CHAPTER

6

FUNDAMENTALS OF CONVECTION

So far, we have considered *conduction,* which is the mechanism of heat transfer through a solid or a quiescent fluid. We now consider *convection,* which is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion.

Convection is classified as *natural* (or *free*) and *forced convection,* depending on how the fluid motion is initiated. In forced convection, the fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect, which manifests itself as the rise of warmer fluid and the fall of the cooler fluid. Convection is also classified as *external* and *internal,* depending on whether the fluid is forced to flow over a surface or in a channel.

We start this chapter with a general physical description of the convection mechanism. We then discuss the *velocity* and *thermal boundary layers,* and *laminar and turbulent flows.* We continue with the discussion of the dimensionless *Reynolds, Prandtl,* and *Nusselt numbers,* and their physical significance. Next we derive the *convection equations* of on the basis of mass, momentum, and energy conservation, and obtain solutions for *flow over a flat plate.* We then nondimensionalize the convection equations, and obtain functional forms of friction and convection coefficients. Finally, we present analogies between momentum and heat transfer.

CONTENTS

6–1 ▪ PHYSICAL MECHANISM OF CONVECTION

We mentioned earlier that there are three basic mechanisms of heat transfer: conduction, convection, and radiation. Conduction and convection are similar in that both mechanisms require the presence of a material medium. But they are different in that convection requires the presence of fluid motion.

Heat transfer through a solid is always by conduction, since the molecules of a solid remain at relatively fixed positions. Heat transfer through a liquid or gas, however, can be by conduction or convection, depending on the presence of any bulk fluid motion. Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it. Therefore, conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid (Fig. 6–1).

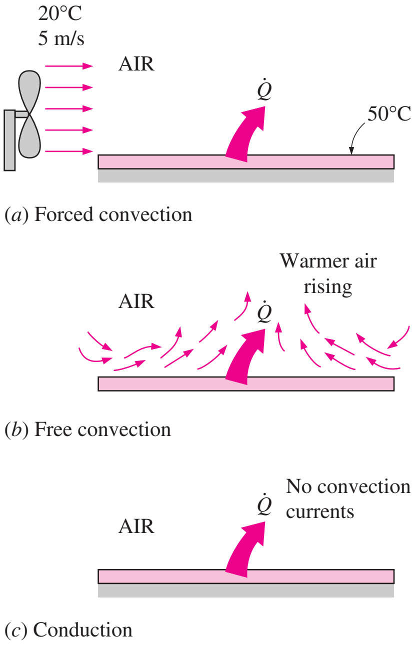


FIGURE 6–1
Heat transfer from a hot surface to the surrounding fluid by convection and conduction.

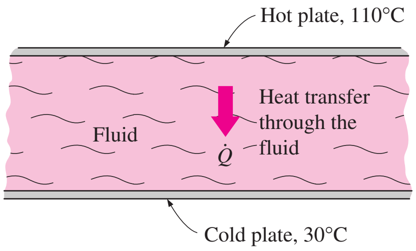


FIGURE 6–2
Heat transfer through a fluid sandwiched between two parallel plates.

Convection heat transfer is complicated by the fact that it involves fluid motion as well as heat conduction. The fluid motion enhances heat transfer, since it brings hotter and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid. Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction. In fact, the higher the fluid velocity, the higher the rate of heat transfer.

To clarify this point further, consider steady heat transfer through a fluid contained between two parallel plates maintained at different temperatures, as shown in Figure 6–2. The temperatures of the fluid and the plate will be the same at the points of contact because of the continuity of temperature. Assuming no fluid motion, the energy of the hotter fluid molecules near the hot plate will be transferred to the adjacent cooler fluid molecules. This energy will then be transferred to the next layer of the cooler fluid molecules. This energy will then be transferred to the next layer of the cooler fluid, and so on, until it is finally transferred to the other plate. This is what happens during conduction through a fluid. Now let us use a syringe to draw some fluid near the hot plate and inject it near the cold plate repeatedly. You can imagine that this will speed up the heat transfer process considerably, since some energy is carried to the other side as a result of fluid motion.

Consider the cooling of a hot iron block with a fan blowing air over its top surface, as shown in Figure 6–3. We know that heat will be transferred from the hot block to the surrounding cooler air, and the block will eventually cool. We also know that the block will cool faster if the fan is switched to a higher speed. Replacing air by water will enhance the convection heat transfer even more.

Experience shows that convection heat transfer strongly depends on the fluid properties *dynamic viscosity* μ, *thermal conductivity k, density* ρ, and *specific heat* C_p, as well as the *fluid velocity* $\mathcal{V}$. It also depends on the *geometry* and the *roughness* of the solid surface, in addition to the *type of fluid flow* (such as being streamlined or turbulent). Thus, we expect the convection heat transfer relations to be rather complex because of the dependence of convection on so many variables. This is not surprising, since convection is the most complex mechanism of heat transfer.

Despite the complexity of convection, the rate of convection heat transfer is observed to be proportional to the temperature difference and is conveniently expressed by **Newton's law of cooling** as

$$\dot{q}_{conv} = h(T_s - T_\infty) \qquad (W/m^2) \tag{6-1}$$

or

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad (W) \tag{6-2}$$

where

h = convection heat transfer coefficient, W/m^2 · °C
A_s = heat transfer surface area, m^2
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C

Judging from its units, the **convection heat transfer coefficient** h can be defined as *the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference.*

You should not be deceived by the simple appearance of this relation, because the convection heat transfer coefficient h depends on the several of the mentioned variables, and thus is difficult to determine.

When a fluid is forced to flow over a solid surface that is nonporous (i.e., impermeable to the fluid), it is observed that the fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface. That is, the fluid layer in direct contact with a solid surface "sticks" to the surface and there is no slip. In fluid flow, this phenomenon is known as the **no-slip condition,** and it is due to the viscosity of the fluid (Fig. 6–4).

The no-slip condition is responsible for the development of the velocity profile for flow. Because of the friction between the fluid layers, the layer that sticks to the wall slows the adjacent fluid layer, which slows the next layer, and so on. A consequence of the no-slip condition is that all velocity profiles must have zero values at the points of contact between a fluid and a solid. The only exception to the no-slip condition occurs in extremely rarified gases.

A similar phenomenon occurs for the temperature. When two bodies at different temperatures are brought into contact, heat transfer occurs until both bodies assume the same temperature at the point of contact. Therefore, a fluid and a solid surface will have the same temperature at the point of contact. This is known as **no-temperature-jump condition.**

An implication of the no-slip and the no-temperature jump conditions is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by *pure conduction,* since the fluid layer is motionless, and can be expressed as

$$\dot{q}_{conv} = \dot{q}_{cond} = -k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad (W/m^2) \tag{6-3}$$

where T represents the temperature distribution in the fluid and $(\partial T/\partial y)_{y=0}$ is the *temperature gradient* at the surface. This heat is then *convected away* from the surface as a result of fluid motion. Note that convection heat transfer from a solid surface to a fluid is merely the conduction heat transfer from the solid surface to the fluid layer adjacent to the surface. Therefore, we can equate Eqs. 6-1 and 6-3 for the heat flux to obtain

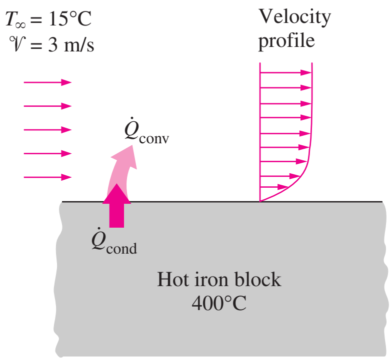


FIGURE 6–3
The cooling of a hot block by forced convection.

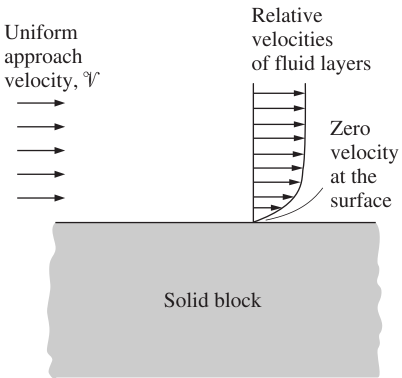


FIGURE 6–4
A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C}) \tag{6-4}$$

for the determination of the *convection heat transfer coefficient* when the temperature distribution within the fluid is known.

The convection heat transfer coefficient, in general, varies along the flow (or x-) direction. The *average* or *mean* convection heat transfer coefficient for a surface in such cases is determined by properly averaging the *local* convection heat transfer coefficients over the entire surface.

Nusselt Number

In convection studies, it is common practice to nondimensionalize the governing equations and combine the variables, which group together into *dimensionless numbers* in order to reduce the number of total variables. It is also common practice to nondimensionalize the heat transfer coefficient h with the Nusselt number, defined as

$$\text{Nu} = \frac{hL_c}{k} \tag{6-5}$$

where k is the thermal conductivity of the fluid and L_c is the *characteristic length.* The Nusselt number is named after Wilhelm Nusselt, who made significant contributions to convective heat transfer in the first half of the twentieth century, and it is viewed as the *dimensionless convection heat transfer coefficient.*

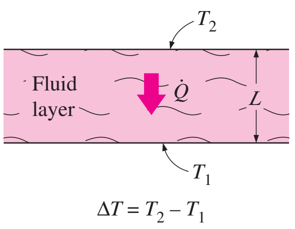


FIGURE 6–5
Heat transfer through a fluid layer of thickness L and temperature difference ΔT.

