fin efficiency is known, or vice versa.

The rate of heat transfer from a sufficiently *long* fin of *uniform* cross section under steady conditions is given by Eq. 3–61. Substituting this relation into Eq. 3–72, the effectiveness of such a long fin is determined to be

$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}} \tag{3-74}$$

since $A_c = A_b$ in this case. We can draw several important conclusions from the fin effectiveness relation above for consideration in the design and selection of the fins:

- The *thermal conductivity k* of the fin material should be as high as possible. Thus it is no coincidence that fins are made from metals, with copper, aluminum, and iron being the most common ones. Perhaps the most widely used fins are made of aluminum because of its low cost and weight and its resistance to corrosion.

- The ratio of the *perimeter* to the *cross-sectional area* of the fin p/A_c should be as high as possible. This criterion is satisfied by *thin* plate fins and *slender* pin fins.

- The use of fins is *most effective* in applications involving a *low convection heat transfer coefficient.* Thus, the use of fins is more easily justified when the medium is a *gas* instead of a liquid and the heat transfer is by *natural convection* instead of by forced convection. Therefore, it is no coincidence that in liquid-to-gas heat exchangers such as the car radiator, fins are placed on the *gas* side.

When determining the rate of heat transfer from a finned surface, we must consider the *unfinned portion* of the surface as well as the *fins.* Therefore, the rate of heat transfer for a surface containing *n* fins can be expressed as

$$\begin{aligned}
\dot{Q}_{\text{total, fin}} &= \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}} \\
&= hA_{\text{unfin}}\,(T_b - T_\infty) + \eta_{\text{fin}}\,hA_{\text{fin}}\,(T_b - T_\infty) \\
&= h(A_{\text{unfin}} + \eta_{\text{fin}}A_{\text{fin}})(T_b - T_\infty)
\end{aligned} \tag{3-75}$$

We can also define an **overall effectiveness** for a finned surface as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}}A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}\,(T_b - T_\infty)} \tag{3-76}$$

where $A_{\text{no fin}}$ is the area of the surface when there are no fins, A_{fin} is the total surface area of all the fins on the surface, and A_{unfin} is the area of the unfinned portion of the surface (Fig. 3–45). Note that the overall fin effectiveness depends on the fin density (number of fins per unit length) as well as the effectiveness of the individual fins. The overall effectiveness is a better measure of the performance of a finned surface than the effectiveness of the individual fins.
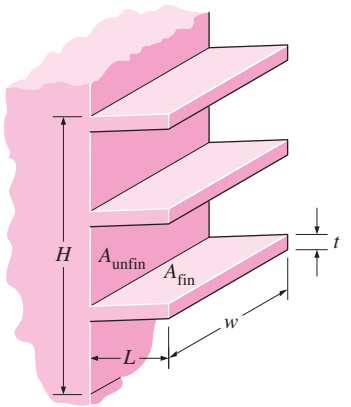
$A_{\text{no fin}} = w \times H$
$A_{\text{unfin}} = w \times H - 3 \times (t \times w)$
$A_{\text{fin}} = 2 \times L \times w + t \times w$ (one fin)
$\qquad \approx 2 \times L \times w$

FIGURE 3–45
Various surface areas associated with a rectangular surface with three fins.
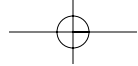
Proper Length of a Fin

An important step in the design of a fin is the determination of the appropriate length of the fin once the fin material and the fin cross section are specified. You may be tempted to think that the longer the fin, the larger the surface area and thus the higher the rate of heat transfer. Therefore, for maximum heat transfer, the fin should be infinitely long. However, the temperature drops along the fin exponentially and reaches the environment temperature at some length. The part of the fin beyond this length does not contribute to heat transfer since it is at the temperature of the environment, as shown in Fig. 3–46. Therefore, designing such an "extra long" fin is out of the question since it results in material waste, excessive weight, and increased size and thus increased cost with no benefit in return (in fact, such a long fin will hurt performance since it will suppress fluid motion and thus reduce the convection heat transfer coefficient). Fins that are so long that the temperature approaches the environment temperature cannot be recommended either since the little increase in heat transfer at the tip region cannot justify the large increase in the weight and cost.

To get a sense of the proper length of a fin, we compare heat transfer from a fin of finite length to heat transfer from an infinitely long fin under the same conditions. The ratio of these two heat transfers is

Heat transfer ratio:
$$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{\sqrt{hpkA_c}\,(T_b - T_\infty)} = \tanh aL \qquad \text{(3-77)}$$

Using a hand calculator, the values of tanh aL are evaluated for some values of aL and the results are given in Table 3–3. We observe from the table that heat transfer from a fin increases with aL almost linearly at first, but the curve reaches a plateau later and reaches a value for the infinitely long fin at about $aL = 5$. Therefore, a fin whose length is $L = \frac{1}{5}a$ can be considered to be an infinitely long fin. We also observe that reducing the fin length by half in that case (from $aL = 5$ to $aL = 2.5$) causes a drop of just 1 percent in heat transfer. We certainly would not hesitate sacrificing 1 percent in heat transfer performance in return for 50 percent reduction in the size and possibly the cost of the fin. In practice, a fin length that corresponds to about $aL = 1$ will transfer 76.2 percent of the heat that can be transferred by an infinitely long fin, and thus it should offer a good compromise between heat transfer performance and the fin size.

A common approximation used in the analysis of fins is to assume the fin temperature varies in one direction only (along the fin length) and the temperature variation along other directions is negligible. Perhaps you are wondering if this one-dimensional approximation is a reasonable one. This is certainly the case for fins made of thin metal sheets such as the fins on a car radiator, but we wouldn't be so sure for fins made of thick materials. Studies have shown that the error involved in one-dimensional fin analysis is negligible (less than about 1 percent) when
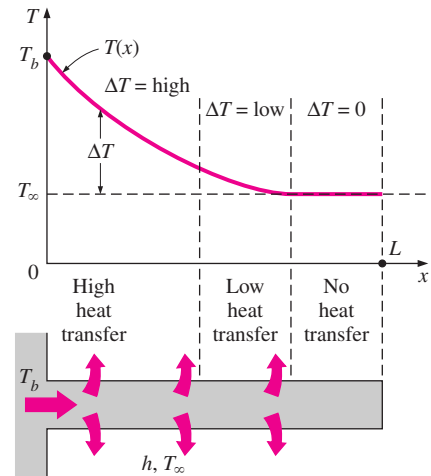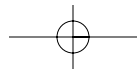
$$\frac{h\delta}{k} < 0.2$$

FIGURE 3–46
Because of the gradual temperature drop along the fin, the region near the fin tip makes little or no contribution to heat transfer.

TABLE 3–3

The variation of heat transfer from a fin relative to that from an infinitely long fin

aL	$\dfrac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \tanh aL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

where δ is the characteristic thickness of the fin, which is taken to be the plate thickness t for rectangular fins and the diameter D for cylindrical ones.

Specially designed finned surfaces called *heat sinks,* which are commonly used in the cooling of electronic equipment, involve one-of-a-kind complex geometries, as shown in Table 3–4. The heat transfer performance of heat sinks is usually expressed in terms of their *thermal resistances R* in °C/W, which is defined as

$$\dot{Q}_{\text{fin}} = \frac{T_b - T_\infty}{R} = hA_{\text{fin}}\,\eta_{\text{fin}}\,(T_b - T_\infty) \tag{3-78}$$

A small value of thermal resistance indicates a small temperature drop across the heat sink, and thus a high fin efficiency.

EXAMPLE 3–10   **Maximum Power Dissipation of a Transistor**

Power transistors that are commonly used in electronic devices consume large amounts of electric power. The failure rate of electronic components increases almost exponentially with operating temperature. As a rule of thumb, the failure rate of electronic components is halved for each 10°C reduction in the junction operating temperature. Therefore, the operating temperature of electronic components is kept below a safe level to minimize the risk of failure.

The sensitive electronic circuitry of a power transistor at the junction is protected by its case, which is a rigid metal enclosure. Heat transfer characteristics of a power transistor are usually specified by the manufacturer in terms of the case-to-ambient thermal resistance, which accounts for both the natural convection and radiation heat transfers.

The case-to-ambient thermal resistance of a power transistor that has a maximum power rating of 10 W is given to be 20°C/W. If the case temperature of the transistor is not to exceed 85°C, determine the power at which this transistor can be operated safely in an environment at 25°C.

SOLUTION   The maximum power rating of a transistor whose case temperature is not to exceed 85°C is to be determined.

Assumptions   **1** Steady operating conditions exist. **2** The transistor case is isothermal at 85°C.

Properties   The case-to-ambient thermal resistance is given to be 20°C/W.

Analysis   The power transistor and the thermal resistance network associated with it are shown in Fig. 3–47. We notice from the thermal resistance network that there is a single resistance of 20°C/W between the case at $T_c = 85$°C and the ambient at $T_\infty = 25$°C, and thus the rate of heat transfer is

$$\dot{Q} = \left(\frac{\Delta T}{R}\right)_{\text{case-ambient}} = \frac{T_c - T_\infty}{R_{\text{case-ambient}}} = \frac{(85 - 25)\text{°C}}{20\text{°C/W}} = \textbf{3 W}$$

Therefore, this power transistor should not be operated at power levels above 3 W if its case temperature is not to exceed 85°C.

Discussion   This transistor can be used at higher power levels by attaching it to a heat sink (which lowers the thermal resistance by increasing the heat transfer surface area, as discussed in the next example) or by using a fan (which lowers the thermal resistance by increasing the convection heat transfer coefficient).
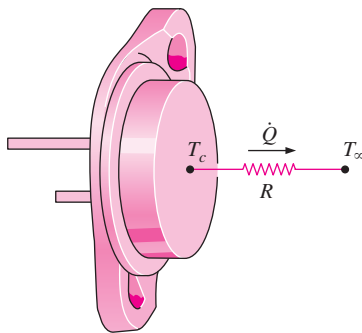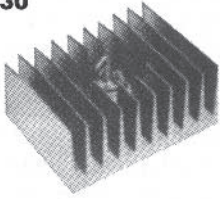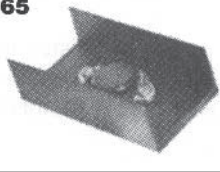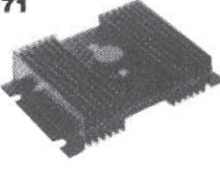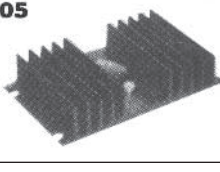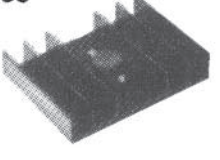

FIGURE 3–47
Schematic for Example 3–10.

TABLE 3–4

Combined natural convection and radiation thermal resistance of various heat sinks used in the cooling of electronic devices between the heat sink and the surroundings. All fins are made of aluminum 6063T-5, are black anodized, and are 76 mm (3 in.) long (courtesy of Vemaline Products, Inc.).

HS 5030

$R = 0.9°C/W$ (vertical)
$R = 1.2°C/W$ (horizontal)

Dimensions: 76 mm × 105 mm × 44 mm
Surface area: 677 cm^2

HS 6065

$R = 5°C/W$

Dimensions: 76 mm × 38 mm × 24 mm
Surface area: 387 cm^2

HS 6071

$R = 1.4°C/W$ (vertical)
$R = 1.8°C/W$ (horizontal)

Dimensions: 76 mm × 92 mm × 26 mm
Surface area: 968 cm^2

HS 6105

$R = 1.8°C/W$ (vertical)
$R = 2.1°C/W$ (horizontal)

Dimensions: 76 mm × 127 mm × 91 mm
Surface area: 677 cm^2

HS 6115

$R = 1.1°C/W$ (vertical)
$R = 1.3°C/W$ (horizontal)

Dimensions: 76 mm × 102 mm × 25 mm
Surface area: 929 cm^2

HS 7030

$R = 2.9°C/W$ (vertical)
$R = 3.1°C/W$ (horizontal)

Dimensions: 76 mm × 97 mm × 19 mm
Surface area: 290 cm^2

EXAMPLE 3–11   **Selecting a Heat Sink for a Transistor**

A 60-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 3–4. Select a heat sink that will allow the case temperature of the transistor not to exceed 90°C in the ambient air at 30°C.

SOLUTION A commercially available heat sink from Table 3–4 is to be selected to keep the case temperature of a transistor below 90°C.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 90°C. **3** The contact resistance between the transistor and the heat sink is negligible.

Analysis The rate of heat transfer from a 60-W transistor at full power is $\dot{Q} = 60$ W. The thermal resistance between the transistor attached to the heat sink and the ambient air for the specified temperature difference is determined to be

$$\dot{Q} = \frac{\Delta T}{R} \quad \longrightarrow \quad R = \frac{\Delta T}{\dot{Q}} = \frac{(90 - 30)°C}{60\ W} = 1.0°C/W$$

Therefore, the thermal resistance of the heat sink should be below 1.0°C/W. An examination of Table 3–4 reveals that the HS 5030, whose thermal resistance is 0.9°C/W in the vertical position, is the only heat sink that will meet this requirement.

$r_2 = 3$ cm $\quad r_1 = 1.5$ cm
T_∞
h
T_b
$t = 2$ mm
$S = 3$ mm

FIGURE 3–48
Schematic for Example 3–12.

EXAMPLE 3–12 **Effect of Fins on Heat Transfer from Steam Pipes**

Steam in a heating system flows through tubes whose outer diameter is $D_1 = 3$ cm and whose walls are maintained at a temperature of 120°C. Circular aluminum fins ($k = 180$ W/m · °C) of outer diameter $D_2 = 6$ cm and constant thickness $t = 2$ mm are attached to the tube, as shown in Fig. 3–48. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at $T_\infty = 25$°C, with a combined heat transfer coefficient of $h = 60$ W/m^2 · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

SOLUTION Circular aluminum fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is uniform over the entire fin surfaces. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be $k = 180$ W/m · °C.

Analysis In the case of no fins, heat transfer from the tube per meter of its length is determined from Newton's law of cooling to be

$$A_{no\ fin} = \pi D_1 L = \pi(0.03\ m)(1\ m) = 0.0942\ m^2$$
$$\dot{Q}_{no\ fin} = hA_{no\ fin}(T_b - T_\infty)$$
$$= (60\ W/m^2 · °C)(0.0942\ m^2)(120 - 25)°C$$
$$= 537\ W$$

The efficiency of the circular fins attached to a circular tube is plotted in Fig. 3–43. Noting that $L = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.06 - 0.03) = 0.015$ m in this case, we have

$$\frac{r_2 + \frac{1}{2}t}{r_1} = \frac{(0.03 + \frac{1}{2} \times 0.002) \text{ m}}{0.015 \text{ m}} = 2.07$$

$$(L + \tfrac{1}{2}t)\sqrt{\frac{h}{kt}} = (0.015 + \tfrac{1}{2} \times 0.002) \text{ m} \times \sqrt{\frac{60 \text{ W/m}^2 \cdot \text{°C}}{(180 \text{ W/m} \cdot \text{°C})(0.002 \text{ m})}} = 0.207 \left.\right\} \quad \eta_{\text{fin}} = 0.95$$

$$A_{\text{fin}} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t$$
$$= 2\pi[(0.03 \text{ m})^2 - (0.015 \text{ m})^2] + 2\pi(0.03 \text{ m})(0.002 \text{ m})$$
$$= 0.00462 \text{ m}^2$$

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}}hA_{\text{fin}}(T_b - T_\infty)$$
$$= 0.95(60 \text{ W/m}^2 \cdot \text{°C})(0.00462 \text{ m}^2)(120 - 25)\text{°C}$$
$$= 25.0 \text{ W}$$

Heat transfer from the unfinned portion of the tube is

$$A_{\text{unfin}} = \pi D_1 S = \pi(0.03 \text{ m})(0.003 \text{ m}) = 0.000283 \text{ m}^2$$
$$\dot{Q}_{\text{unfin}} = hA_{\text{unfin}}(T_b - T_\infty)$$
$$= (60 \text{ W/m}^2 \cdot \text{°C})(0.000283 \text{ m}^2)(120 - 25)\text{°C}$$
$$= 1.60 \text{ W}$$

Noting that there are 200 fins and thus 200 interfin spacings per meter length of the tube, the total heat transfer from the finned tube becomes

$$\dot{Q}_{\text{total, fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 200(25.0 + 1.6) \text{ W} = 5320 \text{ W}$$

Therefore, the increase in heat transfer from the tube per meter of its length as a result of the addition of fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total, fin}} - \dot{Q}_{\text{no fin}} = 5320 - 537 = \textbf{4783 W} \qquad \text{(per m tube length)}$$

Discussion   The overall effectiveness of the finned tube is

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{5320 \text{ W}}{537 \text{ W}} = 9.9$$

That is, the rate of heat transfer from the steam tube increases by a factor of almost 10 as a result of adding fins. This explains the widespread use of finned surfaces.

3-7 ▪ HEAT TRANSFER IN COMMON CONFIGURATIONS

So far, we have considered heat transfer in *simple* geometries such as large plane walls, long cylinders, and spheres. This is because heat transfer in such geometries can be approximated as *one-dimensional,* and simple analytical solutions can be obtained easily. But many problems encountered in practice are two- or three-dimensional and involve rather complicated geometries for which no simple solutions are available.

An important class of heat transfer problems for which simple solutions are obtained encompasses those involving two surfaces maintained at *constant* temperatures T_1 and T_2. The steady rate of heat transfer between these two surfaces is expressed as

$$Q = Sk(T_1 - T_2) \tag{3-79}$$

where S is the **conduction shape factor,** which has the dimension of *length,* and k is the thermal conductivity of the medium between the surfaces. The conduction shape factor depends on the *geometry* of the system only.

Conduction shape factors have been determined for a number of configurations encountered in practice and are given in Table 3–5 for some common cases. More comprehensive tables are available in the literature. Once the value of the shape factor is known for a specific geometry, the total steady heat transfer rate can be determined from the equation above using the specified two constant temperatures of the two surfaces and the thermal conductivity of the medium between them. Note that conduction shape factors are applicable only when heat transfer between the two surfaces is by *conduction.* Therefore, they cannot be used when the medium between the surfaces is a liquid or gas, which involves natural or forced convection currents.

A comparison of Equations 3-4 and 3-79 reveals that the conduction shape factor S is related to the thermal resistance R by $R = 1/kS$ or $S = 1/kR$. Thus, these two quantities are the inverse of each other when the thermal conductivity of the medium is unity. The use of the conduction shape factors is illustrated with examples 3–13 and 3–14.
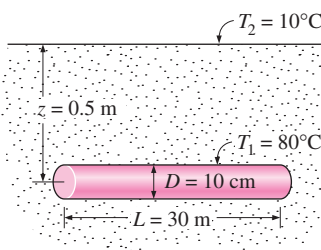
FIGURE 3–49
Schematic for Example 3–13.

EXAMPLE 3–13   **Heat Loss from Buried Steam Pipes**

A 30-m-long, 10-cm-diameter hot water pipe of a district heating system is buried in the soil 50 cm below the ground surface, as shown in Figure 3–49. The outer surface temperature of the pipe is 80°C. Taking the surface temperature of the earth to be 10°C and the thermal conductivity of the soil at that location to be 0.9 W/m · °C, determine the rate of heat loss from the pipe.