To understand the physical significance of the Nusselt number, consider a fluid layer of thickness L and temperature difference $\Delta T = T_2 - T_1$, as shown in Fig. 6–5. Heat transfer through the fluid layer will be by *convection* when the fluid involves some motion and by *conduction* when the fluid layer is motionless. Heat flux (the rate of heat transfer per unit time per unit surface area) in either case will be

$$\dot{q}_{\text{conv}} = h\Delta T \tag{6-6}$$

and

$$\dot{q}_{\text{cond}} = k\frac{\Delta T}{L} \tag{6-7}$$

Taking their ratio gives

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu} \tag{6-8}$$

which is the Nusselt number. Therefore, the Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. The larger the Nusselt number, the more effective the convection. A Nusselt number of Nu = 1 for a fluid layer represents heat transfer across the layer by pure conduction.

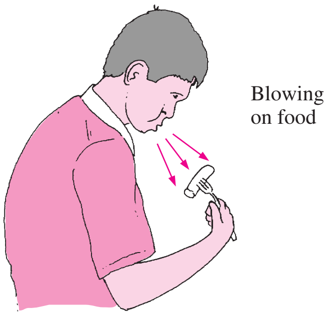


FIGURE 6–6
We resort to forced convection whenever we need to increase the rate of heat transfer.

We use forced convection in daily life more often than you might think (Fig. 6–6). We resort to forced convection whenever we want to increase the rate of heat transfer from a hot object. For example, we turn on the fan on hot summer days to help our body cool more effectively. The higher the fan speed, the better we feel. We *stir* our soup and *blow* on a hot slice of pizza to make them cool faster. The air on *windy* winter days feels much colder than it actually is. The simplest solution to heating problems in electronics packaging is to use a large enough fan.

6–2 ▪ CLASSIFICATION OF FLUID FLOWS

Convection heat transfer is closely tied with fluid mechanics, which is the science that deals with the behavior of fluids at rest or in motion, and the interaction of fluids with solids or other fluids at the boundaries. There are a wide variety of fluid flow problems encountered in practice, and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups. There are many ways to classify the fluid flow problems, and below we present some general categories.

Viscous versus Inviscid Flow

When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer. This internal resistance to flow is called the **viscosity,** which is a measure of internal stickiness of the fluid. Viscosity is caused by cohesive forces between the molecules in liquids, and by the molecular collisions in gases. There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree. Flows in which the effects of viscosity are significant are called **viscous flows.** The effects of viscosity are very small in some flows, and neglecting those effects greatly simplifies the analysis without much loss in accuracy. Such idealized flows of zero-viscosity fluids are called frictionless or **inviscid flows.**

Internal versus External Flow

A fluid flow is classified as being internal and external, depending on whether the fluid is forced to flow in a confined channel or over a surface. The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is **external flow.** The flow in a pipe or duct is **internal flow** if the fluid is completely bounded by solid surfaces. Water flow in a pipe, for example, is internal flow, and air flow over an exposed pipe during a windy day is external flow (Fig. 6–7). The flow of liquids in a pipe is called *open-channel flow* if the pipe is partially filled with the liquid and there is a free surface. The flow of water in rivers and irrigation ditches are examples of such flows.

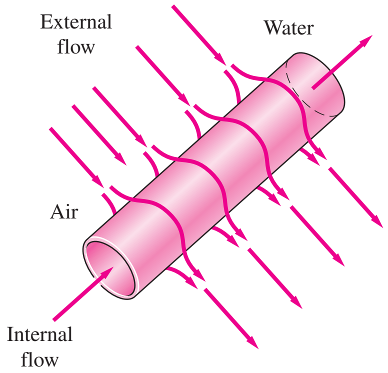


FIGURE 6–7
Internal flow of water in a pipe and the external flow of air over the same pipe.

Compressible versus Incompressible Flow

A fluid flow is classified as being *compressible* or *incompressible,* depending on the density variation of the fluid during flow. The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. Therefore, liquids are usually classified as *incompressible substances.* A pressure of 210 atm, for example, will cause the density of liquid water at 1 atm to change by just 1 percent. Gases, on the other hand, are highly compressible. A pressure change of just 0.01 atm, for example, will cause a change of 1 percent in the density of atmospheric air. However, gas flows can be treated as incompressible if the density changes are under about 5 percent, which is usually the case when the flow velocity is less than 30 percent of the velocity of sound in that gas (i.e., the Mach number of flow is less than 0.3). The velocity of sound in air at room temperature is 346 m/s. Therefore, the compressibility effects of air can be neglected at speeds under 100 m/s. Note that the flow of a gas is not necessarily a compressible flow.

Laminar versus Turbulent Flow

Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth streamlines is called **laminar.** The flow of high-viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities characterized by velocity fluctuations is called **turbulent.** The flow of low-viscosity fluids such as air at high velocities is typically turbulent. The flow regime greatly influences the heat transfer rates and the required power for pumping.

Natural (or Unforced) versus Forced Flow

A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated. In **forced flow,** a fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In **natural flows,** any fluid motion is due to a natural means such as the buoyancy effect, which manifests itself as the rise of the warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid. This thermosiphoning effect is commonly used to replace pumps in solar water heating systems by placing the water tank sufficiently above the solar collectors (Fig. 6–8).

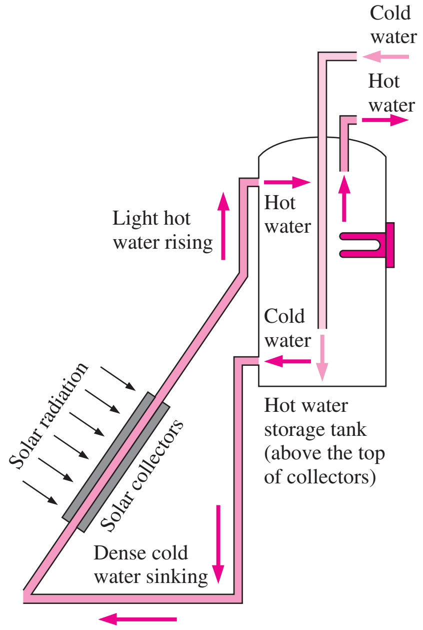


FIGURE 6–8
Natural circulation of water in a solar water heater by thermosiphoning.

Steady versus Unsteady (Transient) Flow

The terms *steady* and *uniform* are used frequently in engineering, and thus it is important to have a clear understanding of their meanings. The term **steady** implies *no change with time.* The opposite of steady is **unsteady,** or **transient.** The term *uniform,* however, implies *no change with location* over a specified region.

Many devices such as turbines, compressors, boilers, condensers, and heat exchangers operate for long periods of time under the same conditions, and they are classified as *steady-flow devices.* During steady flow, the fluid properties can change from point to point within a device, but at any fixed point they remain constant.

One-, Two-, and Three-Dimensional Flows

A flow field is best characterized by the velocity distribution, and thus a flow is said to be one-, two-, or three-dimensional if the flow velocity $\mathcal{V}$ varies in one, two, or three primary dimensions, respectively. A typical fluid flow involves a three-dimensional geometry and the velocity may vary in all three dimensions rendering the flow three-dimensional [$\mathcal{V}(x, y, z)$ in rectangular or $\mathcal{V}(r, \theta, z)$ in cylindrical coordinates]. However, the variation of velocity in certain direction can be small relative to the variation in other directions, and can be ignored with negligible error. In such cases, the flow can be modeled conveniently as being one- or two-dimensional, which is easier to analyze.

When the entrance effects are disregarded, fluid flow in a circular pipe is *one-dimensional* since the velocity varies in the radial r direction but not in the angular θ- or axial z-directions (Fig. 6–9). That is, the velocity profile is the same at any axial z-location, and it is symmetric about the axis of the pipe. Note that even in this simplest flow, the velocity cannot be uniform across the cross section of the pipe because of the no-slip condition. However, for convenience in calculations, the velocity can be assumed to be constant and thus *uniform* at a cross section. Fluid flow in a pipe usually approximated as *one-dimensional uniform flow.*

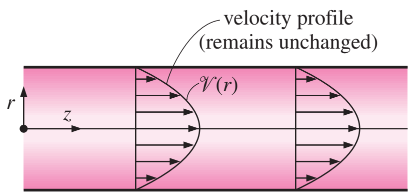


FIGURE 6–9
One-dimensional flow in a circular pipe.

6–3 ▪ VELOCITY BOUNDARY LAYER

Consider the parallel flow of a fluid over a *flat plate,* as shown in Fig. 6–10. Surfaces that are slightly contoured such as turbine blades can also be approximated as flat plates with reasonable accuracy. The x-coordinate is measured along the plate surface from the *leading edge* of the plate in the direction of the flow, and y is measured from the surface in the normal direction. The fluid approaches the plate in the x-direction with a uniform upstream velocity of $\mathcal{V}$, which is practically identical to the free-stream velocity u_∞ over the plate away from the surface (this would not be the case for cross flow over blunt bodies such as a cylinder).