SOLUTION   The hot water pipe of a district heating system is buried in the soil. The rate of heat loss from the pipe is to be determined.
Assumptions   **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant.
Properties   The thermal conductivity of the soil is given to be k = 0.9 W/m · °C.
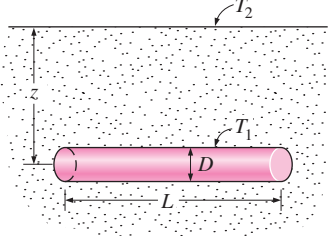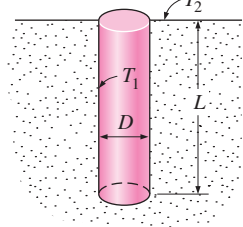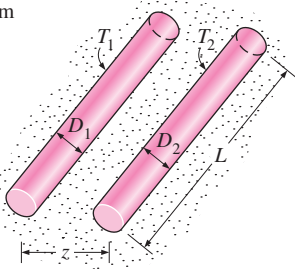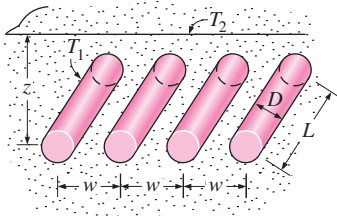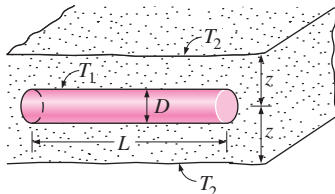*Analysis*   The shape factor for this configuration is given in Table 3–5 to be
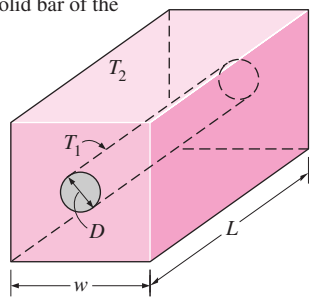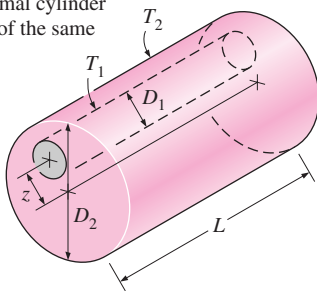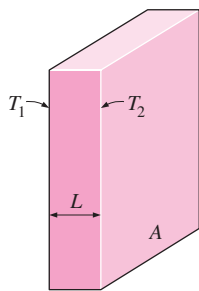
$$S = \frac{2\pi L}{\ln(4z/D)}$$

since $z > 1.5D$, where z is the distance of the pipe from the ground surface, and D is the diameter of the pipe. Substituting,

$$S = \frac{2\pi \times (30 \text{ m})}{\ln(4 \times 0.5/0.1)} = 62.9 \text{ m}$$

TABLE 3-5

Conduction shape factors S for several configurations for use in $\dot{Q} = kS(T_1 - T_2)$ to determine the steady rate of heat transfer through a medium of thermal conductivity k between the surfaces at temperatures T_1 and T_2

(1) Isothermal cylinder of length L
   buried in a semi-infinite medium
   ($L \gg D$ and $z > 1.5D$)

$$S = \frac{2\pi L}{\ln(4z/D)}$$

(2) Vertical isothermal cylinder of length L
   buried in a semi-infinite medium
   ($L \gg D$)

$$S = \frac{2\pi L}{\ln(4L/D)}$$

(3) Two parallel isothermal cylinders
   placed in an infinite medium
   ($L \gg D_1, D_2, z$)

$$S = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{4z^2 - D_1^2 - D_2^2}{2D_1 D_2}\right)}$$

(4) A row of equally spaced parallel isothermal
   cylinders buried in a semi-infinite medium
   ($L \gg D, z$ and $w > 1.5D$)

$$S = \frac{2\pi L}{\ln\left(\dfrac{2w}{\pi D}\ \sinh\dfrac{2\pi z}{w}\right)}$$

(per cylinder)

(5) Circular isothermal cylinder of length L
   in the midplane of an infinite wall
   ($z > 0.5D$)

$$S = \frac{2\pi L}{\ln(8z/\pi D)}$$

(6) Circular isothermal cylinder of length L
   at the center of a square solid bar of the
   same length

$$S = \frac{2\pi L}{\ln(1.08w/D)}$$

(7) Eccentric circular isothermal cylinder
   of length L in a cylinder of the same
   length ($L > D_2$)

$$S = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{D_1^2 + D_2^2 - 4z^2}{2D_1 D_2}\right)}$$

(8) Large plane wall
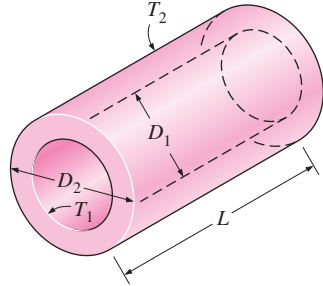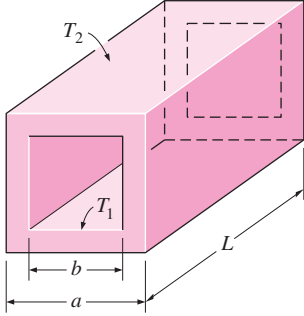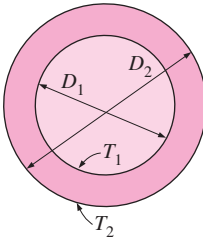
$$S = \frac{A}{L}$$

(continued)
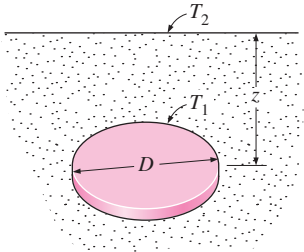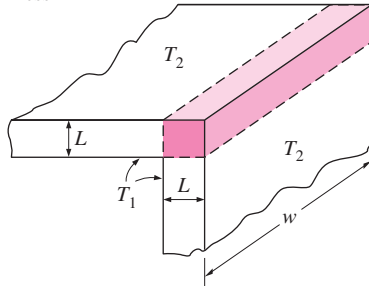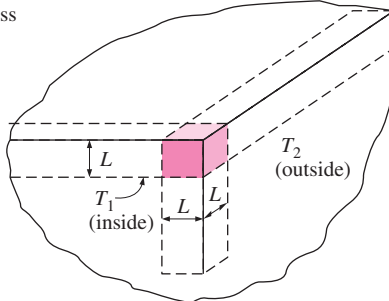
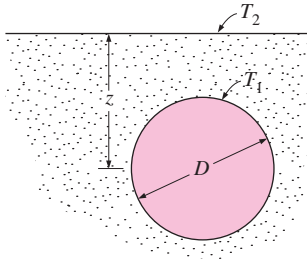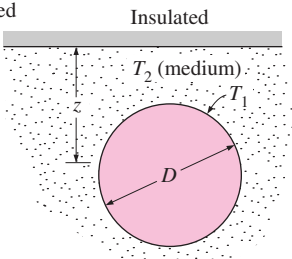## TABLE 3-5 (CONCLUDED)

(9) A long cylindrical layer

$$S = \frac{2\pi L}{\ln (D_2/D_1)}$$

(10) A square flow passage

(a) For $a/b > 1.4$,

$$S = \frac{2\pi L}{0.93 \ln (0.948\,a/b)}$$

(b) For $a/b < 1.41$,

$$S = \frac{2\pi L}{0.785 \ln (a/b)}$$

(11) A spherical layer

$$S = \frac{2\pi D_1 D_2}{D_2 - D_1}$$

(12) Disk buried parallel to the surface in a semi-infinite medium $(z \gg D)$

$$S = 4D$$

$(S = 2D$ when $z = 0)$

(13) The edge of two adjoining walls of equal thickness

$$S = 0.54\,w$$

(14) Corner of three walls of equal thickness

$$S = 0.15L$$

(15) Isothermal sphere buried in a semi-infinite medium

$$S = \frac{2\pi D}{1 - 0.25D/z}$$

(16) Isothermal sphere buried in a semi-infinite medium at T_2 whose surface is insulated

$$S = \frac{2\pi D}{1 + 0.25D/z}$$
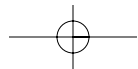
Then the steady rate of heat transfer from the pipe becomes

$$\dot{Q} = Sk(T_1 - T_2) = (62.9 \text{ m})(0.9 \text{ W/m} \cdot °\text{C})(80 - 10)°\text{C} = \textbf{3963 W}$$

Discussion   Note that this heat is conducted from the pipe surface to the surface of the earth through the soil and then transferred to the atmosphere by convection and radiation.

EXAMPLE 3–14   **Heat Transfer between Hot and Cold Water Pipes**

A 5-m-long section of hot and cold water pipes run parallel to each other in a thick concrete layer, as shown in Figure 3–50. The diameters of both pipes are 5 cm, and the distance between the centerline of the pipes is 30 cm. The surface temperatures of the hot and cold pipes are 70°C and 15°C, respectively. Taking the thermal conductivity of the concrete to be $k = 0.75$ W/m · °C, determine the rate of heat transfer between the pipes.

SOLUTION   Hot and cold water pipes run parallel to each other in a thick concrete layer. The rate of heat transfer between the pipes is to be determined.
Assumptions   **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.
Properties   The thermal conductivity of concrete is given to be $k = 0.75$ W/m · °C.
Analysis   The shape factor for this configuration is given in Table 3–5 to be

$$S = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{4z^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$$

where z is the distance between the centerlines of the pipes and L is their length. Substituting,

$$S = \frac{2\pi \times (5 \text{ m})}{\cosh^{-1}\left(\dfrac{4 \times 0.3^2 - 0.05^2 - 0.05^2}{2 \times 0.05 \times 0.05}\right)} = 6.34 \text{ m}$$

Then the steady rate of heat transfer between the pipes becomes

$$\dot{Q} = Sk(T_1 - T_2) = (6.34 \text{ m})(0.75 \text{ W/m} \cdot °\text{C})(70 - 15°)\text{C} = \textbf{262 W}$$

Discussion   We can reduce this heat loss by placing the hot and cold water pipes further away from each other.

$T_1 = 70°\text{C}$ $T_2 = 15°\text{C}$
$D_1 = 5$ cm $D_2 = 5$ cm $L = 5$ m
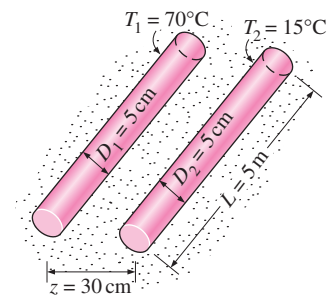$z = 30$ cm

FIGURE 3–50
Schematic for Example 3-14.

It is well known that insulation reduces heat transfer and saves energy and money. Decisions on the right amount of insulation are based on a heat transfer analysis, followed by an economic analysis to determine the "monetary value" of energy loss. This is illustrated with Example 3–15.
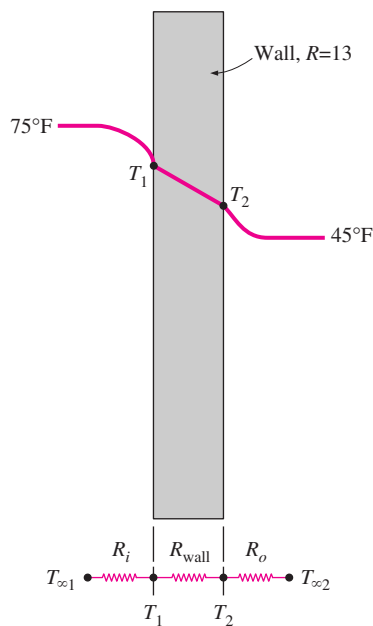
EXAMPLE 3–15   Cost of Heat Loss through Walls in Winter

Consider an electrically heated house whose walls are 9 ft high and have an R-value of insulation of 13 (i.e., a thickness-to-thermal conductivity ratio of $L/k = 13$ h · ft^2 · °F/Btu). Two of the walls of the house are 40 ft long and the others are 30 ft long. The house is maintained at 75°F at all times, while the temperature of the outdoors varies. Determine the amount of heat lost through the walls of the house on a certain day during which the average temperature of the outdoors is 45°F. Also, determine the cost of this heat loss to the homeowner if the unit cost of electricity is $0.075/kWh. For combined convection and radiation heat transfer coefficients, use the ASHRAE (American Society of Heating, Refrigeration, and Air Conditioning Engineers) recommended values of $h_i = 1.46$ Btu/h · ft^2 · °F for the inner surface of the walls and $h_o = 4.0$ Btu/h · ft^2 · °F for the outer surface of the walls under 15 mph wind conditions in winter.

SOLUTION   An electrically heated house with R-13 insulation is considered. The amount of heat lost through the walls and its cost are to be determined.
Assumptions   **1** The indoor and outdoor air temperatures have remained at the given values for the entire day so that heat transfer through the walls is steady. **2** Heat transfer through the walls is one-dimensional since any significant temperature gradients in this case will exist in the direction from the indoors to the outdoors. **3** The radiation effects are accounted for in the heat transfer coefficients.
Analysis   This problem involves conduction through the wall and convection at its surfaces and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 3–51. The heat transfer area of the walls is

$$A = \text{Circumference} \times \text{Height} = (2 \times 30 \text{ ft} + 2 \times 40 \text{ ft})(9 \text{ ft}) = 1260 \text{ ft}^2$$

Then the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv},\, i} = \frac{1}{h_i A} = \frac{1}{(1.46 \text{ Btu/h} \cdot \text{ft}^2 \cdot °\text{F})(1260 \text{ ft}^2)} = 0.00054 \text{ h} \cdot °\text{F/Btu}$$

$$R_{\text{wall}} = \frac{L}{kA} = \frac{R\text{-value}}{A} = \frac{13 \text{ h} \cdot \text{ft}^2 \cdot °\text{F/Btu}}{1260 \text{ ft}^2} = 0.01032 \text{ h} \cdot °\text{F/Btu}$$

$$R_o = R_{\text{conv},\, o} = \frac{1}{h_c A} = \frac{1}{(4.0 \text{ Btu/h} \cdot \text{ft}^2 \cdot °\text{F})(1260 \text{ ft}^2)} = 0.00020 \text{ h} \cdot °\text{F/Btu}$$

Noting that all three resistances are in series, the total resistance is

$$R_{\text{total}} = R_i + R_{\text{wall}} + R_o = 0.00054 + 0.01032 + 0.00020 = 0.01106 \text{ h} \cdot °\text{F/Btu}$$

Then the steady rate of heat transfer through the walls of the house becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(75 - 45)°\text{F}}{0.01106 \text{ h} \cdot °\text{F/Btu}} = 2712 \text{ Btu/h}$$

Finally, the total amount of heat lost through the walls during a 24-h period and its cost to the home owner are

$$Q = \dot{Q}\ \Delta t = (2712 \text{ Btu/h})(24\text{-h/day}) = \textbf{65,099 Btu/day} = \textbf{19.1 kWh/day}$$

FIGURE 3–51
Schematic for Example 3–15.

since 1 kWh = 3412 Btu, and

Heating cost = (Energy lost)(Cost of energy) = (19.1 kWh/day)($0.075/kWh)
= **$1.43/day**

Discussion   The heat losses through the walls of the house that day will cost the home owner $1.43 worth of electricity.

TOPIC OF SPECIAL INTEREST*

Heat Transfer Through Walls and Roofs

Under steady conditions, the rate of heat transfer through any section of a building wall or roof can be determined from

$$\dot{Q} = UA(T_i - T_o) = \frac{A(T_i - T_o)}{R} \tag{3-80}$$

where T_i and T_o are the indoor and outdoor air temperatures, A is the heat transfer area, U is the overall heat transfer coefficient (the U-factor), and $R = 1/U$ is the overall unit thermal resistance (the R-value). Walls and roofs of buildings consist of various layers of materials, and the structure and operating conditions of the walls and the roofs may differ significantly from one building to another. Therefore, it is not practical to list the R-values (or U-factors) of different kinds of walls or roofs under different conditions. Instead, the overall R-value is determined from the thermal resistances of the individual components using the thermal resistance network. The overall thermal resistance of a structure can be determined most accurately in a lab by actually assembling the unit and testing it as a whole, but this approach is usually very time consuming and expensive. The analytical approach described here is fast and straightforward, and the results are usually in good agreement with the experimental values.

The unit thermal resistance of a plane layer of thickness L and thermal conductivity k can be determined from $R = L/k$. The thermal conductivity and other properties of common building materials are given in the appendix. The unit thermal resistances of various components used in building structures are listed in Table 3–6 for convenience.

Heat transfer through a wall or roof section is also affected by the convection and radiation heat transfer coefficients at the exposed surfaces. The effects of convection and radiation on the inner and outer surfaces of walls and roofs are usually combined into the *combined convection and radiation heat transfer coefficients* (also called *surface conductances*) h_i and h_o, respectively, whose values are given in Table 3–7 for ordinary surfaces ($\varepsilon = 0.9$) and reflective surfaces ($\varepsilon = 0.2$ or 0.05). Note that surfaces having a low emittance also have a low surface conductance due to the reduction in radiation heat transfer. The values in the table are based on a surface

*This section can be skipped without a loss of continuity.

TABLE 3–7

Combined convection and radiation heat transfer coefficients at window, wall, or roof surfaces (from ASHRAE *Handbook of Fundamentals,* Ref. 1, Chap. 22, Table 1).

		h, W/m² · °C*		
	Direc-tion of	Surface Emittance, ε		
Posi-tion	Heat Flow	0.90	0.20	0.05
Still air (both indoors and outdoors)				
Horiz.	Up ↑	9.26	5.17	4.32
Horiz.	Down ↓	6.13	2.10	1.25
45° slope	Up ↑	9.09	5.00	4.15
45° slope	Down ↓	7.50	3.41	2.56
Vertical	Horiz. →	8.29	4.20	3.35
Moving air (any position, any direction)				
Winter condition (winds at 15 mph or 24 km/h)		34.0	—	—
Summer condition (winds at 7.5 mph or 12 km/h)		22.7	—	—

*Multiply by 0.176 to convert to Btu/h · ft² · °F. Surface resistance can be obtained from $R = 1/h$.

TABLE 3–6

Unit thermal resistance (the R-value) of common components used in buildings

Component	R-value m² · °C/W	R-value ft² · h · °F/Btu	Component	R-value m² · °C/W	R-value ft² · h · °F/Btu
Outside surface (winter)	0.030	0.17	Wood stud, nominal 2 in. ×		
Outside surface (summer)	0.044	0.25	6 in. (5.5 in. or 140 mm wide)	0.98	5.56
Inside surface, still air	0.12	0.68	Clay tile, 100 mm (4 in.)	0.18	1.01
Plane air space, vertical, ordinary surfaces ($\varepsilon_{eff} = 0.82$):			Acoustic tile	0.32	1.79
13 mm ($\frac{1}{2}$ in.)	0.16	0.90	Asphalt shingle roofing	0.077	0.44
20 mm ($\frac{3}{4}$ in.)	0.17	0.94	Building paper	0.011	0.06
40 mm (1.5 in.)	0.16	0.90	Concrete block, 100 mm (4 in.):		
90 mm (3.5 in.)	0.16	0.91	Lightweight	0.27	1.51
Insulation, 25 mm (1 in.)			Heavyweight	0.13	0.71
Glass fiber	0.70	4.00	Plaster or gypsum board,		
Mineral fiber batt	0.66	3.73	13 mm ($\frac{1}{2}$ in.)	0.079	0.45
Urethane rigid foam	0.98	5.56	Wood fiberboard, 13 mm ($\frac{1}{2}$ in.)	0.23	1.31
Stucco, 25 mm (1 in.)	0.037	0.21	Plywood, 13 mm ($\frac{1}{2}$ in.)	0.11	0.62
Face brick, 100 mm (4 in.)	0.075	0.43	Concrete, 200 mm (8 in.):		
Common brick, 100 mm (4 in.)	0.12	0.79	Lightweight	1.17	6.67
Steel siding	0.00	0.00	Heavyweight	0.12	0.67
Slag, 13 mm ($\frac{1}{2}$ in.)	0.067	0.38	Cement mortar, 13 mm (1/2 in.)	0.018	0.10
Wood, 25 mm (1 in.)	0.22	1.25	Wood bevel lapped siding,		
Wood stud, nominal 2 in. ×			13 mm × 200 mm		
4 in. (3.5 in. or 90 mm wide)	0.63	3.58	(1/2 in. × 8 in.)	0.14	0.81

temperature of 21°C (72°F) and a surface–air temperature difference of 5.5°C (10°F). Also, the equivalent surface temperature of the environment is assumed to be equal to the ambient air temperature. Despite the convenience it offers, this assumption is not quite accurate because of the additional radiation heat loss from the surface to the clear sky. The effect of sky radiation can be accounted for approximately by taking the outside temperature to be the average of the outdoor air and sky temperatures.

The inner surface heat transfer coefficient h_i remains fairly constant throughout the year, but the value of h_o varies considerably because of its dependence on the orientation and wind speed, which can vary from less than 1 km/h in calm weather to over 40 km/h during storms. The commonly used values of h_i and h_o for peak load calculations are

$$h_i = 8.29 \text{ W/m}^2 \cdot °C = 1.46 \text{ Btu/h} \cdot \text{ft}^2 \cdot °F \qquad \text{(winter and summer)}$$

$$h_o = \begin{cases} 34.0 \text{ W/m}^2 \cdot °C = 6.0 \text{ Btu/h} \cdot \text{ft}^2 \cdot °F & \text{(winter)} \\ 22.7 \text{ W/m}^2 \cdot °C = 4.0 \text{ Btu/h} \cdot \text{ft}^2 \cdot °F & \text{(summer)} \end{cases}$$

which correspond to design wind conditions of 24 km/h (15 mph) for winter and 12 km/h (7.5 mph) for summer. The corresponding surface thermal resistances (R-values) are determined from $R_i = 1/h_i$ and $R_o = 1/h_o$. The surface conductance values under still air conditions can be used for interior surfaces as well as exterior surfaces in calm weather.

Building components often involve *trapped air spaces* between various layers. Thermal resistances of such air spaces depend on the thickness of the layer, the temperature difference across the layer, the mean air temperature, the emissivity of each surface, the orientation of the air layer, and the direction of heat transfer. The emissivities of surfaces commonly encountered in buildings are given in Table 3–8. The **effective emissivity** of a plane-parallel air space is given by

$$\frac{1}{\varepsilon_{\text{effective}}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \qquad \text{(3-81)}$$

where ε_1 and ε_2 are the emissivities of the surfaces of the air space. Table 3–8 also lists the effective emissivities of air spaces for the cases where (1) the emissivity of one surface of the air space is ε while the emissivity of the other surface is 0.9 (a building material) and (2) the emissivity of both surfaces is ε. Note that the effective emissivity of an air space between building materials is $0.82/0.03 = 27$ times that of an air space between surfaces covered with aluminum foil. For specified surface temperatures, radiation heat transfer through an air space is proportional to effective emissivity, and thus the rate of radiation heat transfer in the ordinary surface case is 27 times that of the reflective surface case.

Table 3–9 lists the thermal resistances of 20-mm-, 40-mm-, and 90-mm- (0.75-in., 1.5-in., and 3.5-in.) thick air spaces under various conditions. The thermal resistance values in the table are applicable to air spaces of uniform thickness bounded by plane, smooth, parallel surfaces with no air leakage. Thermal resistances for other temperatures, emissivities, and air spaces can be obtained by interpolation and moderate extrapolation. Note that the presence of a low-emissivity surface reduces radiation heat transfer across an air space and thus significantly increases the thermal resistance. The thermal effectiveness of a low-emissivity surface will decline, however, if the condition of the surface changes as a result of some effects such as condensation, surface oxidation, and dust accumulation.

The *R*-value of a wall or roof structure that involves layers of uniform thickness is determined easily by simply adding up the unit thermal resistances of the layers that are in series. But when a structure involves components such as wood studs and metal connectors, then the thermal resistance network involves parallel connections and possible two-dimensional effects. The overall *R*-value in this case can be determined by assuming (1) parallel heat flow paths through areas of different construction or (2) isothermal planes normal to the direction of heat transfer. The first approach usually overpredicts the overall thermal resistance, whereas the second approach usually underpredicts it. The parallel heat flow path approach is more suitable for wood frame walls and roofs, whereas the isothermal planes approach is more suitable for masonry or metal frame walls.

The thermal contact resistance between different components of building structures ranges between 0.01 and 0.1 $\text{m}^2 \cdot {}^\circ\text{C/W}$, which is negligible in most cases. However, it may be significant for metal building components such as steel framing members.

TABLE 3–9

Unit thermal resistances (*R*-values) of well-sealed plane air spaces (from ASHRAE *Handbook of Fundamentals,* Ref. 1, Chap. 22, Table 2)

(*a*) SI units (in m² · °C/W)

Position of Air Space	Direction of Heat Flow	Mean Temp., °C	Temp. Diff., °C	20-mm Air Space Effective Emissivity, ε_{eff}				40-mm Air Space Effective Emissivity, ε_{eff}				90-mm Air Space Effective Emissivity, ε_{eff}			
				0.03	0.05	0.5	0.82	0.03	0.05	0.5	0.82	0.03	0.05	0.5	0.82
Horizontal Up ↑		32.2	5.6	0.41	0.39	0.18	0.13	0.45	0.42	0.19	0.14	0.50	0.47	0.20	0.14
		10.0	16.7	0.30	0.29	0.17	0.14	0.33	0.32	0.18	0.14	0.27	0.35	0.19	0.15
		10.0	5.6	0.40	0.39	0.20	0.15	0.44	0.42	0.21	0.16	0.49	0.47	0.23	0.16
		−17.8	11.1	0.32	0.32	0.20	0.16	0.35	0.34	0.22	0.17	0.40	0.38	0.23	0.18
45° slope Up ↑		32.2	5.6	0.52	0.49	0.20	0.14	0.51	0.48	0.20	0.14	0.56	0.52	0.21	0.14
		10.0	16.7	0.35	0.34	0.19	0.14	0.38	0.36	0.20	0.15	0.40	0.38	0.20	0.15
		10.0	5.6	0.51	0.48	0.23	0.17	0.51	0.48	0.23	0.17	0.55	0.52	0.24	0.17
		−17.8	11.1	0.37	0.36	0.23	0.18	0.40	0.39	0.24	0.18	0.43	0.41	0.24	0.19
Vertical	Horizontal →	32.2	5.6	0.62	0.57	0.21	0.15	0.70	0.64	0.22	0.15	0.65	0.60	0.22	0.15
		10.0	16.7	0.51	0.49	0.23	0.17	0.45	0.43	0.22	0.16	0.47	0.45	0.22	0.16
		10.0	5.6	0.65	0.61	0.25	0.18	0.67	0.62	0.26	0.18	0.64	0.60	0.25	0.18
		−17.8	11.1	0.55	0.53	0.28	0.21	0.49	0.47	0.26	0.20	0.51	0.49	0.27	0.20
45° slope Down ↓		32.2	5.6	0.62	0.58	0.21	0.15	0.89	0.80	0.24	0.16	0.85	0.76	0.24	0.16
		10.0	16.7	0.60	0.57	0.24	0.17	0.63	0.59	0.25	0.18	0.62	0.58	0.25	0.18
		10.0	5.6	0.67	0.63	0.26	0.18	0.90	0.82	0.28	0.19	0.83	0.77	0.28	0.19
		−17.8	11.1	0.66	0.63	0.30	0.22	0.68	0.64	0.31	0.22	0.67	0.64	0.31	0.22
Horizontal Down ↓		32.2	5.6	0.62	0.58	0.21	0.15	1.07	0.94	0.25	0.17	1.77	1.44	0.28	0.18
		10.0	16.7	0.66	0.62	0.25	0.18	1.10	0.99	0.30	0.20	1.69	1.44	0.33	0.21
		10.0	5.6	0.68	0.63	0.26	0.18	1.16	1.04	0.30	0.20	1.96	1.63	0.34	0.22
		−17.8	11.1	0.74	0.70	0.32	0.23	1.24	1.13	0.39	0.26	1.92	1.68	0.43	0.29

(*b*) English units (in h · ft² · °F/Btu)

Position of Air Space	Direction of Heat Flow	Mean Temp., °F	Temp. Diff., °F	0.75-in. Air Space Effective Emissivity, ε_{eff}				1.5-in. Air Space Effective Emissivity, ε_{eff}				3.5-in. Air Space Effective Emissivity, ε_{eff}			
				0.03	0.05	0.5	0.82	0.03	0.05	0.5	0.82	0.03	0.05	0.5	0.82
Horizontal Up ↑		90	10	2.34	2.22	1.04	0.75	2.55	2.41	1.08	0.77	2.84	2.66	1.13	0.80
		50	30	1.71	1.66	0.99	0.77	1.87	1.81	1.04	0.80	2.09	2.01	1.10	0.84
		50	10	2.30	2.21	1.16	0.87	2.50	2.40	1.21	0.89	2.80	2.66	1.28	0.93
		0	20	1.83	1.79	1.16	0.93	2.01	1.95	1.23	0.97	2.25	2.18	1.32	1.03
45° slope Up ↑		90	10	2.96	2.78	1.15	0.81	2.92	2.73	1.14	0.80	3.18	2.96	1.18	0.82
		50	30	1.99	1.92	1.08	0.82	2.14	2.06	1.12	0.84	2.26	2.17	1.15	0.86
		50	10	2.90	2.75	1.29	0.94	2.88	2.74	1.29	0.94	3.12	2.95	1.34	0.96
		0	20	2.13	2.07	1.28	1.00	2.30	2.23	1.34	1.04	2.42	2.35	1.38	1.06
Vertical	Horizontal →	90	10	3.50	3.24	1.22	0.84	3.99	3.66	1.27	0.87	3.69	3.40	1.24	0.85
		50	30	2.91	2.77	1.30	0.94	2.58	2.46	1.23	0.90	2.67	2.55	1.25	0.91
		50	10	3.70	3.46	1.43	1.01	3.79	3.55	1.45	1.02	3.63	3.40	1.42	1.01
		0	20	3.14	3.02	1.58	1.18	2.76	2.66	1.48	1.12	2.88	2.78	1.51	1.14
45° slope Down ↓		90	10	3.53	3.27	1.22	0.84	5.07	4.55	1.36	0.91	4.81	4.33	1.34	0.90
		50	30	3.43	3.23	1.39	0.99	3.58	3.36	1.42	1.00	3.51	3.30	1.40	1.00
		50	10	3.81	3.57	1.45	1.02	5.10	4.66	1.60	1.09	4.74	4.36	1.57	1.08
		0	20	3.75	3.57	1.72	1.26	3.85	3.66	1.74	1.27	3.81	3.63	1.74	1.27
Horizontal Down ↓		90	10	3.55	3.29	1.22	0.85	6.09	5.35	1.43	0.94	10.07	8.19	1.57	1.00
		50	30	3.77	3.52	1.44	1.02	6.27	5.63	1.70	1.14	9.60	8.17	1.88	1.22
		50	10	3.84	3.59	1.45	1.02	6.61	5.90	1.73	1.15	11.15	9.27	1.93	1.24
		0	20	4.18	3.96	1.81	1.30	7.03	6.43	2.19	1.49	10.90	9.52	2.47	1.62

EXAMPLE 3–16   The *R*-Value of a Wood Frame Wall

Determine the overall unit thermal resistance (the *R*-value) and the overall heat transfer coefficient (the *U*-factor) of a wood frame wall that is built around 38-mm × 90-mm (2 × 4 nominal) wood studs with a center-to-center distance of 400 mm. The 90-mm-wide cavity between the studs is filled with glass fiber insulation. The inside is finished with 13-mm gypsum wallboard and the outside with 13-mm wood fiberboard and 13-mm × 200-mm wood bevel lapped siding. The insulated cavity constitutes 75 percent of the heat transmission area while the studs, plates, and sills constitute 21 percent. The headers constitute 4 percent of the area, and they can be treated as studs.

Also, determine the rate of heat loss through the walls of a house whose perimeter is 50 m and wall height is 2.5 m in Las Vegas, Nevada, whose winter design temperature is −2°C. Take the indoor design temperature to be 22°C and assume 20 percent of the wall area is occupied by glazing.

SOLUTION   The *R*-value and the *U*-factor of a wood frame wall as well as the rate of heat loss through such a wall in Las Vegas are to be determined.

Assumptions   **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties   The *R*-values of different materials are given in Table 3–6.

Analysis   The schematic of the wall as well as the different elements used in its construction are shown here. Heat transfer through the insulation and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the insulation and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}}$$

where

$$U_{\text{overall}} = (U \times f_{\text{area}})_{\text{insulation}} + (U \times f_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.75 for the insulation section and 0.25 for the stud section since the headers that constitute a small part of the wall are to be treated as studs. Using the available *R*-values from Table 3–6 and calculating others, the total *R*-values for each section can be determined in a systematic manner in the table in this sample.

We conclude that the overall unit thermal resistance of the wall is 2.23 m² · °C/W, and this value accounts for the effects of the studs and headers. It corresponds to an *R*-value of 2.23 × 5.68 = 12.7 (or nearly *R*-13) in English units. Note that if there were no wood studs and headers in the wall, the overall thermal resistance would be 3.05 m² · °C/W, which is 37 percent greater than 2.23 m² · °C/W. Therefore, the wood studs and headers in this case serve as thermal bridges in wood frame walls, and their effect must be considered in the thermal analysis of buildings.
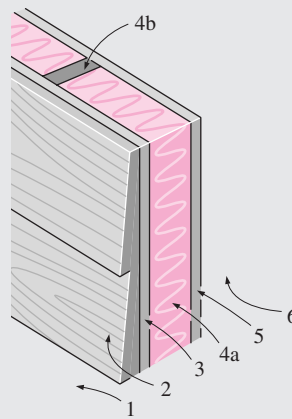
Schematic			R-value, $m^2 \cdot °C/W$	
		Construction	Between Studs	At Studs
		1. Outside surface, 24 km/h wind	0.030	0.030
		2. Wood bevel lapped siding	0.14	0.14
		3. Wood fiberboard sheeting, 13 mm	0.23	0.23
		4a. Glass fiber insulation, 90 mm	2.45	—
		4b. Wood stud, 38 mm × 90 mm	—	0.63
		5. Gypsum wallboard, 13 mm	0.079	0.079
		6. Inside surface, still air	0.12	0.12

Total unit thermal resistance of each section, R (in $m^2 \cdot °C/W$)   3.05   1.23
The U-factor of each section, $U = 1/R$, in $W/m^2 \cdot °C$ 0.328   0.813
Area fraction of each section, f_{area} 0.75   0.25
Overall U-factor: $U = \Sigma f_{area, i} U_i = 0.75 \times 0.328 + 0.25 \times 0.813$
   $= $ **0.449 W/m² · °C**
Overall unit thermal resistance: $R = 1/U = $ **2.23 m² · °C/W**

The perimeter of the building is 50 m and the height of the walls is 2.5 m. Noting that glazing constitutes 20 percent of the walls, the total wall area is

$$A_{wall} = 0.80(\text{Perimeter})(\text{Height}) = 0.80(50 \text{ m})(2.5 \text{ m}) = 100 \text{ m}^2$$

Then the rate of heat loss through the walls under design conditions becomes

$$\dot{Q}_{wall} = (UA)_{wall} (T_i - T_o)$$
$$= (0.449 \text{ W/m}^2 \cdot °C)(100 \text{ m}^2)[22 - (-2)°C]$$
$$= \textbf{1078 W}$$

Discussion   Note that a 1-kW resistance heater in this house will make up almost all the heat lost through the walls, except through the doors and windows, when the outdoor air temperature drops to $-2°C$.

EXAMPLE 3–17   **The *R*-Value of a Wall with Rigid Foam**

The 13-mm-thick wood fiberboard sheathing of the wood stud wall discussed in the previous example is replaced by a 25-mm-thick rigid foam insulation. Determine the percent increase in the R-value of the wall as a result.

SOLUTION The overall R-value of the existing wall was determined in Example 3–16 to be 2.23 $m^2 \cdot$ °C/W. Noting that the R-values of the fiberboard and the foam insulation are 0.23 $m^2 \cdot$ °C/W and 0.98 $m^2 \cdot$ °C/W, respectively, and the added and removed thermal resistances are in series, the overall R-value of the wall after modification becomes

$$R_{\text{new}} = R_{\text{old}} - R_{\text{removed}} + R_{\text{added}}$$
$$= 2.23 - 0.23 + 0.98$$
$$= 2.98 \text{ m}^2 \cdot \text{°C/W}$$

This represents an increase of $(2.98 - 2.23)/2.23 = 0.34$ or **34 percent** in the R-value of the wall. This example demonstrated how to evaluate the new R-value of a structure when some structural members are added or removed.

EXAMPLE 3–18   **The R-Value of a Masonry Wall**

Determine the overall unit thermal resistance (the R-value) and the overall heat transfer coefficient (the U-factor) of a masonry cavity wall that is built around 6-in.-thick concrete blocks made of lightweight aggregate with 3 cores filled with perlite ($R = 4.2$ h $\cdot$ ft$^2 \cdot$ °F/Btu). The outside is finished with 4-in. face brick with $\frac{1}{2}$-in. cement mortar between the bricks and concrete blocks. The inside finish consists of $\frac{1}{2}$ in. gypsum wallboard separated from the concrete block by $\frac{3}{4}$-in.-thick (1-in. $\times$ 3-in. nominal) vertical furring ($R = 4.2$ h $\cdot$ ft$^2 \cdot$ °F/Btu) whose center-to-center distance is 16 in. Both sides of the $\frac{3}{4}$-in.-thick air space between the concrete block and the gypsum board are coated with reflective aluminum foil ($\varepsilon = 0.05$) so that the effective emissivity of the air space is 0.03. For a mean temperature of 50°F and a temperature difference of 30°F, the R-value of the air space is 2.91 h $\cdot$ ft$^2 \cdot$ °F/Btu. The reflective air space constitutes 80 percent of the heat transmission area, while the vertical furring constitutes 20 percent.

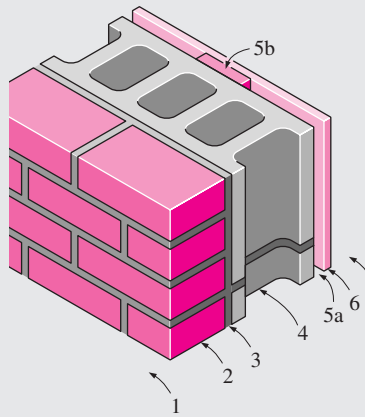**SOLUTION** The R-value and the U-factor of a masonry cavity wall are to be determined.
**Assumptions** **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.
Properties The R-values of different materials are given in Table 3–6.
Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Following the approach described here and using the available R-values from Table 3–6, the overall R-value of the wall is determined in this table.

Schematic		R-value, h · ft^2 · °F/Btu	
	Construction	Between Furring	At Furring
	1. Outside surface, 15 mph wind	0.17	0.17
	2. Face brick, 4 in.	0.43	0.43
	3. Cement mortar, 0.5 in.	0.10	0.10
	4. Concrete block, 6 in.	4.20	4.20
	5a. Reflective air space, $\frac{3}{4}$ in.	2.91	—
	5b. Nominal 1 × 3 vertical furring	—	0.94
	6. Gypsum wallboard, 0.5 in.	0.45	0.45
	7. Inside surface, still air	0.68	0.68

Total unit thermal resistance of each section, R 8.94   6.97
The U-factor of each section, $U = 1/R$, in Btu/h · ft^2 · °F   0.112   0.143
Area fraction of each section, f_{area} 0.80   0.20
Overall U-factor: $U = \Sigma f_{area,\,i} U_i = 0.80 \times 0.112 + 0.20 \times 0.143$
   = **0.118 Btu/h · ft^2 · °F**
Overall unit thermal resistance: $R = 1/U =$ **8.46 h · ft^2 · °F/Btu**

Therefore, the overall unit thermal resistance of the wall is 8.46 h · ft^2 · °F/Btu and the overall U-factor is 0.118 Btu/h · ft^2 · °F. These values account for the effects of the vertical furring.

EXAMPLE 3–19   The *R*-Value of a Pitched Roof

Determine the overall unit thermal resistance (the R-value) and the overall heat transfer coefficient (the U-factor) of a 45° pitched roof built around nominal 2-in. × 4-in. wood studs with a center-to-center distance of 16 in. The 3.5-in.-wide air space between the studs does not have any reflective surface and thus its effective emissivity is 0.84. For a mean temperature of 90°F and a temperature difference of 30°F, the R-value of the air space is 0.86 h · ft^2 · °F/Btu. The lower part of the roof is finished with $\frac{1}{2}$-in. gypsum wallboard and the upper part with $\frac{5}{8}$-in. plywood, building paper, and asphalt shingle roofing. The air space constitutes 75 percent of the heat transmission area, while the studs and headers constitute 25 percent.

SOLUTION The R-value and the U-factor of a 45° pitched roof are to be determined.
Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the roof is one-dimensional. **3** Thermal properties of the roof and the heat transfer coefficients are constant.
Properties The R-values of different materials are given in Table 3–6.
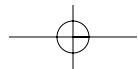
Analysis   The schematic of the pitched roof as well as the different elements used in its construction are shown below. Following the approach described above and using the available *R*-values from Table 3–6, the overall *R*-value of the roof can be determined in the table here.

Schematic
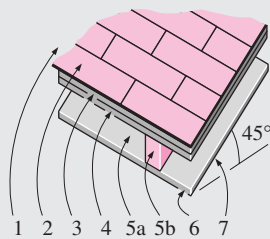
	Construction	*R*-value, h · ft^2 · °F/Btu Between Studs	At Studs
1.	Outside surface, 15 mph wind	0.17	0.17
2.	Asphalt shingle roofing	0.44	0.44
3.	Building paper	0.10	0.10
4.	Plywood deck, $\frac{5}{8}$ in.	0.78	0.78
5a.	Nonreflective air space, 3.5 in.	0.86	—
5b.	Wood stud, 2 in. by 4 in.	—	3.58
6.	Gypsum wallboard, 0.5 in.	0.45	0.45
7.	Inside surface, 45° slope, still air	0.63	0.63

Total unit thermal resistance of each section, *R*   3.43   6.15
The *U*-factor of each section, $U = 1/R$, in Btu/h · ft^2 · °F   0.292   0.163
Area fraction of each section, f_{area} 0.75   0.25
Overall *U*-factor: $U = \Sigma f_{area, i} U_i = 0.75 \times 0.292 + 0.25 \times 0.163$
  = **0.260 Btu/h · ft^2 · °F**
Overall unit thermal resistance: $R = 1/U = $ **3.85 h · ft^2 · °F/Btu**

Therefore, the overall unit thermal resistance of this pitched roof is 3.85 h · ft^2 · °F/Btu and the overall *U*-factor is 0.260 Btu/h · ft^2 · °F. Note that the wood studs offer much larger thermal resistance to heat flow than the air space between the studs.

The construction of wood frame flat ceilings typically involve 2-in. × 6-in. joists on 400-mm (16-in.) or 600-mm (24-in.) centers. The fraction of framing is usually taken to be 0.10 for joists on 400-mm centers and 0.07 for joists on 600-mm centers.