For the sake of discussion, we can consider the fluid to consist of adjacent layers piled on top of each other. The velocity of the particles in the first fluid layer adjacent to the plate becomes zero because of the no-slip condition. This motionless layer slows down the particles of the neighboring fluid layer as a result of friction between the particles of these two adjoining fluid layers at different velocities. This fluid layer then slows down the molecules of the next layer, and so on. Thus, the presence of the plate is felt up to some normal distance δ from the plate beyond which the free-stream velocity u_∞ remains essentially unchanged. As a result, the x-component of the fluid velocity, $u,$ will vary from 0 at $y = 0$ to nearly u_∞ at $y = \delta$ (Fig. 6–11).

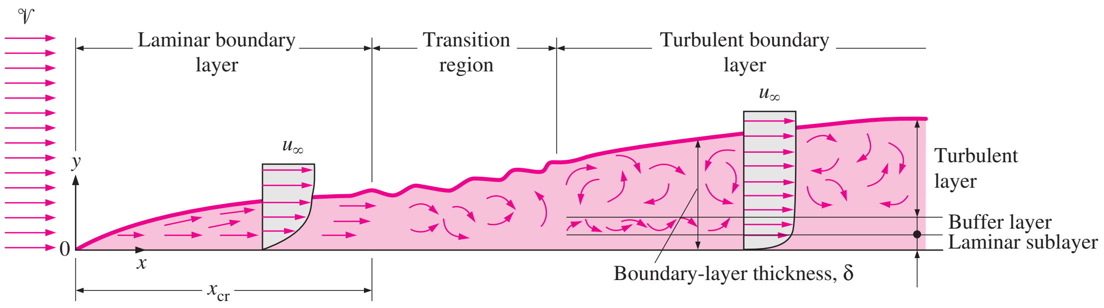


FIGURE 6–10
The development of the boundary layer for flow over a flat plate, and the different flow regimes.

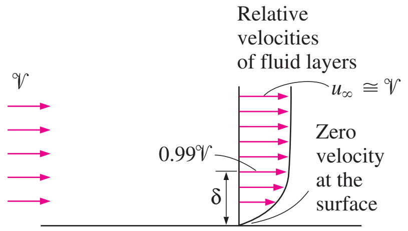


FIGURE 6–11
The development of a boundary layer on a surface is due to the no-slip condition.

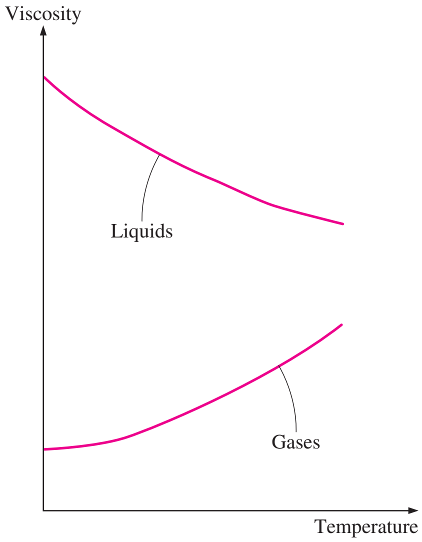


FIGURE 6–12
The viscosity of liquids decreases and the viscosity of gases increases with temperature.

The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the **velocity boundary layer.** The *boundary layer thickness,* δ, is typically defined as the distance y from the surface at which $u = 0.99u_\infty$.

The hypothetical line of $u = 0.99u_\infty$ divides the flow over a plate into two regions: the **boundary layer region,** in which the viscous effects and the velocity changes are significant, and the **inviscid flow region,** in which the frictional effects are negligible and the velocity remains essentially constant.

Surface Shear Stress

Consider the flow of a fluid over the surface of a plate. The fluid layer in contact with the surface will try to drag the plate along via friction, exerting a *friction force* on it. Likewise, a faster fluid layer will try to drag the adjacent slower layer and exert a friction force because of the friction between the two layers. Friction force per unit area is called **shear stress,** and is denoted by τ. Experimental studies indicate that the shear stress for most fluids is proportional to the *velocity gradient,* and the shear stress at the wall surface is as

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad (\text{N/m}^2) \tag{6-9}$$

where the constant of proportionality μ is called the **dynamic viscosity** of the fluid, whose unit is kg/m · s (or equivalently, N · s/m^2, or Pa · s, or poise = 0.1 Pa · s).

The fluids that that obey the linear relationship above are called **Newtonian fluids,** after Sir Isaac Newton who expressed it first in 1687. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids. Blood and liquid plastics are examples of non-Newtonian fluids. In this text we will consider Newtonian fluids only.

In fluid flow and heat transfer studies, the ratio of dynamic viscosity to density appears frequently. For convenience, this ratio is given the name **kinematic viscosity** ν and is expressed as $\nu = \mu/\rho$. Two common units of kinematic viscosity are m^2/s and *stoke* (1 stoke = 1 cm^2/s = 0.0001 m^2/s).

The viscosity of a fluid is a measure of its *resistance to flow,* and it is a strong function of temperature. The viscosities of liquids *decrease* with temperature, whereas the viscosities of gases *increase* with temperature (Fig. 6–12). The viscosities of some fluids at 20°C are listed in Table 6–1. Note that the viscosities of different fluids differ by several orders of magnitude.

The determination of the surface shear stress τ_s from Eq. 6-9 is not practical since it requires a knowledge of the flow velocity profile. A more practical approach in external flow is to relate τ_s to the upstream velocity $\mathcal{V}$ as

$$\tau_s = C_f \frac{\rho \mathcal{V}^2}{2} \qquad (\text{N/m}^2) \tag{6-10}$$

where C_f is the dimensionless **friction coefficient,** whose value in most cases is determined experimentally, and ρ is the density of the fluid. Note that the friction coefficient, in general, will vary with location along the surface. Once the average friction coefficient over a given surface is available, the friction force over the entire surface is determined from

$$F_f = C_f A_s \frac{\rho \mathcal{V}^2}{2} \quad \text{(N)} \tag{6-11}$$

where A_s is the surface area.

The friction coefficient is an important parameter in heat transfer studies since it is directly related to the heat transfer coefficient and the power requirements of the pump or fan.

6–4 ▪ THERMAL BOUNDARY LAYER

We have seen that a velocity boundary layer develops when a fluid flows over a surface as a result of the fluid layer adjacent to the surface assuming the surface velocity (i.e., zero velocity relative to the surface). Also, we defined the velocity boundary layer as the region in which the fluid velocity varies from zero to $0.99u_\infty$. Likewise, a *thermal boundary layer* develops when a fluid at a specified temperature flows over a surface that is at a different temperature, as shown in Fig. 6–13.

Consider the flow of a fluid at a uniform temperature of T_∞ over an isothermal flat plate at temperature T_s. The fluid particles in the layer adjacent to the surface will reach thermal equilibrium with the plate and assume the surface temperature T_s. These fluid particles will then exchange energy with the particles in the adjoining-fluid layer, and so on. As a result, a temperature profile will develop in the flow field that ranges from T_s at the surface to T_∞ sufficiently far from the surface. The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is the **thermal boundary layer.** The *thickness* of the thermal boundary layer δ_t at any location along the surface is defined as *the distance from the surface at which the temperature difference* $T - T_s$ *equals* $0.99(T_\infty - T_s)$. Note that for the special case of $T_s = 0$, we have $T = 0.99T_\infty$ at the outer edge of the thermal boundary layer, which is analogous to $u = 0.99u_\infty$ for the velocity boundary layer.

The thickness of the thermal boundary layer increases in the flow direction, since the effects of heat transfer are felt at greater distances from the surface further down stream.

The convection heat transfer rate anywhere along the surface is directly related to the temperature gradient at that location. Therefore, the shape of the temperature profile in the thermal boundary layer dictates the convection heat transfer between a solid surface and the fluid flowing over it. In flow over a heated (or cooled) surface, both velocity and thermal boundary layers will develop simultaneously. Noting that the fluid velocity will have a strong influence on the temperature profile, the development of the velocity boundary layer relative to the thermal boundary layer will have a strong effect on the convection heat transfer.

Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless* parameter **Prandtl number,** defined as

$$\text{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \tag{6-12}$$

TABLE 6–1

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

Fluid	Dynamic viscosity μ, kg/m · s
Glycerin:	
−20°C	134.0
0°C	12.1
20°C	1.49
40°C	0.27
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.0003
100°C (vapor)	0.000013
Blood, 37°C	0.0004
Gasoline	0.00029
Ammonia	0.00022
Air	0.000018
Hydrogen, 0°C	0.000009

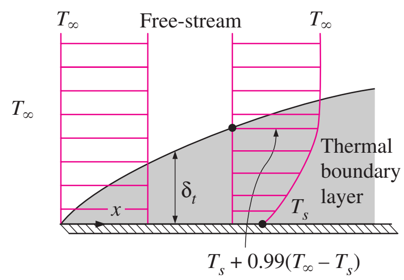


FIGURE 6–13

Thermal boundary layer on a flat plate (the fluid is hotter than the plate surface).

TABLE 6–2

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

It is named after Ludwig Prandtl, who introduced the concept of boundary layer in 1904 and made significant contributions to boundary layer theory. The Prandtl numbers of fluids range from less than 0.01 for liquid metals to more than 100,000 for heavy oils (Table 6–2). Note that the Prandtl number is in the order of 10 for water.

The Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate. Heat diffuses very quickly in liquid metals ($Pr \ll 1$) and very slowly in oils ($Pr \gg 1$) relative to momentum. Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

6–5 ▪ LAMINAR AND TURBULENT FLOWS

If you have been around smokers, you probably noticed that the cigarette smoke rises in a smooth plume for the first few centimeters and then starts fluctuating randomly in all directions as it continues its journey toward the lungs of others (Fig. 6–14). Likewise, a careful inspection of flow in a pipe reveals that the fluid flow is streamlined at low velocities but turns chaotic as the velocity is increased above a critical value, as shown in Figure 6–15. The flow regime in the first case is said to be **laminar,** characterized by *smooth streamlines* and *highly-ordered motion,* and **turbulent** in the second case, where it is characterized by *velocity fluctuations* and *highly-disordered motion.* The **transition** from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.

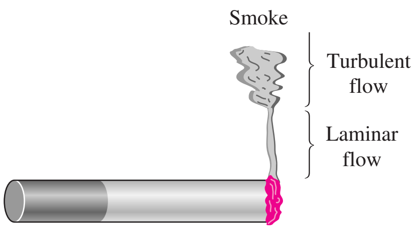


FIGURE 6–14
Laminar and turbulent flow regimes of cigarette smoke.

We can verify the existence of these laminar, transition, and turbulent flow regimes by injecting some dye streak into the flow in a glass tube, as the British scientist Osborn Reynolds (1842–1912) did over a century ago. We will observe that the dye streak will form a *straight and smooth line* at low velocities when the flow is laminar (we may see some blurring because of molecular diffusion), will have *bursts of fluctuations* in the transition regime, and will *zigzag rapidly and randomly* when the flow becomes fully turbulent. These zigzags and the dispersion of the dye are indicative of the fluctuations in the main flow and the rapid mixing of fluid particles from adjacent layers.

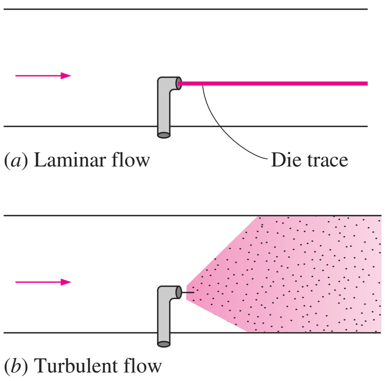


FIGURE 6–15
The behavior of colored fluid injected into the flow in laminar and turbulent flows in a tube.

Typical velocity profiles in laminar and turbulent flow are also given in Figure 6–10. Note that the velocity profile is approximately parabolic in laminar flow and becomes flatter in turbulent flow, with a sharp drop near the surface. The turbulent boundary layer can be considered to consist of three layers. The very thin layer next to the wall where the viscous effects are dominant is the **laminar sublayer.** The velocity profile in this layer is nearly linear, and the flow is streamlined. Next to the laminar sublayer is the **buffer layer,** in which the turbulent effects are significant but not dominant of the diffusion effects, and next to it is the **turbulent layer,** in which the turbulent effects dominate.

The *intense mixing* of the fluid in turbulent flow as a result of rapid fluctuations enhances heat and momentum transfer between fluid particles, which increases the friction force on the surface and the convection heat transfer rate. It also causes the boundary layer to enlarge. Both the friction and heat transfer coefficients reach maximum values when the flow becomes *fully turbulent.* So it will come as no surprise that a special effort is made in the design of heat transfer coefficients associated with turbulent flow. The enhancement in heat transfer in turbulent flow does not come for free, however. It may be necessary to use a larger pump to overcome the larger friction forces accompanying the higher heat transfer rate.

Reynolds Number

The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, free-stream velocity, surface temperature,* and *type of fluid,* among other things. After exhaustive experiments in the 1880s, Osborn Reynolds discovered that the flow regime depends mainly on the ratio of the *inertia forces* to *viscous forces* in the fluid. This ratio is called the **Reynolds number,** which is a *dimensionless* quantity, and is expressed for external flow as (Fig. 6–16)

$$\text{Re} = \frac{\text{Inertia forces}}{\text{Viscous}} = \frac{\mathcal{V} L_c}{\nu} = \frac{\rho \mathcal{V} L_c}{\mu} \tag{6-13}$$

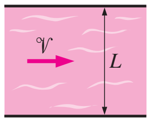
where $\mathcal{V}$ is the upstream velocity (equivalent to the free-stream velocity u_∞ for a flat plate), L_c is the characteristic length of the geometry, and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid. For a flat plate, the characteristic length is the distance x from the leading edge. Note that kinematic viscosity has the unit m^2/s, which is identical to the unit of thermal diffusivity, and can be viewed as *viscous diffusivity* or *diffusivity for momentum.*

At *large* Reynolds numbers, the inertia forces, which are proportional to the density and the velocity of the fluid, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid. At *small* Reynolds numbers, however, the viscous forces are large enough to overcome the inertia forces and to keep the fluid "in line." Thus the flow is *turbulent* in the first case and *laminar* in the second.

The Reynolds number at which the flow becomes turbulent is called the **critical Reynolds number.** The value of the critical Reynolds number is different for different geometries. For flow over a flat plate, the generally accepted value of the critical Reynolds number is $\text{Re}_{cr} = \mathcal{V} x_{cr}/\nu = u_\infty x_{cr}/\nu = 5 \times 10^5$, where x_{cr} is the distance from the leading edge of the plate at which transition from laminar to turbulent flow occurs. The value of Re_{cr} may change substantially, however, depending on the level of turbulence in the free stream.

6–6 ▪ HEAT AND MOMENTUM TRANSFER IN TURBULENT FLOW

Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress and heat transfer. Turbulent flow is characterized by random and rapid fluctuations of groups of fluid particles, called *eddies,* throughout the boundary layer. These fluctuations provide an additional mechanism for momentum and heat transfer. In laminar flow, fluid particles flow in an orderly manner along streamlines, and both momentum and heat are transferred across streamlines by molecular diffusion. In turbulent flow, the transverse motion of eddies transport momentum and heat to other regions of flow before they mix with the rest of the fluid and lose their identity, greatly enhancing momentum and heat



$$\text{Re} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{\rho \mathcal{V}^2/L}{\mu \mathcal{V}/L^2} = \frac{\rho \mathcal{V} L}{\mu} = \frac{\mathcal{V} L}{\nu}$$

FIGURE 6–16
The Reynolds number can be viewed as the ratio of the inertia forces to viscous forces acting on a fluid volume element.

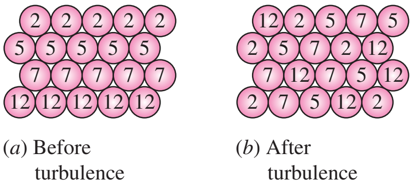


FIGURE 6–17
The intense mixing in turbulent flow brings fluid particles at different temperatures into close contact, and thus enhances heat transfer.

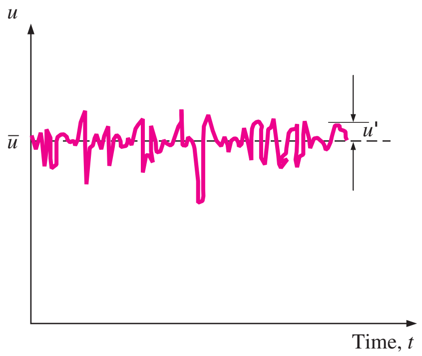


FIGURE 6–18
Fluctuations of the velocity component u with time at a specified location in turbulent flow.

transfer. As a result, turbulent flow is associated with much higher values of friction and heat transfer coefficients (Fig. 6–17).

Even when the mean flow is steady, the eddying motion in turbulent flow causes significant fluctuations in the values of velocity, temperature, pressure, and even density (in compressible flow). Figure 6–18 shows the variation of the instantaneous velocity component u with time at a specified location, as can be measured with a hot-wire anemometer probe or other sensitive device. We observe that the instantaneous values of the velocity fluctuate about a mean value, which suggests that the velocity can be expressed as the sum of a *mean value* $\bar{u}$ and a *fluctuating component* u',

$$u = \bar{u} + u' \tag{6-14}$$

This is also the case for other properties such as the velocity component v in the y direction, and thus $v = \bar{v} + v'$, $P = \bar{P} + P'$, and $T = \bar{T} + T'$. The mean value of a property at some location is determined by averaging it over a time interval that is sufficiently large so that the net effect of fluctuations is zero. Therefore, the time average of fluctuating components is zero, e.g., $\bar{u}' = 0$. The magnitude of u' is usually just a few percent of $\bar{u}$, but the high frequencies of eddies (in the order of a thousand per second) makes them very effective for the transport of momentum and thermal energy. In *steady* turbulent flow, the mean values of properties (indicated by an overbar) are independent of time.

Consider the upward eddy motion of a fluid during flow over a surface. The mass flow rate of fluid per unit area normal to flow is $\rho v'$. Noting that $h = C_pT$ represents the energy of the fluid and T' is the eddy temperature relative to the mean value, the rate of thermal energy transport by turbulent eddies is $\dot{q}_t = \rho C_p v'T'$. By a similar argument on momentum transfer, the turbulent shear stress can be shown to be $\tau_t = -\rho\overline{u'v'}$. Note that $\overline{u'v'} \neq 0$ even though $\overline{u'} = 0$ and $\overline{v'} = 0$, and experimental results show that $\overline{u'v'}$ is a negative quantity. Terms such as $-\rho\overline{u'v'}$ are called **Reynolds stresses.**

The random eddy motion of groups of particles resembles the random motion of molecules in a gas—colliding with each other after traveling a certain distance and exchanging momentum and heat in the process. Therefore, momentum and heat transport by eddies in turbulent boundary layers is analogous to the molecular momentum and heat diffusion. Then turbulent wall shear stress and turbulent heat transfer can be expressed in an analogous manner as

$$\tau_t = -\rho\overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y} \quad \text{and} \quad \dot{q}_t = \rho C_p \overline{v'T'} = -k_t \frac{\partial \bar{T}}{\partial y} \tag{6-15}$$

where μ_t is called the **turbulent viscosity,** which accounts for momentum transport by turbulent eddies, and k_t is called the **turbulent thermal conductivity,** which accounts for thermal energy transport by turbulent eddies. Then the total shear stress and total heat flux can be expressed conveniently as

$$\tau_{\text{total}} = (\mu + \mu_t)\frac{\partial \bar{u}}{\partial y} = \rho(\nu + \varepsilon_M)\frac{\partial \bar{u}}{\partial y} \tag{6-16}$$

and

$$\dot{q}_{\text{total}} = -(k + k_t)\frac{\partial \bar{T}}{\partial y} = -\rho C_p(\alpha + \varepsilon_H)\frac{\partial \bar{T}}{\partial y} \tag{6-17}$$

where $\varepsilon_M = \mu_t/\rho$ is the **eddy diffusivity of momentum** and $\varepsilon_H = k_t/\rho C_p$ is the **eddy diffusivity of heat.**

Eddy motion and thus eddy diffusivities are much larger than their molecular counterparts in the core region of a turbulent boundary layer. The eddy motion loses its intensity close to the wall, and diminishes at the wall because of the no-slip condition. Therefore, the velocity and temperature profiles are nearly uniform in the core region of a turbulent boundary layer, but very steep in the thin layer adjacent to the wall, resulting in large velocity and temperature gradients at the wall surface. So it is no surprise that the wall shear stress and wall heat flux are much larger in turbulent flow than they are in laminar flow (Fig. 6–19).

Note that molecular diffusivities ν and α (as well as μ and k) are fluid properties, and their values can be found listed in fluid handbooks. Eddy diffusivities ε_M and ε_H (as well as μ_t and k_t), however are *not* fluid properties and their values depend on flow conditions. Eddy diffusivities ε_M and ε_H decrease towards the wall, becoming zero at the wall.

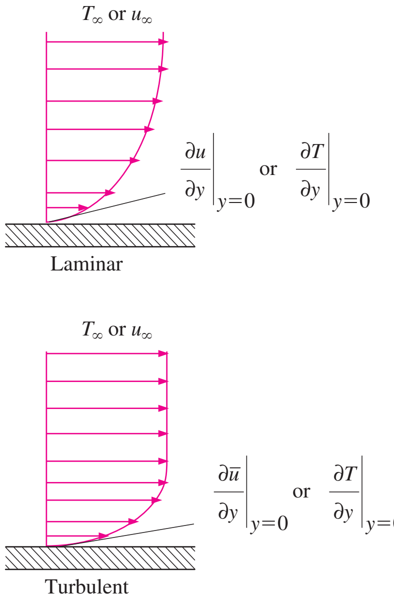


FIGURE 6–19
The velocity and temperature gradients at the wall, and thus the wall shear stress and heat transfer rate, are much larger for turbulent flow than they are for laminar flow (T is shown relative to T_s).

6–7 ▪ DERIVATION OF DIFFERENTIAL CONVECTION EQUATIONS*

In this section we derive the governing equations of fluid flow in the boundary layers. To keep the analysis at a manageable level, we assume the flow to be steady and two-dimensional, and the fluid to be Newtonian with constant properties (density, viscosity, thermal conductivity, etc.).

Consider the parallel flow of a fluid over a surface. We take the flow direction along the surface to be x and the direction normal to the surface to be y, and we choose a differential volume element of length dx, height dy, and unit depth in the z-direction (normal to the paper) for analysis (Fig. 6–20). The fluid flows over the surface with a uniform free-stream velocity u_∞, but the velocity within boundary layer is two-dimensional: the x-component of the velocity is u, and the y-component is v. Note that $u = u(x, y)$ and $v = v(x, y)$ in steady two-dimensional flow.

Next we apply three fundamental laws to this fluid element: Conservation of mass, conservation of momentum, and conservation of energy to obtain the continuity, momentum, and energy equations for laminar flow in boundary layers.

Conservation of Mass Equation

The conservation of mass principle is simply a statement that mass cannot be created or destroyed, and all the mass must be accounted for during an analysis. In steady flow, the amount of mass within the control volume remains constant, and thus the conservation of mass can be expressed as

$$\left(\begin{array}{c}\text{Rate of mass flow}\\ \text{into the control volume}\end{array}\right) = \left(\begin{array}{c}\text{Rate of mass flow}\\ \text{out of the control volume}\end{array}\right) \tag{6-18}$$

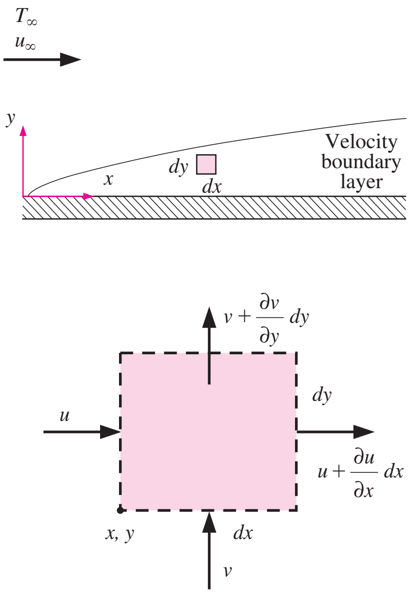


FIGURE 6–20
Differential control volume used in the derivation of mass balance in velocity boundary layer in two-dimensional flow over a surface.

*This and the upcoming sections of this chapter deal with theoretical aspects of convection, and can be skipped and be used as a reference if desired without a loss in continuity.

Noting that mass flow rate is equal to the product of density, mean velocity, and cross-sectional area normal to flow, the rate at which fluid enters the control volume from the left surface is $\rho u(dy \cdot 1)$. The rate at which the fluid leaves the control volume from the right surface can be expressed as

$$\rho\left(u + \frac{\partial u}{\partial x}dx\right)(dy \cdot 1) \tag{6-19}$$

Repeating this for the y direction and substituting the results into Eq. 6-18, we obtain

$$\rho u(dy \cdot 1) + \rho v(dx \cdot 1) = \rho\left(u + \frac{\partial u}{\partial x}dx\right)(dy \cdot 1) + \rho\left(v + \frac{\partial v}{\partial y}dy\right)(dx \cdot 1) \tag{6-20}$$

Simplifying and dividing by $dx \cdot dy \cdot 1$ gives

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6-21}$$

This is the *conservation of mass* relation, also known as the **continuity equation,** or **mass balance** for steady two-dimensional flow of a fluid with constant density.

Conservation of Momentum Equations

The differential forms of the equations of motion in the velocity boundary layer are obtained by applying Newton's second law of motion to a differential control volume element in the boundary layer. Newton's second law is an expression for the conservation of momentum, and can be stated as *the net force acting on the control volume is equal to the mass times the acceleration of the fluid element within the control volume, which is also equal to the net rate of momentum outflow from the control volume.*

The forces acting on the control volume consist of *body forces* that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and are proportional to the volume of the body, and *surface forces* that act on the control surface (such as the pressure forces due to hydrostatic pressure and shear stresses due to viscous effects) and are proportional to the surface area. The surface forces appear as the control volume is isolated from its surroundings for analysis, and the effect of the detached body is replaced by a force at that location. Note that pressure represents the compressive force applied on the fluid element by the surrounding fluid, and is always directed to the surface.

We express Newton's second law of motion for the control volume as

$$(\text{Mass})\begin{pmatrix}\text{Acceleration}\\ \text{in a specified direction}\end{pmatrix} = \begin{pmatrix}\text{Net force (body and surface)}\\ \text{acting in that direction}\end{pmatrix} \tag{6-22}$$

or

$$\delta m \cdot a_x = F_{\text{surface}, x} + F_{\text{body}, x} \tag{6-23}$$

where the mass of the fluid element within the control volume is

$$\delta m = \rho(dx \cdot dy \cdot 1) \tag{6-24}$$

Noting that flow is steady and two-dimensional and thus $u = u(x, y)$, the total differential of u is

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy \qquad \textbf{(6-25)}$$

Then the acceleration of the fluid element in the x direction becomes

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \qquad \textbf{(6-26)}$$

You may be tempted to think that acceleration is zero in steady flow since acceleration is the rate of change of velocity with time, and in steady flow there is no change with time. Well, a garden hose nozzle will tell us that this understanding is not correct. Even in steady flow and thus constant mass flow rate, water will accelerate through the nozzle (Fig. 6–21). *Steady* simply means no change with time at a specified location (and thus $\partial u/\partial t = 0$), but the value of a quantity may change from one location to another (and thus $\partial u/\partial x$ and $\partial u/\partial y$ may be different from zero). In the case of a nozzle, the velocity of water remains constant at a specified point, but it changes from inlet to the exit (water accelerates along the nozzle, which is the reason for attaching a nozzle to the garden hose in the first place).