Most buildings have a combination of a ceiling and a roof with an attic space in between, and the determination of the *R*-value of the roof–attic–ceiling combination depends on whether the attic is vented or not. For adequately ventilated attics, the attic air temperature is practically the same as the outdoor air temperature, and thus heat transfer through the roof is governed by the *R*-value of the ceiling only. However, heat is also transferred between the roof and the ceiling by radiation, and it needs to be considered (Fig. 3–52). The major function of the roof in this case is to serve as a radiation shield by blocking off solar radiation. Effectively ventilating the attic in summer should not lead one to believe that heat gain to the building through the attic is greatly reduced. This is because most of the heat transfer through the attic is by radiation.
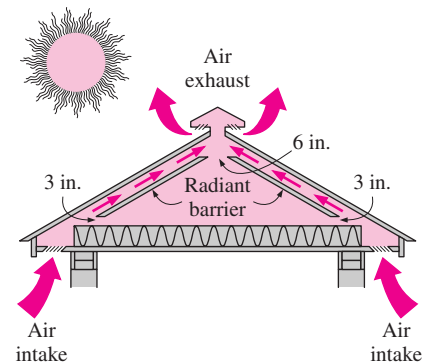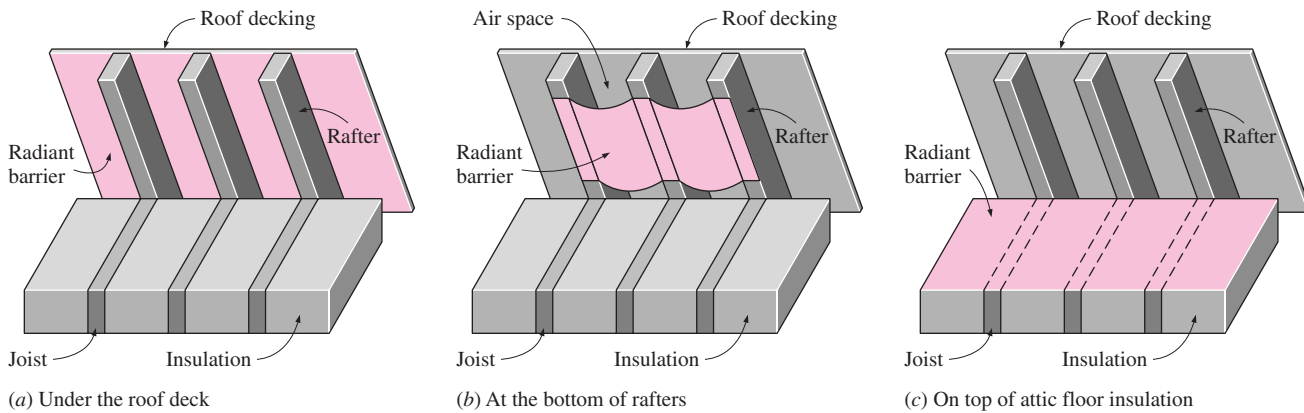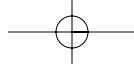
FIGURE 3–52
Ventilation paths for a naturally ventilated attic and the appropriate size of the flow areas around the radiant barrier for proper air circulation (from DOE/CE-0335P, U.S. Dept. of Energy).

Roof decking

Radiant barrier

Rafter

Joist   Insulation

(*a*) Under the roof deck

Air space   Roof decking

Radiant barrier

Rafter

Joist   Insulation

(*b*) At the bottom of rafters

Roof decking

Radiant barrier

Rafter

Joist   Insulation

(*c*) On top of attic floor insulation

FIGURE 3–53

Three possible locations for an attic radiant barrier (from DOE/CE-0335P, U.S. Dept. of Energy).

T_o

Shingles

R_{roof}

A_{roof}

Rafter

$A_{ceiling}$

T_{attic}

Attic

Deck

Ceiling joist

$R_{ceiling}$

T_i

FIGURE 3–54

Thermal resistance network for a pitched roof–attic–ceiling combination for the case of an unvented attic.

Radiation heat transfer between the ceiling and the roof can be minimized by covering at least one side of the attic (the roof or the ceiling side) by a reflective material, called *radiant barrier,* such as aluminum foil or aluminum-coated paper. Tests on houses with R-19 attic floor insulation have shown that radiant barriers can reduce summer ceiling heat gains by 16 to 42 percent compared to an attic with the same insulation level and no radiant barrier. Considering that the ceiling heat gain represents about 15 to 25 percent of the total cooling load of a house, radiant barriers will reduce the air conditioning costs by 2 to 10 percent. Radiant barriers also reduce the heat loss in winter through the ceiling, but tests have shown that the percentage reduction in heat losses is less. As a result, the percentage reduction in heating costs will be less than the reduction in the air-conditioning costs. Also, the values given are for new and undusted radiant barrier installations, and percentages will be lower for aged or dusty radiant barriers.

Some possible locations for attic radiant barriers are given in Figure 3–53. In whole house tests on houses with R-19 attic floor insulation, radiant barriers have reduced the ceiling heat gain by an average of 35 percent when the radiant barrier is installed on the attic floor, and by 24 percent when it is attached to the bottom of roof rafters. Test cell tests also demonstrated that the best location for radiant barriers is the attic floor, provided that the attic is not used as a storage area and is kept clean.

For unvented attics, any heat transfer must occur through (1) the ceiling, (2) the attic space, and (3) the roof (Fig. 3–54). Therefore, the overall R-value of the roof–ceiling combination with an unvented attic depends on the combined effects of the R-value of the ceiling and the R-value of the roof as well as the thermal resistance of the attic space. The attic space can be treated as an air layer in the analysis. But a more practical way of accounting for its effect is to consider surface resistances on the roof and ceiling surfaces facing each other. In this case, the R-values of the ceiling and the roof are first determined separately (by using convection resistances for the still-air case for the attic surfaces). Then it can be shown that the overall R-value of the ceiling–roof combination per unit area of the ceiling can be expressed as
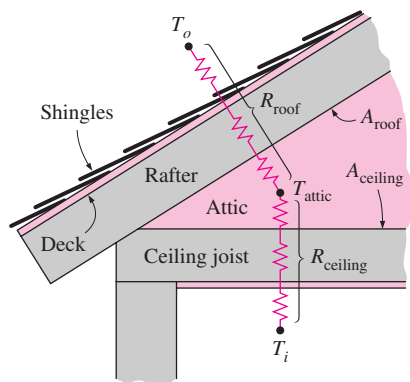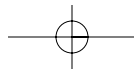
$$R = R_{\text{ceiling}} + R_{\text{roof}} \left(\frac{A_{\text{ceiling}}}{A_{\text{roof}}} \right) \qquad \text{(3-82)}$$

where A_{ceiling} and A_{roof} are the ceiling and roof areas, respectively. The area ratio is equal to 1 for flat roofs and is less than 1 for pitched roofs. For a 45° pitched roof, the area ratio is $A_{\text{ceiling}}/A_{\text{roof}} = 1/\sqrt{2} = 0.707$. Note that the pitched roof has a greater area for heat transfer than the flat ceiling, and the area ratio accounts for the reduction in the unit R-value of the roof when expressed per unit area of the ceiling. Also, the direction of heat flow is up in winter (heat loss through the roof) and down in summer (heat gain through the roof).

The R-value of a structure determined by analysis assumes that the materials used and the quality of workmanship meet the standards. Poor workmanship and substandard materials used during construction may result in R-values that deviate from predicted values. Therefore, some engineers use a safety factor in their designs based on experience in critical applications.

SUMMARY

One-dimensional heat transfer through a simple or composite body exposed to convection from both sides to mediums at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \qquad \text{(W)}$$

where R_{total} is the total thermal resistance between the two mediums. For a plane wall exposed to convection on both sides, the total resistance is expressed as

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

This relation can be extended to plane walls that consist of two or more layers by adding an additional resistance for each additional layer. The elementary thermal resistance relations can be expressed as follows:

Conduction resistance (plane wall): $\quad R_{\text{wall}} = \dfrac{L}{kA}$

Conduction resistance (cylinder): $\quad R_{\text{cyl}} = \dfrac{\ln(r_2/r_1)}{2\pi Lk}$

Conduction resistance (sphere): $\quad R_{\text{sph}} = \dfrac{r_2 - r_1}{4\pi r_1 r_2 k}$

Convection resistance: $\quad R_{\text{conv}} = \dfrac{1}{hA}$

Interface resistance: $\quad R_{\text{interface}} = \dfrac{1}{h_c A} = \dfrac{R_c}{A}$

Radiation resistance: $\quad R_{\text{rad}} = \dfrac{1}{h_{\text{rad}} A}$

where h_c is the thermal contact conductance, R_c is the thermal contact resistance, and the radiation heat transfer coefficient is defined as

$$h_{\text{rad}} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}})$$

Once the rate of heat transfer is available, the *temperature drop* across any layer can be determined from

$$\Delta T = \dot{Q} R$$

The thermal resistance concept can also be used to solve steady heat transfer problems involving parallel layers or combined series-parallel arrangements.

Adding insulation to a cylindrical pipe or a spherical shell will increase the rate of heat transfer if the outer radius of the insulation is less than the *critical radius of insulation,* defined as

$$r_{\text{cr, cylinder}} = \frac{k_{\text{ins}}}{h}$$

$$r_{\text{cr, sphere}} = \frac{2k_{\text{ins}}}{h}$$

The effectiveness of an insulation is often given in terms of its *R-value,* the thermal resistance of the material per unit surface area, expressed as

$$R\text{-value} = \frac{L}{k} \qquad \text{(flat insulation)}$$

where L is the thickness and k is the thermal conductivity of the material.

Finned surfaces are commonly used in practice to enhance heat transfer. Fins enhance heat transfer from a surface by exposing a larger surface area to convection. The temperature distribution along the fin for very long fins and for fins with negligible heat transfer at the fin are given by

Very long fin:
$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-x\sqrt{hp/kA_c}}$$

Adiabatic fin tip:
$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh a(L - x)}{\cosh aL}$$

where $a = \sqrt{hp/kA_c}$, p is the perimeter, and A_c is the cross sectional area of the fin. The rates of heat transfer for both cases are given to be

Very long fin:
$$\dot{Q}_{\text{long fin}} = -kA_c \frac{dT}{dx}\bigg|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$

Adiabatic fin tip:
$$\dot{Q}_{\text{insulated tip}} = -kA_c \frac{dT}{dx}\bigg|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL$$

Fins exposed to convection at their tips can be treated as fins with insulated tips by using the corrected length $L_c = L + A_c/p$ instead of the actual fin length.

The temperature of a fin drops along the fin, and thus the heat transfer from the fin will be less because of the decreasing temperature difference toward the fin tip. To account for the effect of this decrease in temperature on heat transfer, we define *fin efficiency* as

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\substack{\text{Ideal heat transfer rate from the fin if}\\ \text{the entire fin were at base temperature}}}$$

When the fin efficiency is available, the rate of heat transfer from a fin can be determined from

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}}hA_{\text{fin}}\,(T_b - T_\infty)$$

The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case and is expressed in terms of the *fin effectiveness* ε_{fin}, defined as

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\substack{\text{Heat transfer rate from}\\ \text{the fin of } base\ area\ A_b}}{\substack{\text{Heat transfer rate from}\\ \text{the surface of } area\ A_b}}$$

Here, A_b is the cross-sectional area of the fin at the base and $\dot{Q}_{\text{no fin}}$ represents the rate of heat transfer from this area if no fins are attached to the surface. The *overall effectiveness* for a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}}A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}\,(T_b - T_\infty)}$$

Fin efficiency and fin effectiveness are related to each other by

$$\varepsilon_{\text{fin}} = \frac{A_{\text{fin}}}{A_b}\,\eta_{\text{fin}}$$

Certain multidimensional heat transfer problems involve two surfaces maintained at constant temperatures T_1 and T_2. The steady rate of heat transfer between these two surfaces is expressed as

$$\dot{Q} = Sk(T_1 - T_2)$$

where S is the *conduction shape factor* that has the dimension of *length* and k is the thermal conductivity of the medium between the surfaces.

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PROBLEMS*

Steady Heat Conduction in Plane Walls

3–1C   Consider one-dimensional heat conduction through a cylindrical rod of diameter D and length L. What is the heat transfer area of the rod if (a) the lateral surfaces of the rod are insulated and (b) the top and bottom surfaces of the rod are insulated?

3–2C   Consider heat conduction through a plane wall. Does the energy content of the wall change during steady heat conduction? How about during transient conduction? Explain.

3–3C   Consider heat conduction through a wall of thickness L and area A. Under what conditions will the temperature distributions in the wall be a straight line?

3–4C   What does the thermal resistance of a medium represent?

3–5C   How is the combined heat transfer coefficient defined? What convenience does it offer in heat transfer calculations?

3–6C   Can we define the convection resistance per unit surface area as the inverse of the convection heat transfer coefficient?

3–7C   Why are the convection and the radiation resistances at a surface in parallel instead of being in series?

3–8C   Consider a surface of area A at which the convection and radiation heat transfer coefficients are h_{conv} and h_{rad}, respectively. Explain how you would determine (a) the single equivalent heat transfer coefficient, and (b) the equivalent thermal resistance. Assume the medium and the surrounding surfaces are at the same temperature.

3–9C   How does the thermal resistance network associated with a single-layer plane wall differ from the one associated with a five-layer composite wall?

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with an EES-CD icon ❀ are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon 🖳 are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

3–10C   Consider steady one-dimensional heat transfer through a multilayer medium. If the rate of heat transfer $\dot{Q}$ is known, explain how you would determine the temperature drop across each layer.

3–11C   Consider steady one-dimensional heat transfer through a plane wall exposed to convection from both sides to environments at known temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with known heat transfer coefficients h_1 and h_2. Once the rate of heat transfer $\dot{Q}$ has been evaluated, explain how you would determine the temperature of each surface.

3–12C   Someone comments that a microwave oven can be viewed as a conventional oven with zero convection resistance at the surface of the food. Is this an accurate statement?

3–13C   Consider a window glass consisting of two 4-mm-thick glass sheets pressed tightly against each other. Compare the heat transfer rate through this window with that of one consisting of a single 8-mm-thick glass sheet under identical conditions.

3–14C   Consider steady heat transfer through the wall of a room in winter. The convection heat transfer coefficient at the outer surface of the wall is three times that of the inner surface as a result of the winds. On which surface of the wall do you think the temperature will be closer to the surrounding air temperature? Explain.

3–15C   The bottom of a pan is made of a 4-mm-thick aluminum layer. In order to increase the rate of heat transfer through the bottom of the pan, someone proposes a design for the bottom that consists of a 3-mm-thick copper layer sandwiched between two 2-mm-thick aluminum layers. Will the new design conduct heat better? Explain. Assume perfect contact between the layers.

3–16C   Consider two cold canned drinks, one wrapped in a blanket and the other placed on a table in the same room. Which drink will warm up faster?

3–17   Consider a 4-m-high, 6-m-wide, and 0.3-m-thick brick wall whose thermal conductivity is $k = 0.8$ W/m · °C . On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 14°C and 6°C, respectively. Determine the rate of heat loss through the wall on that day.
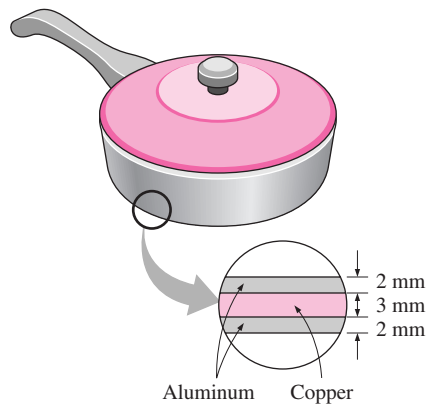
FIGURE P3–15C

3–18 Consider a 1.2-m-high and 2-m-wide glass window whose thickness is 6 mm and thermal conductivity is $k = 0.78$ W/m · °C. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 24°C while the temperature of the outdoors is -5°C. Take the convectio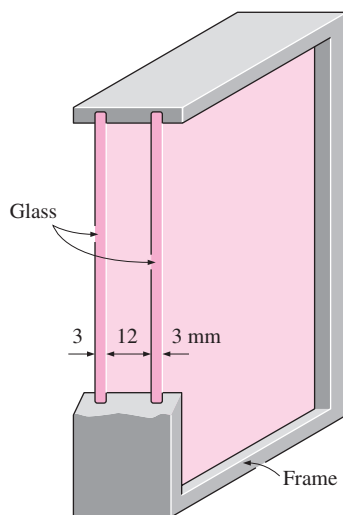n heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2 · °C and $h_2 = 25$ W/m^2 · °C, and disregard any heat transfer by radiation.

3–19 Consider a 1.2-m-high and 2-m-wide double-pane window consisting of two 3-mm-thick layers of glass ($k = 0.78$ W/m · °C) separated by a 12-mm-wide stagnant air space ($k = 0.026$ W/m · °C). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 24°C while the temperature of the outdoors is -5°C. Take the convection heat transfer coefficients on the inner and outer

FIGURE P3–19

surfaces of the window to be $h_1 = 10$ W/m^2 · °C and $h_2 = 25$ W/m^2 · °C, and disregard any heat transfer by radiation.
Answers: 114 W, 19.2°C

3–20 Repeat Problem 3–19, assuming the space between the two glass layers is evacuated.

3–21 [EES] Reconsider Problem 3–19. Using EES (or other) software, plot the rate of heat transfer through the window as a function of the width of air space in the range of 2 mm to 20 mm, assuming pure conduction through the air. Discuss the results.

3–22E Consider an electrically heated brick house ($k = 0.40$ Btu/h · ft · °F) whose walls are 9 ft high and 1 ft thick. Two of the walls of the house are 40 ft long and the others are 30 ft long. The house is maintained at 70°F at all times while the temperature of the outdoors varies. On a certain day, the temperature of the inner surface of the walls is measured to be at 55°F while the average temperature of the outer surface is observed to remain at 45°F during the day for 10 h and at 35°F at night for 14 h. Determine the amount of heat lost from the house that day. Also determine the cost of that heat loss to the homeowner for an electricity price of $0.09/kWh.
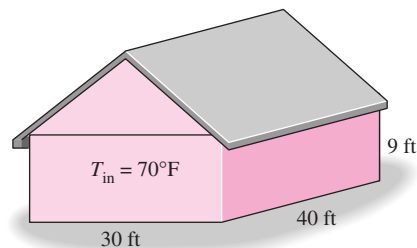
FIGURE P3–22E

3–23 A cylindrical resistor element on a circuit board dissipates 0.15 W of power in an environment at 40°C. The resistor is 1.2 cm long, and has a diameter of 0.3 cm. Assuming heat to be transferred uniformly from all surfaces, determine (a) the amount of heat this resistor dissipates during a 24-h period, (b) the heat flux on the surface of the resistor, in W/m^2, and (c) the surface temperature of the resistor for a combined convection and radiation heat transfer coefficient of 9 W/m^2 · °C.

3–24 Consider a power transistor that dissipates 0.2 W of power in an environment at 30°C. The transistor is 0.4 cm long and has a diameter of 0.5 cm. Assuming heat to be transferred uniformly from all surfaces, determine (a) the amount of heat this resistor dissipates during a 24-h period, in kWh; (b) the heat flux on the surface of the transistor, in W/m^2; and (c) the surface temperature of the resistor for a combined convection and radiation heat transfer coefficient of 18 W/m^2 · °C.

3–25 A 12-cm × 18-cm circuit board houses on its surface 100 closely spaced logic chips, each dissipating 0.07 W in an environment at 40°C. The heat transfer from the back surface of the board is negligible. If the heat transfer coefficient on the

30°C

Power
transistor
0.2 W

0.5 cm

0.4 cm
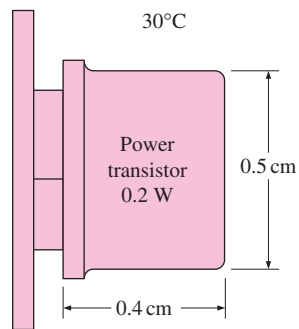
FIGURE P3–24

surface of the board is 10 W/m^2 · °C, determine (*a*) the heat flux on the surface of the circuit board, in W/m^2; (*b*) the surface temperature of the chips; and (*c*) the thermal resistance between the surface of the circuit board and the cooling medium, in °C/W.

3–26 Consider a person standing in a room at 20°C with an exposed surface area of 1.7 m^2. The deep body temperature of the human body is 37°C, and the thermal conductivity of the human tissue near the skin is about 0.3 W/m · °C. The body is losing heat at a rate of 150 W by natural convection and radiation to the surroundings. Taking the body temperature 0.5 cm beneath the skin to be 37°C, determine the skin temperature of the person. *Answer: 35.5° C*

3–27 Water is boiling in a 25-cm-diameter aluminum pan (*k* = 237 W/m · °C) at 95°C. Heat is transferred steadily to the boiling water in the pan through its 0.5-cm-thick flat bottom at a rate of 800 W. If the inner surface temperature of the bottom of the pan is 108°C, determine (*a*) the boiling heat transfer coefficient on the inner surface of the pan, and (*b*) the outer surface temperature of the bottom of the pan.

3–28E A wall is constructed of two layers of 0.5-in-thick sheetrock (*k* = 0.10 Btu/h · ft · °F), which is a plasterboard made of two layers of heavy paper separated by a layer of gypsum, placed 5 in. apart. The space between the sheetrocks

Fiberglass
insulation

Sheetrock

0.5 in.   5 in.   0.5 in.
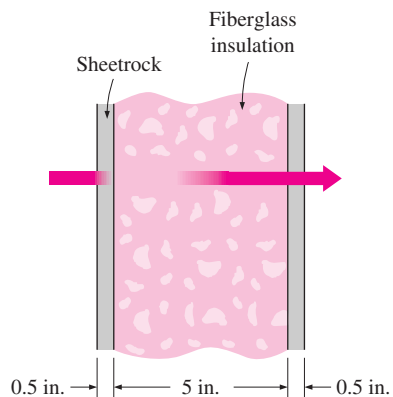
FIGURE P3–28E

is filled with fiberglass insulation (*k* = 0.020 Btu/h · ft · °F). Determine (*a*) the thermal resistance of the wall, and (*b*) its *R*-value of insulation in English units.