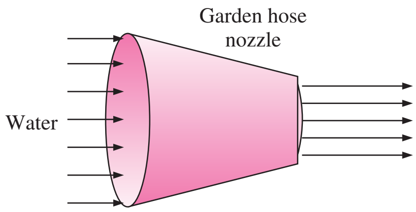


FIGURE 6–21
During steady flow, a fluid may not accelerate in time at a fixed point, but it may accelerate in space.

The forces acting on a surface are due to pressure and viscous effects. In two-dimensional flow, the *viscous stress* at any point on an imaginary surface within the fluid can be resolved into two perpendicular components: one normal to the surface called *normal stress* (which should not be confused with pressure) and another along the surface called *shear stress.* The normal stress is related to the velocity gradients $\partial u/\partial x$ and $\partial v/\partial y$, that are much smaller than $\partial u/\partial y$, to which shear stress is related. Neglecting the normal stresses for simplicity, the surface forces acting on the control volume in the x-direction will be as shown in Fig. 6–22. Then the net surface force acting in the x-direction becomes

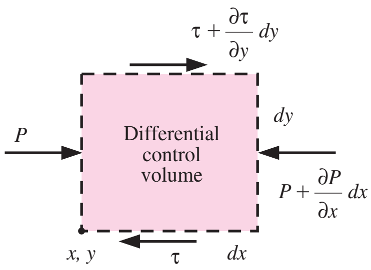


FIGURE 6–22
Differential control volume used in the derivation of x-momentum equation in velocity boundary layer in two-dimensional flow over a surface.

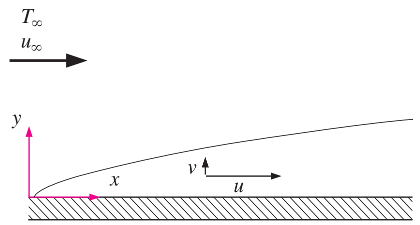
$$\begin{aligned} F_{\text{surface}, x} &= \left(\frac{\partial \tau}{\partial y}dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x}dx\right)(dy \cdot 1) = \left(\frac{\partial \tau}{\partial y} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1) \\ &= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1) \end{aligned} \qquad \textbf{(6-27)}$$

since $\tau = \mu(\partial u/\partial y)$. Substituting Eqs. 6-21, 6-23, and 6-24 into Eq. 6-20 and dividing by $dx \cdot dy \cdot 1$ gives

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \qquad \textbf{(6-28)}$$

This is the relation for the **conservation of momentum** in the x-direction, and is known as the ***x*-momentum equation.** Note that we would obtain the same result if we used momentum flow rates for the left-hand side of this equation instead of mass times acceleration. If there is a body force acting in the x-direction, it can be added to the right side of the equation provided that it is expressed per unit volume of the fluid.

In a boundary layer, the velocity component in the flow direction is much larger than that in the normal direction, and thus $u \gg v$, and $\partial v/\partial x$ and $\partial v/\partial y$ are



1) Velocity components:
$v \ll u$
2) Velocity grandients:
$\frac{\partial v}{\partial x} \approx 0, \frac{\partial v}{\partial y} \approx 0$
$\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$
3) Temperature gradients:
$\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$

FIGURE 6–23
Boundary layer approximations.

negligible. Also, u varies greatly with y in the normal direction from zero at the wall surface to nearly the free-stream value across the relatively thin boundary layer, while the variation of u with x along the flow is typically small. Therefore, $\partial u/\partial y \gg \partial u/\partial x$. Similarly, if the fluid and the wall are at different temperatures and the fluid is heated or cooled during flow, heat conduction will occur primarily in the direction normal to the surface, and thus $\partial T/\partial y \gg \partial T/\partial x$. That is, the velocity and temperature gradients normal to the surface are much greater than those along the surface. These simplifications are known as the **boundary layer approximations.** These approximations greatly simplify the analysis usually with little loss in accuracy, and make it possible to obtain analytical solutions for certain types of flow problems (Fig. 6–23).

When gravity effects and other body forces are negligible and the boundary layer approximations are valid, applying Newton's second law of motion on the volume element in the y-direction gives the *y-momentum equation* to be

$$\frac{\partial P}{\partial y} = 0 \qquad \textbf{(6-29)}$$

That is, *the variation of pressure in the direction normal to the surface is negligible,* and thus $P = P(x)$ and $\partial P/\partial x = dP/dx$. Then it follows that for a given x, the pressure in the boundary layer is equal to the pressure in the free stream, and the pressure determined by a separate analysis of fluid flow in the free stream (which is typically easier because of the absence of viscous effects) can readily be used in the boundary layer analysis.

The velocity components in the free stream region of a flat plate are $u = u_\infty$ = constant and $v = 0$. Substituting these into the x-momentum equations (Eq. 6-28) gives $\partial P/\partial x = 0$. Therefore, for flow over a flat plate, the pressure remains constant over the entire plate (both inside and outside the boundary layer).

Conservation of Energy Equation

The energy balance for any system undergoing any process is expressed as $E_{in} - E_{out} = \Delta E_{system}$, which states that the change in the energy content of a system during a process is equal to the difference between the energy input and the energy output. During a *steady-flow process,* the total energy content of a control volume remains constant (and thus $\Delta E_{system} = 0$), and the amount of energy entering a control volume in all forms must be equal to the amount of energy leaving it. Then the rate form of the general energy equation reduces for a steady-flow process to $\dot{E}_{in} - \dot{E}_{out} = 0$.

Noting that energy can be transferred by heat, work, and mass only, the energy balance for a steady-flow control volume can be written explicitly as

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by heat}} + (\dot{E}_{in} - \dot{E}_{out})_{\text{by work}} + (\dot{E}_{in} - \dot{E}_{out})_{\text{by mass}} = 0 \qquad \textbf{(6-30)}$$

The total energy of a flowing fluid stream per unit mass is $e_{stream} = h + \text{ke} + \text{pe}$ where h is the enthalpy (which is the sum of internal energy and flow energy), $\text{pe} = gz$ is the potential energy, and $\text{ke} = \mathcal{V}^2/2 = (u^2 + v^2)/2$ is the kinetic energy of the fluid per unit mass. The kinetic and potential energies are usually very small relative to enthalpy, and therefore it is common practice to neglect them (besides, it can be shown that if kinetic energy is included in the analysis below, all the terms due to this inclusion cancel each other). We assume the density ρ, specific heat C_p, viscosity μ, and the thermal conductivity k of the fluid to be constant. Then the energy of the fluid per unit mass can be expressed as $e_{stream} = h = C_pT$.

Energy is a scalar quantity, and thus energy interactions in all directions can be combined in one equation. Noting that mass flow rate of the fluid entering the control volume from the left is $\rho u(dy \cdot 1)$, the rate of energy transfer to the control volume by mass in the x-direction is, from Fig. 6–24,

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by mass, }x} = (\dot{m}e_{stream})_x - \left[(\dot{m}e_{stream})_x + \frac{\partial(\dot{m}e_{stream})_x}{\partial x}dx\right]$$
$$= -\frac{\partial[\rho u(dy \cdot 1)C_pT]}{\partial x}dx = -\rho C_p\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dx\,dy \quad \textbf{(6-31)}$$

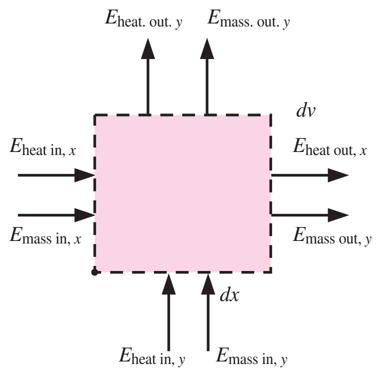


FIGURE 6–24
The energy transfers by heat and mass flow associated with a differential control volume in the thermal boundary layer in steady two-dimensional flow.

Repeating this for the y-direction and adding the results, the net rate of energy transfer to the control volume by mass is determined to be

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by mass}} = -\rho C_p\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dx\,dy - \rho C_p\left(v\frac{\partial T}{\partial y} + T\frac{\partial v}{\partial y}\right)dx\,dy$$
$$= -\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)dx\,dy \quad \textbf{(6-32)}$$

since $\partial u/\partial x + \partial v/\partial y = 0$ from the continuity equation.

The net rate of heat conduction to the volume element in the x-direction is

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by heat, }x} = \dot{Q}_x - \left(\dot{Q}_x + \frac{\partial \dot{Q}_x}{\partial x}dx\right)$$
$$= -\frac{\partial}{\partial x}\left(-k(dy \cdot 1)\frac{\partial T}{\partial x}\right)dx = k\frac{\partial^2 T}{\partial x^2}dx\,dy \quad \textbf{(6-33)}$$

Repeating this for the y-direction and adding the results, the net rate of energy transfer to the control volume by heat conduction becomes

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by heat}} = k\frac{\partial^2 T}{\partial x^2}dx\,dy + k\frac{\partial^2 T}{\partial y^2}dx\,dy = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)dx\,dy \quad \textbf{(6-34)}$$

Another mechanism of energy transfer to and from the fluid in the control volume is the work done by the body and surface forces. The work done by a body force is determined by multiplying this force by the velocity in the direction of the force and the volume of the fluid element, and this work needs to be considered only in the presence of significant gravitational, electric, or magnetic effects. The surface forces consist of the forces due to fluid pressure and the viscous shear stresses. The work done by pressure (the flow work) is already accounted for in the analysis above by using enthalpy for the microscopic energy of the fluid instead of internal energy. The shear stresses that result from viscous effects are usually very small, and can be neglected in many cases. This is especially the case for applications that involve low or moderate velocities.

Then the energy equation for the steady two-dimensional flow of a fluid with constant properties and negligible shear stresses is obtained by substituting Eqs. 6-32 and 6-34 into 6-30 to be

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad \textbf{(6-35)}$$

which states that *the net energy convected by the fluid out of the control volume is equal to the net energy transferred into the control volume by heat conduction.*

When the viscous shear stresses are not negligible, their effect is accounted for by expressing the energy equation as

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi \tag{6-36}$$

where the *viscous dissipation function* Φ is obtained after a lengthy analysis (see an advanced book such as the one by *Schlichting* (Ref. 9) for details) to be

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \tag{6-37}$$

Viscous dissipation may play a dominant role in high-speed flows, especially when the viscosity of the fluid is high (like the flow of oil in journal bearings). This manifests itself as a significant rise in fluid temperature due to the conversion of the kinetic energy of the fluid to thermal energy. Viscous dissipation is also significant for high-speed flights of aircraft.

For the special case of a stationary fluid, $u = v = 0$ and the energy equation reduces, as expected, to the steady two-dimensional heat conduction equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{6-38}$$

EXAMPLE 6–1 Temperature Rise of Oil in a Journal Bearing

The flow of oil in a journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. Such flows are known as Couette flow.

Consider two large isothermal plates separated by 2-mm-thick oil film. The upper plates moves at a constant velocity of 12 m/s, while the lower plate is stationary. Both plates are maintained at 20°C. (*a*) Obtain relations for the velocity and temperature distributions in the oil. (*b*) Determine the maximum temperature in the oil and the heat flux from the oil to each plate (Fig. 6–25).

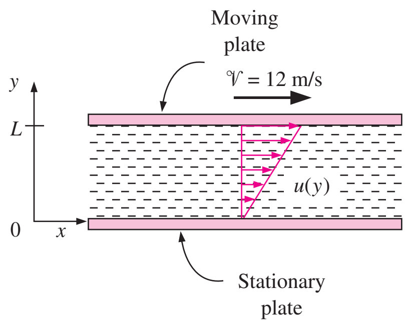


FIGURE 6–25
Schematic for Example 6–1.

SOLUTION Parallel flow of oil between two plates is considered. The velocity and temperature distributions, the maximum temperature, and the total heat transfer rate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible. **4** The plates are large so that there is no variation in the *z* direction.

Properties The properties of oil at 20°C are (Table A-10):

$$k = 0.145 \text{ W/m}\cdot\text{K} \quad \text{and} \quad \mu = 0.800 \text{ kg/m}\cdot\text{s} = 0.800 \text{ N}\cdot\text{s/m}^2$$

Analysis (*a*) We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation (Eq. 6-21) reduces to

$$\textit{Continuity:} \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow \frac{\partial u}{\partial x} = 0 \quad \rightarrow u = u(y)$$

Therefore, the x-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the x-momentum equation (Eq. 6-28) reduces to

$$\textit{x-momentum:}\ \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \quad \rightarrow \quad \frac{d^2u}{dy^2} = 0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y) = C_1 y + C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = \mathcal{V}$, and applying them gives the velocity distribution to be

$$u(y) = \frac{y}{L}\mathcal{V}$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on y only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation (Eqs. 6-36 and 6-37) reduce to

$$\textit{Energy:}\qquad 0 = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \quad \rightarrow \quad k\frac{d^2T}{dy^2} = -\mu\left(\frac{\mathcal{V}}{L}\right)^2$$

since $\partial u/\partial y = \mathcal{V}/L$. Dividing both sides by k and integrating twice give

$$T(y) = -\frac{\mu}{2k}\left(\frac{y}{L}\mathcal{V}\right)^2 + C_3 y + C_4$$

Applying the boundary conditions $T(0) = T_0$ and $T(L) = T_0$ gives the temperature distribution to be

$$T(y) = T_0 + \frac{\mu\mathcal{V}^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

(b) The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy} = \frac{\mu\mathcal{V}^2}{2kL}\left(1 - 2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for y,

$$\frac{dT}{dy} = \frac{\mu\mathcal{V}^2}{2kL}\left(1 - 2\frac{y}{L}\right) = 0 \quad \rightarrow \quad y = \frac{L}{2}$$

Therefore, maximum temperature will occur at mid plane, which is not surprising since both plates are maintained at the same temperature. The maximum temperature is the value of temperature at $y = L/2$,

$$T_{max} = T\left(\frac{L}{2}\right) = T_0 + \frac{\mu\mathcal{V}^2}{2k}\left(\frac{L/2}{L} - \frac{(L/2)^2}{L^2}\right) = T_0 + \frac{\mu\mathcal{V}^2}{8k}$$

$$= 20 + \frac{(0.8\ \text{N}\cdot\text{s/m}^2)(12\text{m/s})^2}{8(0.145\ \text{W/m}\cdot{}^\circ\text{C})}\left(\frac{1\text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{119^\circ C}$$

Heat flux at the plates is determined from the definition of heat flux,

$$\dot{q}_0 = -k \frac{dT}{dy}\bigg|_{y=0} = -k \frac{\mu \mathcal{V}^2}{2kL}(1-0) = -\frac{\mu \mathcal{V}^2}{2L}$$

$$= -\frac{(0.8 \text{ N}\cdot\text{s/m}^2)(12 \text{ m/s})^2}{2(0.002 \text{ m})}\left(\frac{1 W}{1 N \cdot m/s}\right) = -\mathbf{28{,}800 \text{ W/m}^2}$$

$$\dot{q}_L = -k \frac{dT}{dy}\bigg|_{y=L} = -k \frac{\mu \mathcal{V}^2}{2kL}(1-2) = \frac{\mu \mathcal{V}^2}{2L} = -\dot{q}_0 = \mathbf{28{,}800 \text{ W/m}^2}$$

Therefore, heat fluxes at the two plates are equal in magnitude but opposite in sign.

Discussion A temperature rise of 99°C confirms our suspicion that viscous dissipation is very significant. Also, the heat flux is equivalent to the rate of mechanical energy dissipation. Therefore, mechanical energy is being converted to thermal energy at a rate of 57.2 kW/m^2 of plate area to overcome friction in the oil. Finally, calculations are done using oil properties at 20°C, but the oil temperature turned out to be much higher. Therefore, knowing the strong dependence of viscosity on temperature, calculations should be repeated using properties at the average temperature of 70°C to improve accuracy.

6–8 ▪ SOLUTIONS OF CONVECTION EQUATIONS FOR A FLAT PLATE

Consider laminar flow of a fluid over a *flat plate,* as shown in Fig. 6–19. Surfaces that are slightly contoured such as turbine blades can also be approximated as flat plates with reasonable accuracy. The x-coordinate is measured along the plate surface from the leading edge of the plate in the direction of the flow, and y is measured from the surface in the normal direction. The fluid approaches the plate in the x-direction with a uniform upstream velocity, which is equivalent to the free stream velocity u_∞.

When viscous dissipation is negligible, the continuity, momentum, and energy equations (Eqs. 6-21, 6-28, and 6-35) reduce for steady, incompressible, laminar flow of a fluid with constant properties over a flat plate to

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \textbf{(6-39)}$$

Momentum: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad \textbf{(6-40)}$$

Energy: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad \textbf{(6-41)}$$

with the boundary conditions (Fig. 6–26)

$$\begin{aligned} &\text{At } x = 0: && u(0, y) = u_\infty, && T(0, y) = T_\infty \\ &\text{At } y = 0: && u(x, 0) = 0, && v(x, 0) = 0,\ T(x, 0) = T_s \\ &\text{As } y \to \infty: && u(x, \infty) = u_\infty, && T(x, \infty) = T_\infty \end{aligned} \quad \textbf{(6-42)}$$

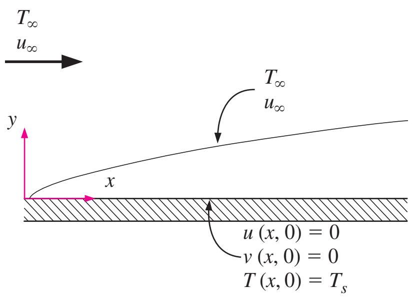


FIGURE 6–26
Boundary conditions for flow over a flat plate.

When fluid properties are assumed to be constant and thus independent of temperature, the first two equations can be solved separately for the velocity components u and v. Once the velocity distribution is available, we can determine the friction coefficient and the boundary layer thickness using their definitions. Also, knowing u and v, the temperature becomes the only unknown in the last equation, and it can be solved for temperature distribution.

The continuity and momentum equations were first solved in 1908 by the German engineer H. Blasius, a student of L. Prandtl. This was done by transforming the two partial differential equations into a single ordinary differential equation by introducing a new independent variable, called the **similarity variable.** The finding of such a variable, assuming it exists, is more of an art than science, and it requires to have a good insight of the problem.