3–29 The roof of a house consists of a 3–cm-thick concrete slab (*k* = 2 W/m · °C) that is 15 m wide and 20 m long. The convection heat transfer coefficients on the inner and outer surfaces of the roof are 5 and 12 W/m^2 · °C, respectively. On a clear winter night, the ambient air is reported to be at 10°C, while the night sky temperature is 100 K. The house and the interior surfaces of the wall are maintained at a constant temperature of 20°C. The emissivity of both surfaces of the concrete roof is 0.9. Considering both radiation and convection heat transfers, determine the rate of heat transfer through the roof, and the inner surface temperature of the roof.

If the house is heated by a furnace burning natural gas with an efficiency of 80 percent, and the price of natural gas is $0.60/therm (1 therm = 105,500 kJ of energy content), determine the money lost through the roof that night during a 14-h period.

T_{sky} = 100 K

T_{air} = 10°C

Concrete
roof

15 cm

20 m

15 m

T_{in} = 20°C
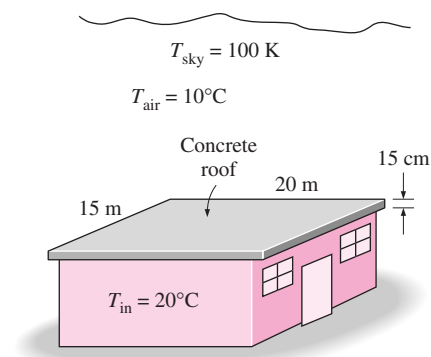
FIGURE P3–29

3–30 A 2-m × 1.5-m section of wall of an industrial furnace burning natural gas is not insulated, and the temperature at the outer surface of this section is measured to be 80°C. The temperature of the furnace room is 30°C, and the combined convection and radiation heat transfer coefficient at the surface of the outer furnace is 10 W/m^2 · °C. It is proposed to insulate this section of the furnace wall with glass wool insulation (*k* = 0.038 W/m · °C) in order to reduce the heat loss by 90 percent. Assuming the outer surface temperature of the metal section still remains at about 80°C, determine the thickness of the insulation that needs to be used.

The furnace operates continuously and has an efficiency of 78 percent. The price of the natural gas is $0.55/therm (1 therm = 105,500 kJ of energy content). If the installation of the insulation will cost $250 for materials and labor, determine how long it will take for the insulation to pay for itself from the energy it saves.

3–31 Repeat Problem. 3–30 for expanded perlite insulation assuming conductivity is *k* = 0.052 W/m · °C.

3–32 Reconsider Problem 3–30. Using EES (or other) software, investigate the effect of thermal conductivity on the required insulation thickness. Plot the thickness of insulation as a function of the thermal conductivity of the insulation in the range of 0.02 W/m · °C to 0.08 W/m · °C, and discuss the results.

3–33E Consider a house whose walls are 12 ft high and 40 ft long. Two of the walls of the house have no windows, while each of the other two walls has four windows made of 0.25-in.-thick glass ($k = 0.45$ Btu/h · ft · °F), 3 ft $\times$ 5 ft in size. The walls are certified to have an R-value of 19 (i.e., an L/k value of 19 h · ft^2 · °F/Btu). Disregarding any direct radiation gain or loss through the windows and taking the heat transfer coefficients at the inner and outer surfaces of the house to be 2 and 4 Btu/h · ft^2 · °F, respectively, determine the ratio of the heat transfer through the walls with and without windows.
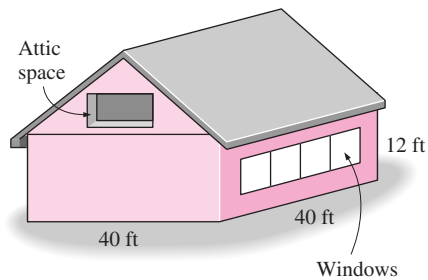
FIGURE P3–33E

3–34 Consider a house that has a 10-m $\times$ 20-m base and a 4-m-high wall. All four walls of the house have an R-value of 2.31 m^2 · °C/W. The two 10-m $\times$ 4-m walls have no windows. The third wall has five windows made of 0.5-cm-thick glass ($k = 0.78$ W/m · °C), 1.2 m $\times$ 1.8 m in size. The fourth wall has the same size and number of windows, but they are double-paned with a 1.5-cm-thick stagnant air space ($k = 0.026$ W/m · °C) enclosed between two 0.5-cm-thick glass layers. The thermostat in the house is set at 22°C and the average temperature outside at that location is 8°C during the seven-month-long heating season. Disregarding any direct radiation gain or loss through the windows and taking the heat transfer coefficients at the inner and outer surfaces of the house to be 7 and 15 W/m^2 · °C, respectively, determine the average rate of heat transfer through each wall.

If the house is electrically heated and the price of electricity is $0.08/kWh, determine the amount of money this household will save per heating season by converting the single-pane windows to double-pane windows.

3–35 The wall of a refrigerator is constructed of fiberglass insulation ($k = 0.035$ W/m · °C) sandwiched between two layers of 1-mm-thick sheet metal ($k = 15.1$ W/m · °C). The refrigerated space is maintained at 3°C, and the average heat transfer coefficients at the inner and outer surfaces of the wall are
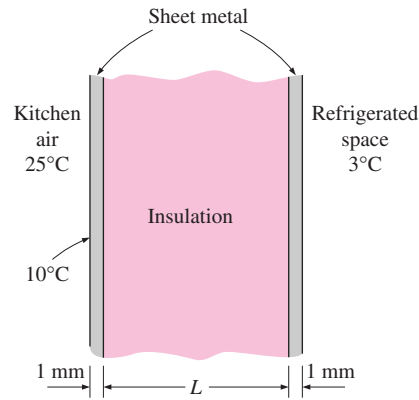
FIGURE P3–35

4 W/m^2 · °C and 9 W/m^2 · °C, respectively. The kitchen temperature averages 25°C. It is observed that condensation occurs on the outer surfaces of the refrigerator when the temperature of the outer surface drops to 20°C. Determine the minimum thickness of fiberglass insulation that needs to be used in the wall in order to avoid condensation on the outer surfaces.

3–36 Reconsider Problem 3–35. Using EES (or other) software, investigate the effects of the thermal conductivities of the insulation material and the sheet metal on the thickness of the insulation. Let the thermal conductivity vary from 0.02 W/m · °C to 0.08 W/m · °C for insulation and 10 W/m · °C to 400 W/m · °C for sheet metal. Plot the thickness of the insulation as the functions of the thermal conductivities of the insulation and the sheet metal, and discuss the results.

3–37 Heat is to be conducted along a circuit board that has a copper layer on one side. The circuit board is 15 cm long and 15 cm wide, and the thicknesses of the copper and epoxy layers are 0.1 mm and 1.2 mm, respectively. Disregarding heat transfer from side surfaces, determine the percentages of heat conduction along the copper ($k = 386$ W/m · °C) and epoxy ($k = 0.26$ W/m · °C) layers. Also determine the effective thermal conductivity of the board.
Answers: 0.8 percent, 99.2 percent, and 29.9 W/m · °C

3–38E A 0.03-in-thick copper plate ($k = 223$ Btu/h · ft · °F) is sandwiched between two 0.1-in.-thick epoxy boards ($k = 0.15$ Btu/h · ft · °F) that are 7 in. $\times$ 9 in. in size. Determine the effective thermal conductivity of the board along its 9-in.-long side. What fraction of the heat conducted along that side is conducted through copper?

Thermal Contact Resistance

3–39C What is thermal contact resistance? How is it related to thermal contact conductance?

3–40C Will the thermal contact resistance be greater for smooth or rough plain surfaces?
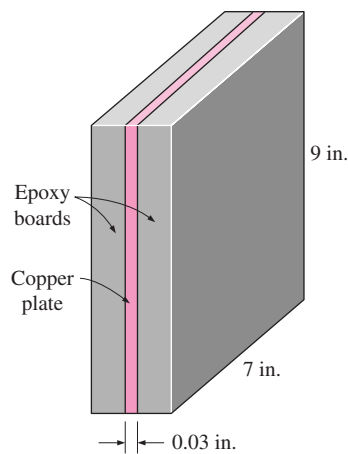
FIGURE P3–38E

3–41C A wall consists of two layers of insulation pressed against each other. Do we need to be concerned about the thermal contact resistance at the interface in a heat transfer analysis or can we just ignore it?

3–42C A plate consists of two thin metal layers pressed against each other. Do we need to be concerned about the thermal contact resistance at the interface in a heat transfer analysis or can we just ignore it?

3–43C Consider two surfaces pressed against each other. Now the air at the interface is evacuated. Will the thermal contact resistance at the interface increase or decrease as a result?

3–44C Explain how the thermal contact resistance can be minimized.

3–45 The thermal contact conductance at the interface of two 1-cm-thick copper plates is measured to be 18,000 W/m^2 · °C. Determine the thickness of the copper plate whose thermal resistance is equal to the thermal resistance of the interface between the plates.

3–46 Six identical power transistors with aluminum casing are attached on one side of a 1.2-cm-thick 20-cm × 30-cm copper plate ($k = 386$ W/m · °C) by screws that exert an average pressure of 10 MPa. The base area of each transistor is 9 cm^2, and each transistor is placed at the center of a 10-cm × 10-cm section of the plate. The interface roughness is estimated to be about 1.4 μm. All transistors are covered by a thick Plexiglas layer, which is a poor conductor of heat, and thus all the heat generated at the junction of the transistor must be dissipated to the ambient at 15°C through the back surface of the copper plate. The combined convection/radiation heat transfer coefficient at the back surface can be taken to be 30 W/m^2 · °C. If the case temperature of the transistor is not to exceed 85°C, determine the maximum power each transistor can dissipate safely, and the temperature jump at the case-plate interface.
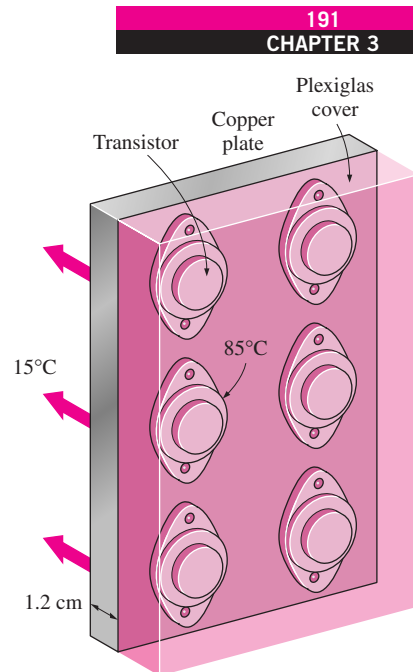
FIGURE P3–46

3–47 Two 5-cm-diameter, 15–cm-long aluminum bars ($k = 176$ W/m · °C) with ground surfaces are pressed against each other with a pressure of 20 atm. The bars are enclosed in an insulation sleeve and, thus, heat transfer from the lateral surfaces is negligible. If the top and bottom surfaces of the two-bar system are maintained at temperatures of 150°C and 20°C, respectively, determine (*a*) the rate of heat transfer along the cylinders under steady conditions and (*b*) the temperature drop at the interface. *Answers: (a)* 142.4 W, *(b)* 6.4°C

3–48 A 1-mm-thick copper plate ($k = 386$ W/m · °C) is sandwiched between two 5-mm-thick epoxy boards ($k = 0.26$ W/m · °C) that are 15 cm × 20 cm in size. If the thermal contact conductance on both sides of the copper plate is estimated to be 6000 W/m · °C, determine the error involved in the total thermal resistance of the plate if the thermal contact conductances are ignored.
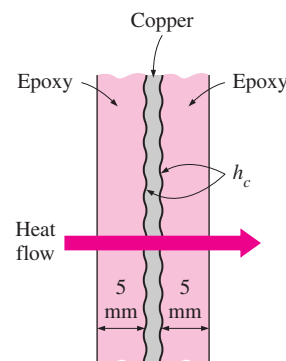
FIGURE P3–48

Generalized Thermal Resistance Networks

3–49C When plotting the thermal resistance network associated with a heat transfer problem, explain when two resistances are in series and when they are in parallel.

3–50C The thermal resistance networks can also be used approximately for multidimensional problems. For what kind of multidimensional problems will the thermal resistance approach give adequate results?

3–51C What are the two approaches used in the development of the thermal resistance network for two-dimensional problems?

3–52 A 4-m-high and 6-m-wide wall consists of a long 18-cm × 30-cm cross section of horizontal bricks ($k = 0.72$ W/m · °C) separated by 3-cm-thick plaster layers ($k = 0.22$ W/m · °C). There are also 2-cm-thick plaster layers on each side of the wall, and a 2-cm-thick rigid foam ($k = 0.026$ W/m · °C) on the inner side of the wall. The indoor and the outdoor temperatures are 22°C and −4°C, and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10$ W/m² · °C and $h_2 = 20$ W/m² · °C, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.
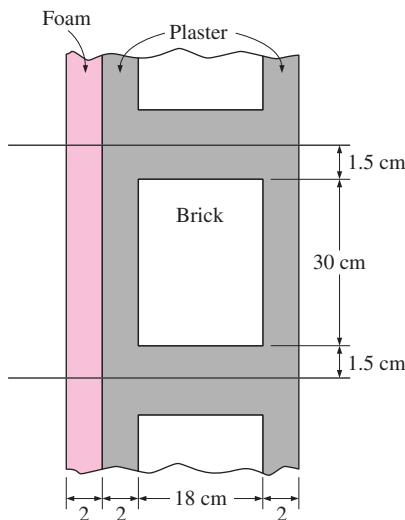
FIGURE P3–52

3–53 Reconsider Problem 3–52. Using EES (or other) software, plot the rate of heat transfer through the wall as a function of the thickness of the rigid foam in the range of 1 cm to 10 cm. Discuss the results.

3–54 A 10-cm-thick wall is to be constructed with 2.5-m-long wood studs ($k = 0.11$ W/m · °C) that have a cross section of 10 cm × 10 cm. At some point the builder ran out of those studs and started using pairs of 2.5-m-long wood studs that have a cross section of 5 cm × 10 cm nailed to each other instead. The manganese steel nails ($k = 50$ W/m · °C) are 10 cm long and have a diameter of 0.4 cm. A total of 50 nails are used to connect the two studs, which are mounted to the wall such that the nails cross the wall. The temperature difference between the inner and outer surfaces of the wall is 8°C. Assuming the thermal contact resistance between the two layers to be negligible, determine the rate of heat transfer (*a*) through a solid stud and (*b*) through a stud pair of equal length and width nailed to each other. (*c*) Also determine the effective conductivity of the nailed stud pair.

3–55 A 12-m-long and 5-m-high wall is constructed of two layers of 1-cm-thick sheetrock ($k = 0.17$ W/m · °C) spaced 12 cm by wood studs ($k = 0.11$ W/m · °C) whose cross section is 12 cm × 5 cm. The studs are placed vertically 60 cm apart, and the space between them is filled with fiberglass insulation ($k = 0.034$ W/m · °C). The house is maintained at 20°C and the ambient temperature outside is −5°C. Taking the heat transfer coefficients at the inner and outer surfaces of the house to be 8.3 and 34 W/m² · °C, respectively, determine (*a*) the thermal resistance of the wall considering a representative section of it and (*b*) the rate of heat transfer through the wall.

3–56E A 10-in.-thick, 30-ft-long, and 10-ft-high wall is to be constructed using 9-in.-long solid bricks ($k = 0.40$ Btu/h · ft · °F) of cross section 7 in. × 7 in., or identical size bricks with nine square air holes ($k = 0.015$ Btu/h · ft · °F) that are 9 in. long and have a cross section of 1.5 in. × 1.5 in. There is a 0.5-in.-thick plaster layer ($k = 0.10$ Btu/h · ft · °F) between two adjacent bricks on all four sides and on both sides of the wall. The house is maintained at 80°F and the ambient temperature outside is 30°F. Taking the heat transfer coefficients at the inner and outer surfaces of the wall to be 1.5 and 4 Btu/h · ft² · °F, respectively, determine the rate of heat transfer through the wall constructed of (*a*) solid bricks and (*b*) bricks with air holes.
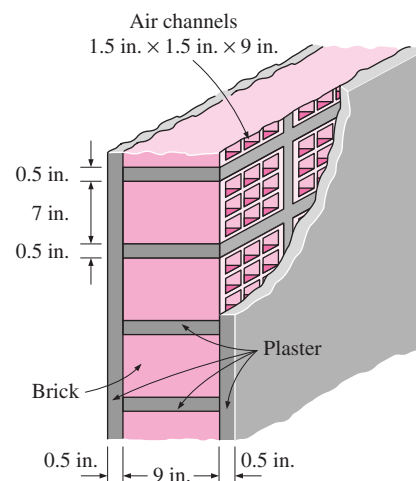

FIGURE P3–56E

3–57 Consider a 5-m-high, 8-m-long, and 0.22-m-thick wall whose representative cross section is as given in the figure. The thermal conductivities of various materials used, in W/m · °C, are $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, and $k_E = 35$. The left and right surfaces of the wall are maintained at uniform temperatures of 300°C and 100°C, respectively. Assuming heat transfer through the wall to be one-dimensional, determine (a) the rate of heat transfer through the wall; (b) the temperature at the point where the sections *B*, *D*, and *E* meet; and (c) the temperature drop across the section *F*. Disregard any contact resistances at the interfaces.
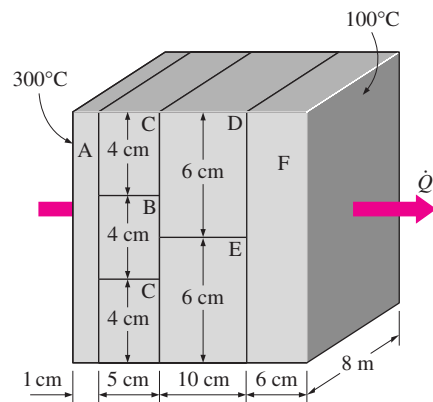
FIGURE P3–57

3–58 Repeat Problem 3–57 assuming that the thermal contact resistance at the interfaces D-F and E-F is 0.00012 m² · °C/W.

3–59 Clothing made of several thin layers of fabric with trapped air in between, often called ski clothing, is commonly used in cold climates because it is light, fashionable, and a very effective thermal insulator. So it is no surprise that such clothing has largely replaced thick and heavy old-fashioned coats.

Consider a jacket made of five layers of 0.1-mm-thick synthetic fabric ($k = 0.13$ W/m · °C) with 1.5-mm-thick air space ($k = 0.026$ W/m · °C) between the layers. Assuming the inner surface temperature of the jacket to be 28°C and the surface area to be 1.1 m², determine the rate of heat loss through the jacket when the temperature of the outdoors is −5°C and the heat transfer coefficient at the outer surface is 25 W/m² · °C.

Multilayered ski jacket
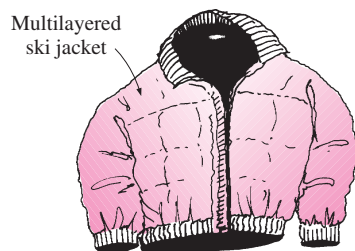
FIGURE P3–59

What would your response be if the jacket is made of a single layer of 0.5-mm-thick synthetic fabric? What should be the thickness of a wool fabric ($k = 0.035$ W/m · °C) if the person is to achieve the same level of thermal comfort wearing a thick wool coat instead of a five-layer ski jacket?

3–60 Repeat Problem 3–59 assuming the layers of the jacket are made of cotton fabric ($k = 0.06$ W/m · °C).

3–61 A 5-m-wide, 4-m-high, and 40-m-long kiln used to cure concrete pipes is made of 20-cm-thick concrete walls and ceiling ($k = 0.9$ W/m · °C). The kiln is maintained at 40°C by injecting hot steam into it. The two ends of the kiln, 4 m × 5 m in size, are made of a 3-mm-thick sheet metal covered with 2-cm-thick Styrofoam ($k = 0.033$ W/m · °C). The convection heat transfer coefficients on the inner and the outer surfaces of the kiln are 3000 W/m² · °C and 25 W/m² · °C, respectively. Disregarding any heat loss through the floor, determine the rate of heat loss from the kiln when the ambient air is at −4°C.
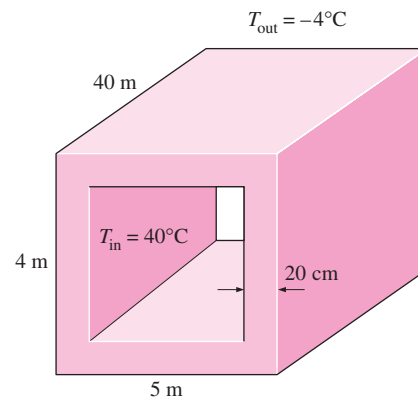
FIGURE P3–61

3–62 Reconsider Problem 3–61. Using EES (or other) software, investigate the effects of the thickness of the wall and the convection heat transfer coefficient on the outer surface of the rate of heat loss from the kiln. Let the thickness vary from 10 cm to 30 cm and the convection heat transfer coefficient from 5 W/m² · °C to 50 W/m² · °C. Plot the rate of heat transfer as functions of wall thickness and the convection heat transfer coefficient, and discuss the results.

3–63E Consider a 6-in. × 8-in. epoxy glass laminate ($k = 0.10$ Btu/h · ft · °F) whose thickness is 0.05 in. In order to reduce the thermal resistance across its thickness, cylindrical copper fillings ($k = 223$ Btu/h · ft · °F) of 0.02 in. diameter are to be planted throughout the board, with a center-to-center distance of 0.06 in. Determine the new value of the thermal resistance of the epoxy board for heat conduction across its thickness as a result of this modification.

Answer: 0.00064 h · °F/Btu

0.02 in.   0.06 in.

Copper filling   Epoxy board

FIGURE P3–63E

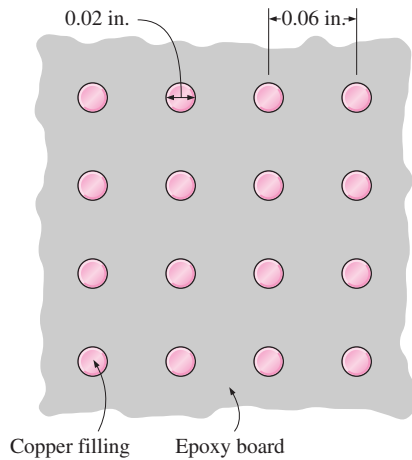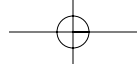## Heat Conduction in Cylinders and Spheres

3–64C   What is an infinitely long cylinder? When is it proper to treat an actual cylinder as being infinitely long, and when is it not?

3–65C   Consider a short cylinder whose top and bottom surfaces are insulated. The cylinder is initially at a uniform temperature T_i and is subjected to convection from its side surface to a medium at temperature T_∞, with a heat transfer coefficient of h. Is the heat transfer in this short cylinder one- or two-dimensional? Explain.

3–66C   Can the thermal resistance concept be used for a solid cylinder or sphere in steady operation? Explain.

3–67   A 5-m-internal-diameter spherical tank made of 1.5-cm-thick stainless steel ($k = 15$ W/m · °C) is used to store iced water at 0°C. The tank is located in a room whose temperature is 30°C. The walls of the room are also at 30°C. The outer surface of the tank is black (emissivity $\varepsilon = 1$), and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are 80 W/m² · °C and 10 W/m² · °C, respectively. Determine (a) the rate of heat transfer to the iced water in the tank and (b) the amount of ice at 0°C that melts during a 24-h period. The heat of fusion of water at atmospheric pressure is $h_{if} = 333.7$ kJ/kg.
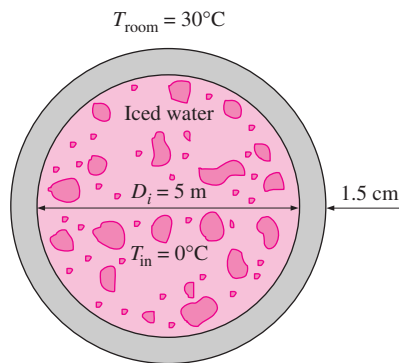
3–68   Steam at 320°C flows in a stainless steel pipe ($k = 15$ W/m · °C) whose inner and outer diameters are 5 cm and 5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation ($k = 0.038$ W/m · °C). Heat is lost to the surroundings at 5°C by natural convection and radiation, with a combined natural convection and radiation heat transfer coefficient of 15 W/m² · °C. Taking the heat transfer coefficient inside the pipe to be 80 W/m² · °C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

3–69 [EES] Reconsider Problem 3–68. Using EES (or other) software, investigate the effect of the thickness of the insulation on the rate of heat loss from the steam and the temperature drop across the insulation layer. Let the insulation thickness vary from 1 cm to 10 cm. Plot the rate of heat loss and the temperature drop as a function of insulation thickness, and discuss the results.

3–70   A 50-m-long section of a steam pipe whose outer diameter is 10 cm passes through an open space at 15°C. The average temperature of the outer surface of the pipe is measured to be 150°C. If the combined heat transfer coefficient on the outer surface of the pipe is 20 W/m² · °C, determine (a) the rate of heat loss from the steam pipe, (b) the annual cost of this energy lost if steam is generated in a natural gas furnace that has an efficiency of 75 percent and the price of natural gas is $0.52/therm (1 therm = 105,500 kJ), and (c) the thickness of fiberglass insulation ($k = 0.035$ W/m · °C) needed in order to save 90 percent of the heat lost. Assume the pipe temperature to remain constant at 150°C.

$T_{\text{air}} = 15°C$

150°C

Steam
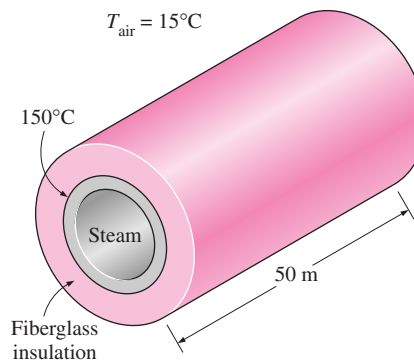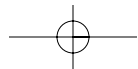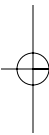
50 m

Fiberglass insulation

FIGURE P3–70

3–71   Consider a 2-m-high electric hot water heater that has a diameter of 40 cm and maintains the hot water at 55°C. The tank is located in a small room whose average temperature is
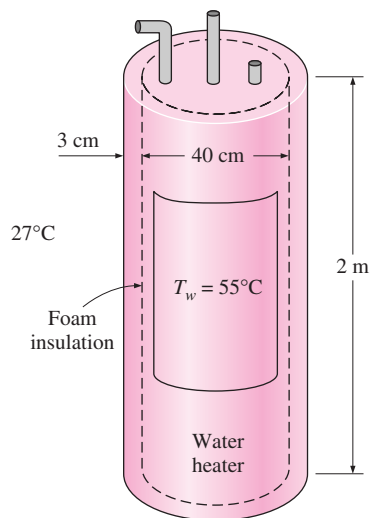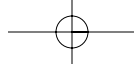
$T_{\text{room}} = 30°C$

Iced water

$D_i = 5$ m

1.5 cm

$T_{\text{in}} = 0°C$

FIGURE P3–67

FIGURE P3–71

27°C, and the heat transfer coefficients on the inner and outer surfaces of the heater are 50 and 12 W/m² · °C, respectively. The tank is placed in another 46-cm-diameter sheet metal tank of negligible thickness, and the space between the two tanks is filled with foam insulation ($k = 0.03$ W/m · °C). The thermal resistances of the water tank and the outer thin sheet metal shell are very small and can be neglected. The price of electricity is $0.08/kWh, and the home owner pays $280 a year for water heating. Determine the fraction of the hot water energy cost of this household that is due to the heat loss from the tank.

Hot water tank insulation kits consisting of 3-cm-thick fiberglass insulation ($k = 0.035$ W/m · °C) large enough to wrap the entire tank are available in the market for about $30. If such an insulation is installed on this water tank by the home owner himself, how long will it take for this additional insulation to pay for itself? *Answers:* 17.5 percent, 1.5 years

3–72 ⟦EES⟧ Reconsider Problem 3–71. Using EES (or other) software, plot the fraction of energy cost of hot water due to the heat loss from the tank as a function of the hot water temperature in the range of 40°C to 90°C. Discuss the results.

3–73 Consider a cold aluminum canned drink that is initially at a uniform temperature of 3°C. The can is 12.5 cm high and has a diameter of 6 cm. If the combined convection/radiation heat transfer coefficient between the can and the surrounding air at 25°C is 10 W/m² · °C, determine how long it will take for the average temperature of the drink to rise to 10°C.

In an effort to slow down the warming of the cold drink, a person puts the can in a perfectly fitting 1-cm-thick cylindrical rubber insulation ($k = 0.13$ W/m · °C). Now how long will it take for the average temperature of the drink to rise to 10°C? Assume the top of the can is not covered.
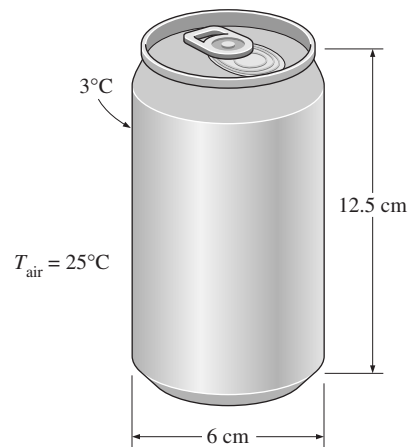

FIGURE P3–73

3–74 Repeat Problem 3–73, assuming a thermal contact resistance of 0.00008 m² · °C/W between the can and the insulation.

3–75E Steam at 450°F is flowing through a steel pipe ($k = 8.7$ Btu/h · ft · °F) whose inner and outer diameters are 3.5 in. and 4.0 in., respectively, in an environment at 55°F. The pipe is insulated with 2-in.-thick fiberglass insulation ($k = 0.020$ Btu/h · ft · °F). If the heat transfer coefficients on the inside and the outside of the pipe are 30 and 5 Btu/h · ft² · °F, respectively, determine the rate of heat loss from the steam per foot length of the pipe. What is the error involved in neglecting the thermal resistance of the steel pipe in calculations?
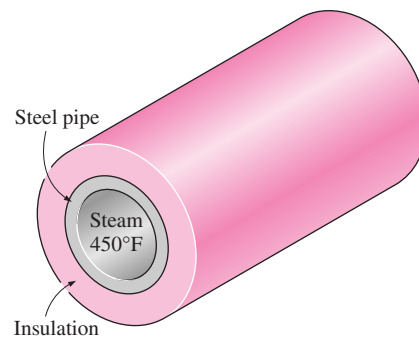

FIGURE P3–75E

3–76 Hot water at an average temperature of 90°C is flowing through a 15-m section of a cast iron pipe ($k = 52$ W/m · °C) whose inner and outer diameters are 4 cm and 4.6 cm, respectively. The outer surface of the pipe, whose emissivity is 0.7, is exposed to the cold air at 10°C in the basement, with a heat transfer coefficient of 15 W/m² · °C. The heat transfer coefficient at the inner surface of the pipe is 120 W/m² · °C. Taking the walls of the basement to be at 10°C also, determine the rate of heat loss from the hot water. Also, determine the average

velocity of the water in the pipe if the temperature of the water drops by 3°C as it passes through the basement.

3–77 Repeat Problem 3–76 for a pipe made of copper ($k = 386$ W/m · °C) instead of cast iron.

3–78E Steam exiting the turbine of a steam power plant at 100°F is to be condensed in a large condenser by cooling water flowing through copper pipes ($k = 223$ Btu/h · ft · °F) of inner diameter 0.4 in. and outer diameter 0.6 in. at an average temperature of 70°F. The heat of vaporization of water at 100°F is 1037 Btu/lbm. The heat transfer coefficients are 1500 Btu/h · ft^2 · °F on the steam side and 35 Btu/h · ft^2 · °F on the water side. Determine the length of the tube required to condense steam at a rate of 120 lbm/h. *Answer:* 1148 ft

Steam, 100°F
120 lbm/h

Cooling water

Liquid water
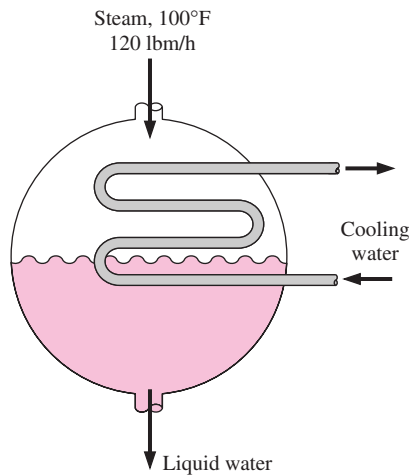
FIGURE P3–78E

3–79E Repeat Problem 3–78E, assuming that a 0.01-in.-thick layer of mineral deposit ($k = 0.5$ Btu/h · ft · °F) has formed on the inner surface of the pipe.

3–80 Reconsider Problem 3–78E. Using EES (or other) software, investigate the effects of the thermal conductivity of the pipe material and the outer diameter of the pipe on the length of the tube required. Let the thermal conductivity vary from 10 Btu/h · ft · °F to 400 Btu/h · ft · °F and the outer diameter from 0.5 in. to 1.0 in. Plot the length of the tube as functions of pipe conductivity and the outer pipe diameter, and discuss the results.

3–81 The boiling temperature of nitrogen at atmospheric pressure at sea level (1 atm pressure) is −196°C. Therefore, nitrogen is commonly used in low-temperature scientific studies since the temperature of liquid nitrogen in a tank open to the atmosphere will remain constant at −196°C until it is depleted. Any heat transfer to the tank will result in the evaporation of some liquid nitrogen, which has a heat of vaporization of 198 kJ/kg and a density of 810 kg/m^3 at 1 atm.
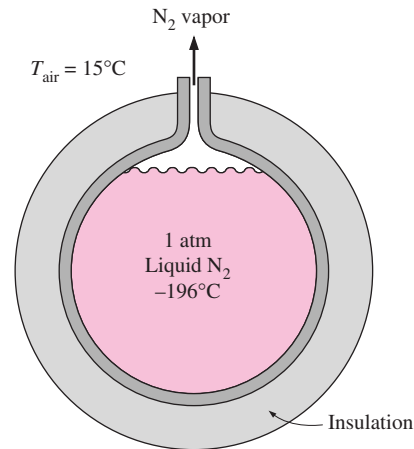
$T_{air} = 15$°C

N$_2$ vapor

1 atm
Liquid N$_2$
−196°C

Insulation

FIGURE P3–81

Consider a 3-m-diameter spherical tank that is initially filled with liquid nitrogen at 1 atm and −196°C. The tank is exposed to ambient air at 15°C, with a combined convection and radiation heat transfer coefficient of 35 W/m^2 · °C. The temperature of the thin-shelled spherical tank is observed to be almost the same as the temperature of the nitrogen inside. Determine the rate of evaporation of the liquid nitrogen in the tank as a result of the heat transfer from the ambient air if the tank is (*a*) not insulated, (*b*) insulated with 5-cm-thick fiberglass insulation ($k = 0.035$ W/m · °C), and (*c*) insulated with 2-cm-thick superinsulation which has an effective thermal conductivity of 0.00005 W/m · °C.

3–82 Repeat Problem 3–81 for liquid oxygen, which has a boiling temperature of −183°C, a heat of vaporization of 213 kJ/kg, and a density of 1140 kg/m^3 at 1 atm pressure.

Critical Radius of Insulation

3–83C What is the critical radius of insulation? How is it defined for a cylindrical layer?

3–84C A pipe is insulated such that the outer radius of the insulation is less than the critical radius. Now the insulation is taken off. Will the rate of heat transfer from the pipe increase or decrease for the same pipe surface temperature?

3–85C A pipe is insulated to reduce the heat loss from it. However, measurements indicate that the rate of heat loss has increased instead of decreasing. Can the measurements be right?

3–86C Consider a pipe at a constant temperature whose radius is greater than the critical radius of insulation. Someone claims that the rate of heat loss from the pipe has increased when some insulation is added to the pipe. Is this claim valid?

3–87C Consider an insulated pipe exposed to the atmosphere. Will the critical radius of insulation be greater on calm days or on windy days? Why?

3–88 A 2-mm-diameter and 10-m-long electric wire is tightly wrapped with a 1-mm-thick plastic cover whose thermal conductivity is $k = 0.15$ W/m · °C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30$°C with a heat transfer coefficient of $h = 24$ W/m^2 · °C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine if doubling the thickness of the plastic cover will increase or decrease this interface temperature.

$T_{air} = 30$°C

Electrical wire

Insulation

10 m
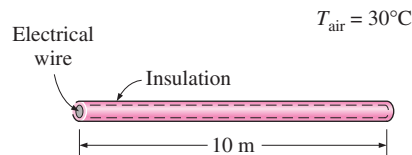
FIGURE P3–88

3–89E A 0.083-in.-diameter electrical wire at 115°F is covered by 0.02-in.-thick plastic insulation ($k = 0.075$ Btu/h · ft · °F). The wire is exposed to a medium at 50°F, with a combined convection and radiation heat transfer coefficient of 2.5 Btu/h · ft^2 · °F. Determine if the plastic insulation on the wire will increase or decrease heat transfer from the wire.
Answer: It helps

3–90E Repeat Problem 3–89E, assuming a thermal contact resistance of 0.001 h · ft^2 · °F/Btu at the interface of the wire and the insulation.

3–91 A 5-mm-diameter spherical ball at 50°C is covered by a 1-mm-thick plastic insulation ($k = 0.13$ W/m · °C). The ball is exposed to a medium at 15°C, with a combined convection and radiation heat transfer coefficient of 20 W/m^2 · °C. Determine if the plastic insulation on the ball will help or hurt heat transfer from the ball.

Plastic insulation

5 mm   1 mm
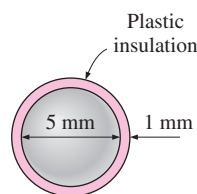
FIGURE P3–91

3–92 Reconsider Problem 3–91. Using EES (or other) software, plot the rate of heat transfer from the ball as a function of the plastic insulation thickness in the range of 0.5 mm to 20 mm. Discuss the results.

Heat Transfer from Finned Surfaces

3–93C What is the reason for the widespread use of fins on surfaces?

3–94C What is the difference between the fin effectiveness and the fin efficiency?

3–95C The fins attached to a surface are determined to have an effectiveness of 0.9. Do you think the rate of heat transfer from the surface has increased or decreased as a result of the addition of these fins?

3–96C Explain how the fins enhance heat transfer from a surface. Also, explain how the addition of fins may actually decrease heat transfer from a surface.

3–97C How does the overall effectiveness of a finned surface differ from the effectiveness of a single fin?

3–98C Hot water is to be cooled as it flows through the tubes exposed to atmospheric air. Fins are to be attached in order to enhance heat transfer. Would you recommend attaching the fins inside or outside the tubes? Why?

3–99C Hot air is to be cooled as it is forced to flow through the tubes exposed to atmospheric air. Fins are to be added in order to enhance heat transfer. Would you recommend attaching the fins inside or outside the tubes? Why? When would you recommend attaching fins both inside and outside the tubes?

3–100C Consider two finned surfaces that are identical except that the fins on the first surface are formed by casting or extrusion, whereas they are attached to the second surface afterwards by welding or tight fitting. For which case do you think the fins will provide greater enhancement in heat transfer? Explain.

3–101C The heat transfer surface area of a fin is equal to the sum of all surfaces of the fin exposed to the surrounding medium, including the surface area of the fin tip. Under what conditions can we neglect heat transfer from the fin tip?

3–102C Does the (*a*) efficiency and (*b*) effectiveness of a fin increase or decrease as the fin length is increased?

3–103C Two pin fins are identical, except that the diameter of one of them is twice the diameter of the other. For which fin will the (*a*) fin effectiveness and (*b*) fin efficiency be higher? Explain.

3–104C Two plate fins of constant rectangular cross section are identical, except that the thickness of one of them is twice the thickness of the other. For which fin will the (*a*) fin effectiveness and (*b*) fin efficiency be higher? Explain.

3–105C Two finned surfaces are identical, except that the convection heat transfer coefficient of one of them is twice that of the other. For which finned surface will the (*a*) fin effectiveness and (*b*) fin efficiency be higher? Explain.

3–106 Obtain a relation for the fin efficiency for a fin of constant cross-sectional area A_c, perimeter p, length L, and thermal conductivity k exposed to convection to a medium at T_∞ with a heat transfer coefficient h. Assume the fins are sufficiently long so that the temperature of the fin at the tip is nearly T_∞. Take the temperature of the fin at the base to be T_b and neglect heat
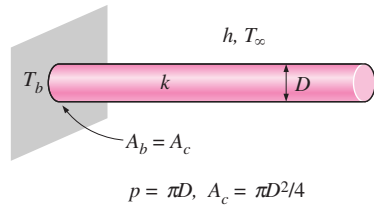
$p = \pi D, \ A_c = \pi D^2/4$

FIGURE P3–106

transfer from the fin tips. Simplify the relation for (*a*) a circular fin of diameter *D* and (*b*) rectangular fins of thickness *t*.

3–107 The case-to-ambient thermal resistance of a power transistor that has a maximum power rating of 15 W is given to be 25°C/W. If the case temperature of the transistor is not to exceed 80°C, determine the power at which this transistor can be operated safely in an environment at 40°C.

3–108 A 40-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 3–4. Select a heat sink that will allow the case temperature of the transistor not to exceed 90° in the ambient air at 20°.
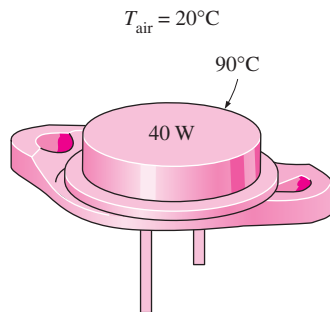
FIGURE P3–108

3–109 A 30-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 3–4. Select a heat sink that will allow the case temperature of the transistor not to exceed 80°C in the ambient air at 35°C.

3–110 Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180°C. Circular aluminum alloy 2024-T6 fins ($k = 186$ W/m · °C) of outer diameter 6 cm and constant thickness 1 mm are attached to the tube. The space between the fins is 3 mm, and thus there are 250 fins per meter length of the tube. Heat is transferred to the surrounding air at $T_\infty = 25$°C, with a heat transfer coefficient of 40 W/m² · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.   *Answer:* 2639 W

3–111E Consider a stainless steel spoon ($k = 8.7$ Btu/h · ft · °F) partially immersed in boiling water at 200°F in a kitchen at 75°F. The handle of the spoon has a cross section of 0.08 in. × 0.5 in., and extends 7 in. in the air from the free
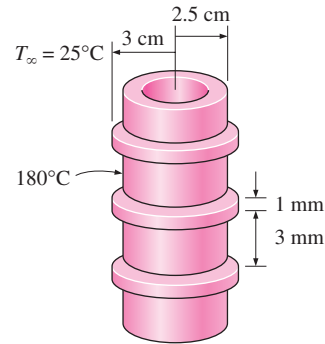
FIGURE P3–110

surface of the water. If the heat transfer coefficient at the exposed surfaces of the spoon handle is 3 Btu/h · ft² · °F, determine the temperature difference across the exposed surface of the spoon handle. State your assumptions.   *Answer:* 124.6°F

FIGURE P3–111E
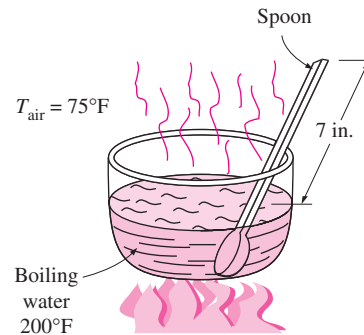
3–112E Repeat Problem 3–111 for a silver spoon ($k = 247$ Btu/h · ft · °F).

3–113E Reconsider Problem 3–111E. Using EES (or other) software, investigate the effects of the thermal conductivity of the spoon material and the length of its extension in the air on the temperature difference across the exposed surface of the spoon handle. Let the thermal conductivity vary from 5 Btu/h · ft · °F to 225 Btu/h · ft · °F and the length from 5 in. to 12 in. Plot the temperature difference as the functions of thermal conductivity and length, and discuss the results.

3–114 A 0.3-cm-thick, 12-cm-high, and 18-cm-long circuit board houses 80 closely spaced logic chips on one side, each dissipating 0.04 W. The board is impregnated with copper fillings and has an effective thermal conductivity of 20 W/m · °C. All the heat generated in the chips is conducted across the circuit board and is dissipated from the back side of the board to a medium at 40°C, with a heat transfer coefficient of 50 W/m² · °C. (*a*) Determine the temperatures on the two sides of the circuit board. (*b*) Now a 0.2-cm-thick, 12-cm-high, and
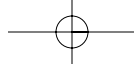
18-cm-long aluminum plate ($k = 237$ W/m · °C) with 864 2-cm-long aluminum pin fins of diameter 0.25 cm is attached to the back side of the circuit board with a 0.02-cm-thick epoxy adhesive ($k = 1.8$ W/m · °C). Determine the new temperatures on the two sides of the circuit board.

3–115 Repeat Problem 3–114 using a copper plate with copper fins ($k = 386$ W/m · °C) instead of aluminum ones.

3–116 A hot surface at 100°C is to be cooled by attaching 3-cm-long, 0.25-cm-diameter aluminum pin fins ($k = 237$ W/m · °C) to it, with a center-to-center distance of 0.6 cm. The temperature of the surrounding medium is 30°C, and the heat transfer coefficient on the surfaces is 35 W/m^2 · °C. Determine the rate of heat transfer from the surface for a 1-m × 1-m section of the plate. Also determine the overall effectiveness of the fins.
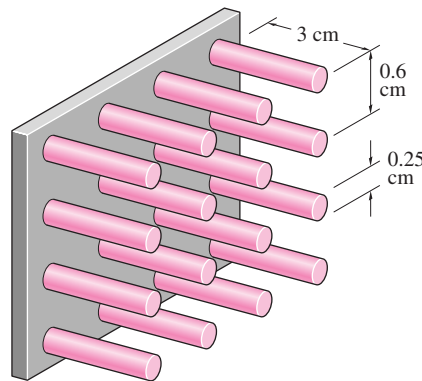
FIGURE P3–116

3–117 Repeat Problem 3–116 using copper fins ($k = 386$ W/m · °C) instead of aluminum ones.

3–118 Reconsider Problem 3–116. Using EES (or other) software, investigate the effect of the center-to-center distance of the fins on the rate of heat transfer from the surface and the overall effectiveness of the fins. Let the center-to-center distance vary from 0.4 cm to 2.0 cm. Plot the rate of heat transfer and the overall effectiveness as a function of the center-to-center distance, and discuss the results.

3–119 Two 3-m-long and 0.4-cm-thick cast iron ($k = 52$ W/m · °C) steam pipes of outer diameter 10 cm are connected to each other through two 1-cm-thick flanges of outer diameter 20 cm. The steam flows inside the pipe at an average temperature of 200°C with a heat transfer coefficient of 180 W/m^2 · °C. The outer surface of the pipe is exposed to an ambient at 12°C, with a heat transfer coefficient of 25 W/m^2 · °C. (*a*) Disregarding the flanges, determine the average outer surface temperature of the pipe. (*b*) Using this temperature for the base of the flange and treating the flanges as the fins, determine the fin efficiency and the rate of heat transfer from the flanges. (*c*) What length of pipe is the flange section equivalent to for heat transfer purposes?
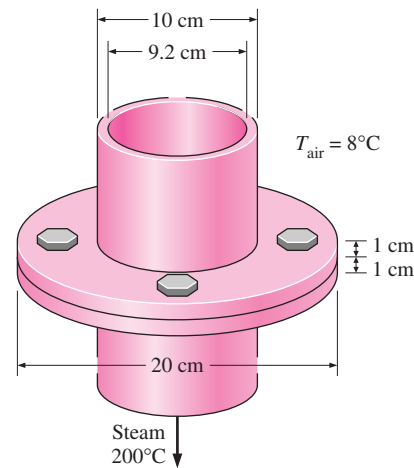

FIGURE P3–119

Heat Transfer in Common Configurations

3-120C What is a conduction shape factor? How is it related to the thermal resistance?

3-121C What is the value of conduction shape factors in engineering?

3-122 A 20-m-long and 8-cm-diameter hot water pipe of a district heating system is buried in the soil 80 cm below the ground surface. The outer surface temperature of the pipe is 60°C. Taking the surface temperature of the earth to be 5°C and the thermal conductivity of the soil at that location to be 0.9 W/m · °C, determine the rate of heat loss from the pipe.
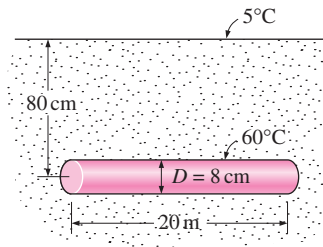

FIGURE P3–122

3–123 Reconsider Problem 3–122. Using EES (or other) software, plot the rate of heat loss from the pipe as a function of the burial depth in the range of 20 cm to 2.0 m. Discuss the results.

3-124 Hot and cold water pipes 8 m long run parallel to each other in a thick concrete layer. The diameters of both pipes are 5 cm, and the distance between the centerlines of the pipes is 40 cm. The surface temperatures of the hot and cold pipes are 60°C and 15°C, respectively. Taking the thermal conductivity of the concrete to be $k = 0.75$ W/m · °C, determine the rate of heat transfer between the pipes. *Answer:* 306 W

3–125 Reconsider Problem 3–124. Using EES (or other) software, plot the rate of heat transfer between the pipes as a function of the distance between the centerlines of the pipes in the range of 10 cm to 1.0 m. Discuss the results.

3-126E A row of 3-ft-long and 1-in.-diameter used uranium fuel rods that are still radioactive are buried in the ground parallel to each other with a center-to-center distance of 8 in. at a depth 15 ft from the ground surface at a location where the thermal conductivity of the soil is 0.6 Btu/h · ft · °F. If the surface temperature of the rods and the ground are 350°F and 60°F, respectively, determine the rate of heat transfer from the fuel rods to the atmosphere through the soil.
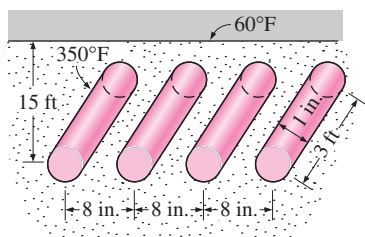
FIGURE P3–126

3-127 Hot water at an average temperature of 60°C and an average velocity of 0.6 m/s is flowing through a 5-m section of a thin-walled hot water pipe that has an outer diameter of 2.5 cm. The pipe passes through the center of a 14-cm-thick wall filled with fiberglass insulation ($k = 0.035$ W/m · °C). If the surfaces of the wall are at 18°C, determine (*a*) the rate of heat transfer from the pipe to the air in the rooms and (*b*) the temperature drop of the hot water as it flows through this 5-m-long section of the wall. *Answers: 23.5 W, 0.02°C*
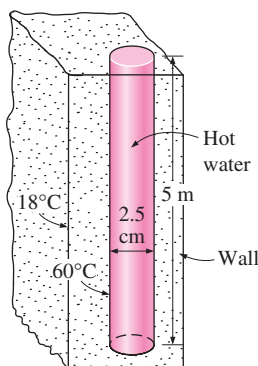

FIGURE P3–127

3-128 Hot water at an average temperature of 80°C and an average velocity of 1.5 m/s is flowing through a 25-m section of a pipe that has an outer diameter of 5 cm. The pipe extends 2 m in the ambient air above the ground, dips into the ground
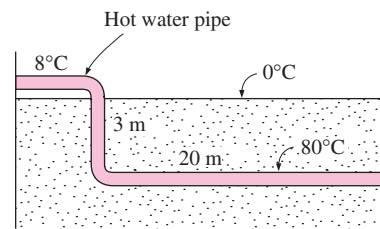

FIGURE P3–128

($k = 1.5$ W/m · °C) vertically for 3 m, and continues horizontally at this depth for 20 m more before it enters the next building. The first section of the pipe is exposed to the ambient air at 8°C, with a heat transfer coefficient of 22 W/m² · °C. If the surface of the ground is covered with snow at 0°C, determine (*a*) the total rate of heat loss from the hot water and (*b*) the temperature drop of the hot water as it flows through this 25-m-long section of the pipe.

3-129 Consider a house with a flat roof whose outer dimensions are 12 m × 12 m. The outer walls of the house are 6 m high. The walls and the roof of the house are made of 20-cm-thick concrete ($k = 0.75$ W/m · °C). The temperatures of the inner and outer surfaces of the house are 15°C and 3°C, respectively. Accounting for the effects of the edges of adjoining surfaces, determine the rate of heat loss from the house through its walls and the roof. What is the error involved in ignoring the effects of the edges and corners and treating the roof as a 12 m × 12 m surface and the walls as 6 m × 12 m surfaces for simplicity?

3-130 Consider a 10-m-long thick-walled concrete duct ($k = 0.75$ W/m · °C) of square cross-section. The outer dimensions of the duct are 20 cm × 20 cm, and the thickness of the duct wall is 2 cm. If the inner and outer surfaces of the duct are at 100°C and 15°C, respectively, determine the rate of heat transfer through the walls of the duct. *Answer: 22.9 kW*
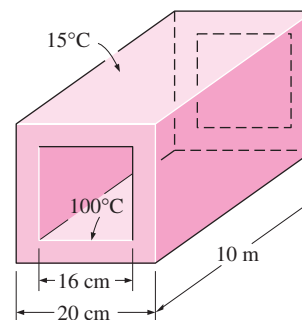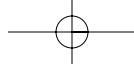

FIGURE P3–130

3-131 A 3-m-diameter spherical tank containing some radioactive material is buried in the ground ($k = 1.4$ W/m · °C). The distance between the top surface of the tank and the ground surface is 4 m. If the surface temperatures of the tank and the

ground are 140°C and 15°C, respectively, determine the rate of heat transfer from the tank.

3–132 [EES] Reconsider Problem 3–131. Using EES (or other) software, plot the rate of heat transfer from the tank as a function of the tank diameter in the range of 0.5 m to 5.0 m. Discuss the results.

3-133 Hot water at an average temperature of 85°C passes through a row of eight parallel pipes that are 4 m long and have an outer diameter of 3 cm, located vertically in the middle of a concrete wall ($k = 0.75$ W/m · °C) that is 4 m high, 8 m long, and 15 cm thick. If the surfaces of the concrete walls are exposed to a medium at 32°C, with a heat transfer coefficient of 12 W/m^2 · °C, determine the rate of heat loss from the hot water and the surface temperature of the wall.

Special Topics:
Heat Transfer through the Walls and Roofs

3–134C What is the R-value of a wall? How does it differ from the unit thermal resistance of the wall? How is it related to the U-factor?

3–135C What is effective emissivity for a plane-parallel air space? How is it determined? How is radiation heat transfer through the air space determined when the effective emissivity is known?

3–136C The unit thermal resistances (R-values) of both 40-mm and 90-mm vertical air spaces are given in Table 3–9 to be 0.22 m^2 · °C/W, which implies that more than doubling the thickness of air space in a wall has no effect on heat transfer through the wall. Do you think this is a typing error? Explain.

3–137C What is a radiant barrier? What kind of materials are suitable for use as radiant barriers? Is it worthwhile to use radiant barriers in the attics of homes?

3–138C Consider a house whose attic space is ventilated effectively so that the air temperature in the attic is the same as the ambient air temperature at all times. Will the roof still have any effect on heat transfer through the ceiling? Explain.

3–139 Determine the summer R-value and the U-factor of a wood frame wall that is built around 38-mm × 140-mm wood studs with a center-to-center distance of 400 mm. The 140-mm-wide cavity between the studs is filled with mineral fiber batt insulation. The inside is finished with 13-mm gypsum wallboard and the outside with 13-mm wood fiberboard and 13-mm × 200-mm wood bevel lapped siding. The insulated cavity constitutes 80 percent of the heat transmission area, while the studs, headers, plates, and sills constitute 20 percent.

Answers: 3.213 m^2 · °C/W, 0.311 W/m^2 · °C

3–140 The 13-mm-thick wood fiberboard sheathing of the wood stud wall in Problem 3–139 is replaced by a 25-mm-thick rigid foam insulation. Determine the percent increase in the R-value of the wall as a result.
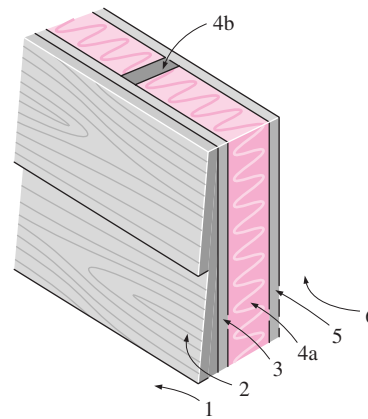

FIGURE P3–139

3–141E Determine the winter R-value and the U-factor of a masonry cavity wall that is built around 4-in.-thick concrete blocks made of lightweight aggregate. The outside is finished with 4-in. face brick with $\frac{1}{2}$-in. cement mortar between the bricks and concrete blocks. The inside finish consists of $\frac{1}{2}$-in. gypsum wallboard separated from the concrete block by $\frac{3}{4}$-in.-thick (1-in. by 3-in. nominal) vertical furring whose center-to-center distance is 16 in. Neither side of the $\frac{3}{4}$-in.-thick air space between the concrete block and the gypsum board is coated with any reflective film. When determining the R-value of the air space, the temperature difference across it can be taken to be 30°F with a mean air temperature of 50°F. The air space constitutes 80 percent of the heat transmission area, while the vertical furring and similar structures constitute 20 percent.
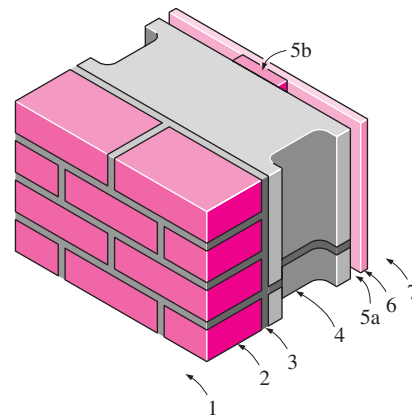

FIGURE P3–141E

3–142 Consider a flat ceiling that is built around 38-mm × 90-mm wood studs with a center-to-center distance of 400 mm. The lower part of the ceiling is finished with 13-mm gypsum wallboard, while the upper part consists of a wood subfloor ($R = 0.166$ m^2 · °C/W), a 13-mm plywood, a layer of felt ($R = 0.011$ m^2 · °C/W), and linoleum ($R = 0.009$ m^2 · °C/W). Both
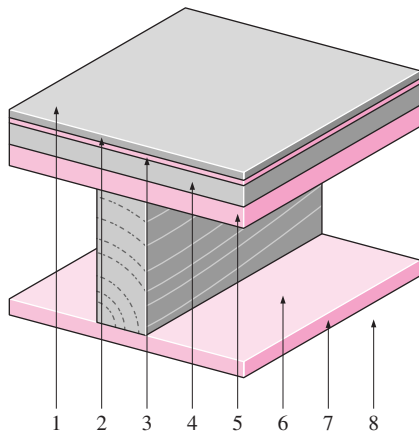
FIGURE P3–142

sides of the ceiling are exposed to still air. The air space constitutes 82 percent of the heat transmission area, while the studs and headers constitute 18 percent. Determine the winter R-value and the U-factor of the ceiling assuming the 90-mm-wide air space between the studs (*a*) does not have any reflective surface, (*b*) has a reflective surface with $\varepsilon = 0.05$ on one side, and (*c*) has reflective surfaces with $\varepsilon = 0.05$ on both sides. Assume a mean temperature of 10°C and a temperature difference of 5.6°C for the air space.

3–143   Determine the winter R-value and the U-factor of a masonry cavity wall that consists of 100-mm common bricks, a 90-mm air space, 100-mm concrete blocks made of lightweight aggregate, 20-mm air space, and 13-mm gypsum wallboard separated from the concrete block by 20-mm-thick (1-in. × 3-in. nominal) vertical furring whose center-to-center distance is 400 mm. Neither side of the two air spaces is coated with any reflective films. When determining the R-value of the air spaces, the temperature difference across them can be taken to be 16.7°C with a mean air temperature of 10°C. The air space constitutes 84 percent of the heat transmission area,
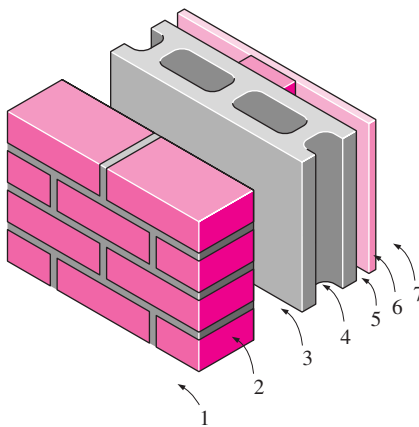

FIGURE P3–143

while the vertical furring and similar structures constitute 16 percent.   *Answers:* 1.02 m² · °C/W, 0.978 W/m² · °C

3–144   Repeat Problem 3–143 assuming one side of both air spaces is coated with a reflective film of $\varepsilon = 0.05$.

3–145   Determine the winter R-value and the U-factor of a masonry wall that consists of the following layers: 100-mm face bricks, 100-mm common bricks, 25-mm urethane rigid foam insulation, and 13-mm gypsum wallboard.
   Answers: 1.404 m² · °C/W, 0.712 W/m² · °C

3–146   The overall heat transfer coefficient (the U-value) of a wall under winter design conditions is $U = 1.55$ W/m² · °C. Determine the U-value of the wall under summer design conditions.

3–147   The overall heat transfer coefficient (the U-value) of a wall under winter design conditions is $U = 2.25$ W/m² · °C. Now a layer of 100-mm face brick is added to the outside, leaving a 20-mm air space between the wall and the bricks. Determine the new U-value of the wall. Also, determine the rate of heat transfer through a 3-m-high, 7-m-long section of the wall after modification when the indoor and outdoor temperatures are 22°C and −5°C, respectively.
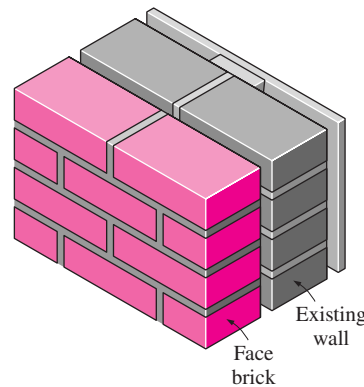

FIGURE P3–147

3–148   Determine the summer and winter R-values, in m² · °C/W, of a masonry wall that consists of 100-mm face bricks, 13-mm of cement mortar, 100-mm lightweight concrete block, 40-mm air space, and 20-mm plasterboard.
   Answers: 0.809 and 0.795 m² · °C/W

3–149E   The overall heat transfer coefficient of a wall is determined to be $U = 0.09$ Btu/h · ft² · °F under the conditions of still air inside and winds of 7.5 mph outside. What will the U-factor be when the wind velocity outside is doubled?
   Answer: 0.0907 Btu/h · ft² · °F

3–150   Two homes are identical, except that the walls of one house consist of 200-mm lightweight concrete blocks, 20-mm air space, and 20-mm plasterboard, while the walls of the other house involve the standard R-2.4 m² · °C/W frame wall construction. Which house do you think is more energy efficient?

3–151 Determine the *R*-value of a ceiling that consists of a layer of 19-mm acoustical tiles whose top surface is covered with a highly reflective aluminum foil for winter conditions. Assume still air below and above the tiles.
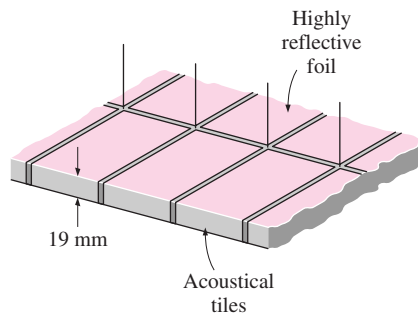

FIGURE P3–151

Review Problems

3–152E Steam is produced in the copper tubes ($k = 223$ Btu/h · ft · °F) of a heat exchanger at a temperature of 250°F by another fluid condensing on the outside surfaces of the tubes at 350°F. The inner and outer diameters of the tube are 1 in. and 1.3 in., respectively. When the heat exchanger was new, the rate of heat transfer per foot length of the tube was 2×10^4 Btu/h. Determine the rate of heat transfer per foot length of the tube when a 0.01-in.-thick layer of limestone ($k = 1.7$ Btu/h · ft · °F) has formed on the inner surface of the tube after extended use.

3–153E Repeat Problem 3–152E, assuming that a 0.01-in.-thick limestone layer has formed on both the inner and outer surfaces of the tube.

3–154 A 1.2-m-diameter and 6-m-long cylindrical propane tank is initially filled with liquid propane whose density is 581 kg/m³. The tank is exposed to the ambient air at 30°C, with a heat transfer coefficient of 25 W/m² · °C. Now a crack develops at the top of the tank and the pressure inside drops to 1 atm while the temperature drops to −42°C, which is the boiling temperature of prop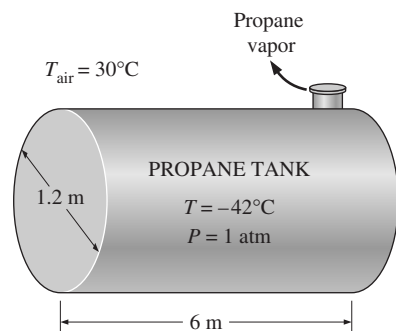ane at 1 atm. The heat of vaporization of propane at 1 atm is 425 kJ/kg. The propane is slowly vaporized as a result of the heat transfer from the ambient air into the tank, and the propane vapor escapes the tank at −42°C through the crack. Assuming the propane tank to be at about the same temperature as the propane inside at all times, determine how long it will take for the propane tank to empty if the tank is (*a*) not insulated and (*b*) insulated with 7.5-cm-thick glass wool insulation ($k = 0.038$ W/m · °C).

3–155 Hot water is flowing at an average velocity of 1.5 m/s through a cast iron pipe ($k = 52$ W/m · °C) whose inner and outer diameters are 3 cm and 3.5 cm, respectively. The pipe passes through a 15-m-long section of a basement whose temperature is 15°C. If the temperature of the water drops from 70°C to 67°C as it passes through the basement and the heat transfer coefficient on the inner surface of the pipe is 400 W/m² · °C, determine the combined convection and radiation heat transfer coefficient at the outer surface of the pipe.
Answer: 272.5 W/m² · °C

3–156 Newly formed concrete pipes are usually cured first overnight by steam in a curing kiln maintained at a temperature of 45°C before the pipes are cured for several days outside. The heat and moisture to the kiln is provided by steam flowing in a pipe whose outer diameter is 12 cm. During a plant inspection, it was noticed that the pipe passes through a 10-m section that is completely exposed to the ambient air before it reaches the kiln. The temperature measurements indicate that the average temperature of the outer surface of the steam pipe is 82°C when the ambient temperature is 8°C. The combined convection and radiation heat transfer coefficient at the outer surface of the pipe is estimated to be 25 W/m² · °C. Determine the amount of heat lost from the steam during a 10-h curing process that night.

Steam is supplied by a gas-fired steam generator that has an efficiency of 80 percent, and the plant pays $0.60/therm of natural gas (1 therm = 105,500 kJ). If the pipe is insulated and 90 percent of the heat loss is saved as a result, determine the amount of money this facility will save a year as a result of insulating the steam pipes. Assume that the concrete pipes are cured 110 nights a year. State your assumptions.
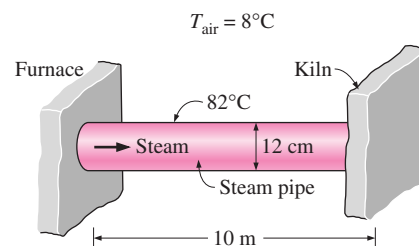

FIGURE P3–156

3–157 Consider an 18-cm × 18-cm multilayer circuit board dissipating 27 W of heat. The board consists of four layers of 0.2-mm-thick copper ($k = 386$ W/m · °C) and three layers of
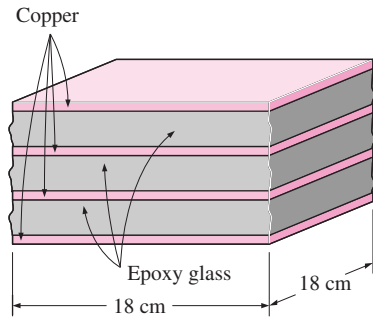

FIGURE P3–154

FIGURE P3–157

1.5-mm-thick epoxy glass ($k = 0.26$ W/m · °C) sandwiched together, as shown in the figure. The circuit board is attached to a heat sink from both ends, and the temperature of the board at those ends is 35°C. Heat is considered to be uniformly generated in the epoxy layers of the board at a rate of 0.5 W per 1-cm $\times$ 18-cm epoxy laminate strip (or 1.5 W per 1-cm $\times$ 18-cm strip of the board). Considering only a portion of the board because of symmetry, determine the magnitude and location of the maximum temperature that occurs in the board. Assume heat transfer from the top and bottom faces of the board to be negligible.

3–158 The plumbing system of a house involves a 0.5-m section of a plastic pipe ($k = 0.16$ W/m · °C) of inner diameter 2 cm and outer diameter 2.4 cm exposed to the ambient air. During a cold and windy night, the ambient air temperature remains at about -5°C for a period of 14 h. The combined convection and radiation heat transfer coefficient on the outer surface of the pipe is estimated to be 40 W/m² · °C, and the heat of fusion of water is 333.7 kJ/kg. Assuming the pipe to contain stationary water initially at 0°C, determine if the water in that section of the pipe will completely freeze that night.
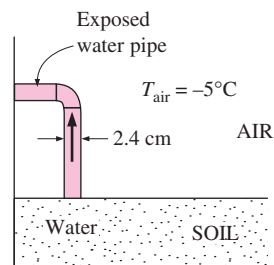

FIGURE P3–158

3–159 Repeat Problem 3–158 for the case of a heat transfer coefficient of 10 W/m² · °C on the outer surface as a result of putting a fence around the pipe that blocks the wind.

3–160E The surface temperature of a 3-in.-diameter baked potato is observed to drop from 300°F to 200°F in 5 minutes in an environment at 70°F. Determine the average heat transfer coefficient between the potato and its surroundings. Using this heat transfer coefficient and the same surface temperature, determine how long it will take for the potato to experience the same temperature drop if it is wrapped completely in a 0.12-in.-thick towel ($k = 0.035$ Btu/h · ft · °F). You may use the properties of water for potato.

3–161E Repeat Problem 3–160E assuming there is a 0.02-in.-thick air space ($k = 0.015$ Btu/h · ft · °F) between the potato and the towel.

3–162 An ice chest whose outer dimensions are 30 cm $\times$ 40 cm $\times$ 50 cm is made of 3-cm-thick Styrofoam ($k = 0.033$ W/m · °C). Initially, the chest is filled with 45 kg of ice at 0°C, and the inner surface temperature of the ice chest can be taken to be 0°C at all times. The heat of fusion of ice at 0°C is 333.7 kJ/kg, and the heat transfer coefficient between the outer surface of the ice chest and surrounding air at 35°C is 18 W/m² · °C. Disregarding any heat transfer from the 40-cm $\times$ 50-cm base of the ice chest, determine how long it will take for the ice in the chest to melt completely.

FIGURE P3–162
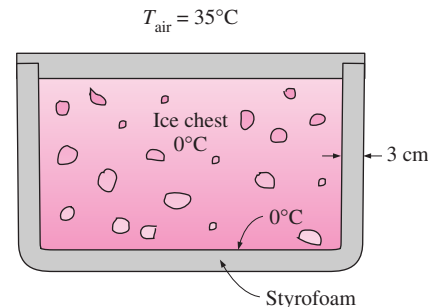
3–163 A 4-m-high and 6-m-long wall is constructed of two large 2-cm-thick steel plates ($k = 15$ W/m · °C) separated by 1-cm-thick and 20-cm-wide steel bars placed 99 cm apart. The
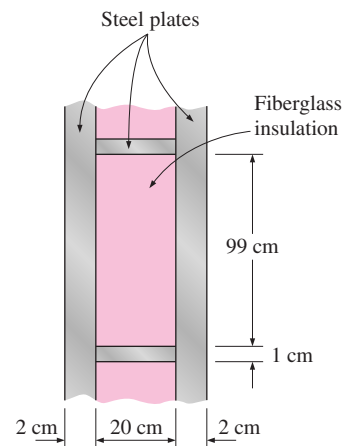

FIGURE P3–163

remaining space between the steel plates is filled with fiber-glass insulation ($k = 0.035$ W/m · °C). If the temperature difference between the inner and the outer surfaces of the walls is 22°C, determine the rate of heat transfer through the wall. Can we ignore the steel bars between the plates in heat transfer analysis since they occupy only 1 percent of the heat transfer surface area?

3–164 A 0.2-cm-thick, 10-cm-high, and 15-cm-long circuit board houses electronic components on one side that dissipate a total of 15 W of heat uniformly. The board is impregnated with conducting metal fillings and has an effective thermal conductivity of 12 W/m · °C. All the heat generated in the components is conducted across the circuit board and is dissipated from the back side of the board to a medium at 37°C, with a heat transfer coefficient of 45 W/m^2 · °C. (*a*) Determine the surface temperatures on the two sides of the circuit board. (*b*) Now a 0.1-cm-thick, 10-cm-high, and 15-cm-long aluminum plate ($k = 237$ W/m · °C) with 20 0.2-cm-thick, 2-cm-long, and 15-cm-wide aluminum fins of rectangular profile are attached to the back side of the circuit board with a 0.03–cm-thick epoxy adhesive ($k = 1.8$ W/m · °C). Determine the new temperatures on the two sides of the circuit board.
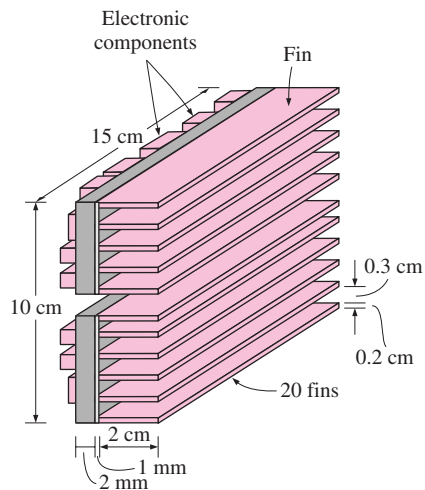

FIGURE P3–164

3–165 Repeat Problem 3–164 using a copper plate with copper fins ($k = 386$ W/m · °C) instead of aluminum ones.

3-166 A row of 10 parallel pipes that are 5 m long and have an outer diameter of 6 cm are used to transport steam at 150°C through the concrete floor ($k = 0.75$ W/m · °C) of a 10-m × 5-m room that is maintained at 25°C. The combined convection and radiation heat transfer coefficient at the floor is 12 W/m^2 · °C. If the surface temperature of the concrete floor is not to exceed 40°C, determine how deep the steam pipes should be buried below the surface of the concrete floor.
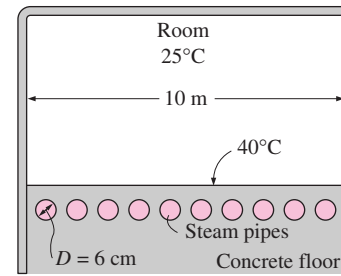

FIGURE P3–166

3–167 Consider two identical people each generating 60 W of metabolic heat steadily while doing sedentary work, and dissipating it by convection and perspiration. The first person is wearing clothes made of 1-mm-thick leather ($k = 0.159$ W/m · °C) that covers half of the body while the second one is wearing clothes made of 1-mm-thick synthetic fabric ($k = 0.13$ W/m · °C) that covers the body completely. The ambient air is at 30°C, the heat transfer coefficient at the outer surface is 15 W/m^2 · °C, and the inner surface temperature of the clothes can be taken to be 32°C. Treating the body of each person as a 25-cm-diameter 1.7-m-long cylinder, determine the fractions of heat lost from each person by perspiration.

3–168 A 6-m-wide 2.8-m-high wall is constructed of one layer of common brick ($k = 0.72$ W/m · °C) of thickness 20 cm, one inside layer of light-weight plaster ($k = 0.36$ W/m · °C) of thickness 1 cm, and one outside layer of cement based covering ($k = 1.40$ W/m · °C) of thickness 2 cm. The inner surface of the wall is maintained at 23°C while the outer surface is exposed to outdoors at 8°C with a combined convection and radiation heat transfer coefficient of 17 W/m^2 · °C. Determine the rate of heat transfer through the wall and temperature drops across the plaster, brick, covering, and surface-ambient air.

3–169 Reconsider Problem 3–168. It is desired to insulate the wall in order to decrease the heat loss by 85 percent. For the same inner surface temperature, determine the thickness of insulation and the outer surface temperature if the wall is insulated with (*a*) polyurethane foam ($k = 0.025$ W/m · °C) and (*b*) glass fiber ($k = 0.036$ W/m · °C).

3–170 Cold conditioned air at 12°C is flowing inside a 1.5-cm-thick square aluminum ($k = 237$ W/m · °C) duct of inner cross section 22 cm × 22 cm at a mass flow rate of 0.8 kg/s. The duct is exposed to air at 33°C with a combined convection-radiation heat transfer coefficient of 8 W/m^2 · °C. The convection heat transfer coefficient at the inner surface is 75 W/m^2 · °C. If the air temperature in the duct should not increase by more than 1°C determine the maximum length of the duct.

3–171 When analyzing heat transfer through windows, it is important to consider the frame as well as the glass area. Consider a 2-m-wide 1.5-m-high wood-framed window with

85 percent of the area covered by 3-mm-thick single-pane glass ($k = 0.7$ W/m · °C). The frame is 5 cm thick, and is made of pine wood ($k = 0.12$ W/m · °C). The heat transfer coefficient is 7 W/m² · °C inside and 13 W/m² · °C outside. The room is maintained at 24°C, and the temperature outdoors is 40°C. Determine the percent error involved in heat transfer when the window is assumed to consist of glass only.

3–172 Steam at 235°C is flowing inside a steel pipe ($k = 61$ W/m · °C) whose inner and outer diameters are 10 cm and 12 cm, respectively, in an environment at 20°C. The heat transfer coefficients inside and outside the pipe are 105 W/m² · °C and 14 W/m² · °C, respectively. Determine (*a*) the thickness of the insulation ($k = 0.038$ W/m · °C) needed to reduce the heat loss by 95 percent and (*b*) the thickness of the insulation needed to reduce the exposed surface temperature of insulated pipe to 40°C for safety reasons.

3–173 When the transportation of natural gas in a pipeline is not feasible for economic or other reasons, it is first liquefied at about −160°C, and then transported in specially insulated tanks placed in marine ships. Consider a 6-m-diameter spherical tank that is filled with liquefied natural gas (LNG) at −160°C. The tank is exposed to ambient air at 18°C with a heat transfer coefficient of 22 W/m² · °C. The tank is thin-shelled and its temperature can be taken to be the same as the LNG temperature. The tank is insulated with 5-cm-thick super insulation that has an effective thermal conductivity of 0.00008 W/m · °C. Taking the density and the specific heat of LNG to be 425 kg/m³ and 3.475 kJ/kg · °C, respectively, estimate how long it will take for the LNG temperature to rise to −150°C.

3–174 A 15-cm × 20-cm hot surface at 85°C is to be cooled by attaching 4-cm-long aluminum ($k = 237$ W/m · °C) fins of 2-mm × 2-mm square cross section. The temperature of surrounding medium is 25°C and the heat transfer coefficient on the surfaces can be taken to be 20 W/m² · °C. If it is desired to triple the rate of heat transfer from the bare hot surface, determine the number of fins that needs to be attached.

3–175 Reconsider Problem 3–174. Using EES (or other) software, plot the number of fins as a function of the increase in the heat loss by fins relative to no fin case (i.e., overall effectiveness of the fins) in the range of 1.5 to 5. Discuss the results. Is it realistic to assume the heat transfer coefficient to remain constant?

3–176 A 1.4-m-diameter spherical steel tank filled with iced water at 0°C is buried underground at a location where the thermal conductivity of the soil is $k = 0.55$ W/m · °C. The distance between the tank center and the ground surface is 2.4 m. For ground surface temperature of 18°C, determine the rate of heat transfer to the iced water in the tank. What would your answer be if the soil temperature were 18°C and the ground surface were insulated?

3–177 A 0.6-m-diameter 1.9-m-long cylindrical tank containing liquefied natural gas (LNG) at −160°C is placed at the center of a 1.9-m-long 1.4-m × 1.4-m square solid bar made of an insulating material with $k = 0.0006$ W/m · °C. If the outer surface temperature of the bar is 20°C, determine the rate of heat transfer to the tank. Also, determine the LNG temperature after one month. Take the density and the specific heat of LNG to be 425 kg/m³ and 3.475 kJ/kg · °C, respectively.

Design and Essay Problems

3–178 The temperature in deep space is close to absolute zero, which presents thermal challenges for the astronauts who do space walks. Propose a design for the clothing of the astronauts that will be most suitable for the thermal environment in space. Defend the selections in your design.

3–179 In the design of electronic components, it is very desirable to attach the electronic circuitry to a substrate material that is a very good thermal conductor but also a very effective electrical insulator. If the high cost is not a major concern, what material would you propose for the substrate?

3–180 Using cylindrical samples of the same material, devise an experiment to determine the thermal contact resistance. Cylindrical samples are available at any length, and the thermal conductivity of the material is known.

3–181 Find out about the wall construction of the cabins of large commercial airplanes, the range of ambient conditions under which they operate, typical heat transfer coefficients on the inner and outer surfaces of the wall, and the heat generation rates inside. Determine the size of the heating and air-conditioning system that will be able to maintain the cabin at 20°C at all times for an airplane capable of carrying 400 people.

3–182 Repeat Problem 3–181 for a submarine with a crew of 60 people.

3–183 A house with 200-m² floor space is to be heated with geothermal water flowing through pipes laid in the ground under the floor. The walls of the house are 4 m high, and there are 10 single-paned windows in the house that are 1.2 m wide and 1.8 m high. The house has R-19 (in h · ft² · °F/Btu) insulation in the walls and R-30 on the ceiling. The floor temperature is not to exceed 40°C. Hot geothermal water is available at 90°C, and the inner and outer diameter of the pipes to be used are 2.4 cm and 3.0 cm. Design such a heating system for this house in your area.

3–184 Using a timer (or watch) and a thermometer, conduct this experiment to determine the rate of heat gain of your refrigerator. First, make sure that the door of the refrigerator is not opened for at least a few hours to make sure that steady operating conditions are established. Start the timer when the refrigerator stops running and measure the time Δt_1 it stays off
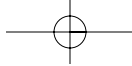
before it kicks in. Then measure the time Δt_2 it stays on. Noting that the heat removed during Δt_2 is equal to the heat gain of the refrigerator during $\Delta t_1 + \Delta t_2$ and using the power consumed by the refrigerator when it is running, determine the average rate of heat gain for your refrigerator, in watts. Take the COP (coefficient of performance) of your refrigerator to be 1.3 if it is not available.

Now, clean the condenser coils of the refrigerator and remove any obstacles on the way of airflow through the coils By replacing these measurements, determine the improvement in the COP of the refrigerator.