Noticing that the general shape of the velocity profile remains the same along the plate, Blasius reasoned that the nondimensional velocity profile u/u_∞ should remain unchanged when plotted against the nondimensional distance y/δ, where δ is the thickness of the local velocity boundary layer at a given x. That is, although both δ and u at a given y vary with x, the velocity u at a fixed y/δ remains constant. Blasius was also aware from the work of Stokes that δ is proportional to $\sqrt{\nu x/u_\infty}$, and thus he defined a *dimensionless similarity variable* as

$$\eta = y\sqrt{\frac{u_\infty}{\nu x}} \tag{6-43}$$

and thus u/u_∞ = function(η). He then introduced a *stream function* $\psi(x, y)$ as

$$u = \frac{\partial \psi}{\partial y} \qquad \text{and} \qquad v = -\frac{\partial \psi}{\partial x} \tag{6-44}$$

so that the continuity equation (Eq. 6-39) is automatically satisfied and thus eliminated (this can be verified easily by direct substitution). He then defined a function $f(\eta)$ as the dependent variable as

$$f(\eta) = \frac{\psi}{u_\infty\sqrt{\nu x/u_\infty}} \tag{6-45}$$

Then the velocity components become

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = u_\infty\sqrt{\frac{\nu x}{u_\infty}}\frac{df}{d\eta}\sqrt{\frac{u_\infty}{\nu x}} = u_\infty\frac{df}{d\eta} \tag{6-46}$$

$$v = -\frac{\partial \psi}{\partial x} = -u_\infty\sqrt{\frac{\nu x}{u_\infty}}\frac{\partial f}{\partial x} - \frac{u_\infty}{2}\sqrt{\frac{\nu}{u_\infty x}}\,f = \frac{1}{2}\sqrt{\frac{u_\infty \nu}{x}}\left(\eta\frac{df}{d\eta} - f\right) \tag{6-47}$$

By differentiating these u and v relations, the derivatives of the velocity components can be shown to be

$$\frac{\partial u}{\partial x} = -\frac{u_\infty}{2x}\eta\frac{d^2 f}{d\eta^2}, \qquad \frac{\partial u}{\partial y} = u_\infty\sqrt{\frac{u_\infty}{\nu x}}\frac{d^2 f}{d\eta^2}, \qquad \frac{\partial^2 u}{\partial y^2} = \frac{u_\infty^2}{\nu x}\frac{d^3 f}{d\eta^3} \tag{6-48}$$

Substituting these relations into the momentum equation and simplifying, we obtain

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0 \tag{6-49}$$

which is a third-order nonlinear differential equation. Therefore, the system of two partial differential equations is transformed into a single ordinary differential equation by the use of a similarity variable. Using the definitions of f and η, the boundary conditions in terms of the similarity variables can be expressed as

$$f(0) = 0, \qquad \left.\frac{df}{d\eta}\right|_{\eta=0} = 0, \qquad \text{and} \qquad \left.\frac{df}{d\eta}\right|_{\eta=\infty} = 1 \tag{6-50}$$

The transformed equation with its associated boundary conditions cannot be solved analytically, and thus an alternative solution method is necessary. The problem was first solved by Blasius in 1908 using a power series expansion approach, and this original solution is known as the *Blasius solution.* The problem is later solved more accurately using different numerical approaches, and results from such a solution are given in Table 6–3. The nondimensional velocity profile can be obtained by plotting u/u_∞ against η. The results obtained by this simplified analysis are in excellent agreement with experimental results.

TABLE 6–3

Similarity function f and its derivatives for laminar boundary layer along a flat plate.

η	f	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
1.5	0.370	0.487	0.303
2.0	0.650	0.630	0.267
2.5	0.996	0.751	0.217
3.0	1.397	0.846	0.161
3.5	1.838	0.913	0.108
4.0	2.306	0.956	0.064
4.5	2.790	0.980	0.034
5.0	3.283	0.992	0.016
5.5	3.781	0.997	0.007
6.0	4.280	0.999	0.002
∞	∞	1	0

Recall that we defined the boundary layer thickness as the distance from the surface for which $u/u_\infty = 0.99$. We observe from Table 6–3 that the value of η corresponding to $u/u_\infty = 0.992$ is $\eta = 5.0$. Substituting $\eta = 5.0$ and $y = \delta$ into the definition of the similarity variable (Eq. 6-43) gives $5.0 = \delta\sqrt{u_\infty/\nu x}$. Then the velocity boundary layer thickness becomes

$$\delta = \frac{5.0}{\sqrt{u_\infty/\nu x}} = \frac{5.0x}{\sqrt{\text{Re}_x}} \tag{6-51}$$

since $\text{Re}_x = u_\infty x/\nu$, where x is the distance from the leading edge of the plate. Note that the boundary layer thickness increases with increasing kinematic viscosity ν and with increasing distance from the leading edge x, but it decreases with increasing free-stream velocity u_∞. Therefore, a large free-stream velocity will suppress the boundary layer and cause it to be thinner.

The shear stress on the wall can be determined from its definition and the $\partial u/\partial y$ relation in Eq. 6-48:

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu u_\infty \sqrt{\frac{u_\infty}{\nu x}} \left.\frac{d^2f}{d\eta^2}\right|_{\eta=0} \tag{6-52}$$

Substituting the value of the second derivative of f at $\eta = 0$ from Table 6–3 gives

$$\tau_w = 0.332u_\infty \sqrt{\frac{\rho\mu u_\infty}{x}} = \frac{0.332\rho u_\infty^2}{\sqrt{\text{Re}_x}} \tag{6-53}$$

Then the local skin friction coefficient becomes

$$C_{f,x} = \frac{\tau_w}{\rho \mathcal{V}^2/2} = \frac{\tau_w}{\rho u_\infty^2/2} = 0.664\ \text{Re}_x^{-1/2} \tag{6-54}$$

Note that unlike the boundary layer thickness, wall shear stress and the skin friction coefficient decrease along the plate as $x^{-1/2}$.

The Energy Equation

Knowing the velocity profile, we are now ready to solve the energy equation for temperature distribution for the case of constant wall temperature T_s. First we introduce the dimensionless temperature θ as

$$\theta(x, y) = \frac{T(x, y) - T_s}{T_\infty - T_s} \tag{6-55}$$

Noting that both T_s and T_∞ are constant, substitution into the energy equation gives

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \alpha \frac{\partial^2 \theta}{\partial y^2} \tag{6-56}$$

Temperature profiles for flow over an isothermal flat plate are similar, just like the velocity profiles, and thus we expect a similarity solution for temperature to exist. Further, the thickness of the thermal boundary layer is proportional to $\sqrt{\nu x/u_\infty}$, just like the thickness of the velocity boundary layer, and thus the similarity variable is also η, and $\theta = \theta(\eta)$. Using the chain rule and substituting the u and v expressions into the energy equation gives

$$u_\infty \frac{df}{d\eta}\frac{d\theta}{d\eta}\frac{\partial \eta}{\partial x} + \frac{1}{2}\sqrt{\frac{u_\infty \nu}{x}}\left(\eta \frac{df}{d\eta} - f\right)\frac{d\theta}{d\eta}\frac{\partial \eta}{\partial y} = \alpha \frac{d^2\theta}{d\eta^2}\left(\frac{\partial \eta}{\partial y}\right)^2 \tag{6-57}$$

Simplifying and noting that $\Pr = \nu/\alpha$ give

$$2\frac{d^2\theta}{d\eta^2} + \Pr f \frac{d\theta}{d\eta} = 0 \tag{6-58}$$

with the boundary conditions $\theta(0) = 0$ and $\theta(\infty) = 1$. Obtaining an equation for θ as a function of η alone confirms that the temperature profiles are similar, and thus a similarity solution exists. Again a closed-form solution cannot be obtained for this boundary value problem, and it must be solved numerically.

It is interesting to note that for $\Pr = 1$, this equation reduces to Eq. 6-49 when θ is replaced by $df/d\eta$, which is equivalent to u/u_∞ (see Eq. 6-46). The boundary conditions for θ and $df/d\eta$ are also identical. Thus we conclude that the velocity and thermal boundary layers coincide, and the nondimensional velocity and temperature profiles (u/u_∞ and θ) are identical for steady, incompressible, laminar flow of a fluid with constant properties and $\Pr = 1$ over an isothermal flat plate (Fig. 6–27). The value of the temperature gradient at the surface ($y = 0$ or $\eta = 0$) in this case is, from Table 6–3, $d\theta/d\eta = d^2f/d\eta^2 = 0.332$.

Equation 6-58 is solved for numerous values of Prandtl numbers. For $\Pr > 0.6$, the nondimensional temperature gradient at the surface is found to be proportional to $\Pr^{1/3}$, and is expressed as

$$\left.\frac{d\theta}{d\eta}\right|_{\eta=0} = 0.332 \Pr^{1/3} \tag{6-59}$$

The temperature gradient at the surface is

$$\begin{aligned} \left.\frac{\partial T}{\partial y}\right|_{y=0} &= (T_\infty - T_s)\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left.\frac{d\theta}{d\eta}\right|_{\eta=0}\left.\frac{\partial \eta}{\partial y}\right|_{y=0} \\ &= 0.332 \Pr^{1/3}(T_\infty - T_s)\sqrt{\frac{u_\infty}{\nu x}} \end{aligned} \tag{6-60}$$

Then the local convection coefficient and Nusselt number become

$$h_x = \frac{q_s}{T_s - T_\infty} = \frac{-k(\partial T/\partial y)|_{y=0}}{T_s - T_\infty} = 0.332 \Pr^{1/3} k\sqrt{\frac{u_\infty}{\nu x}} \tag{6-61}$$

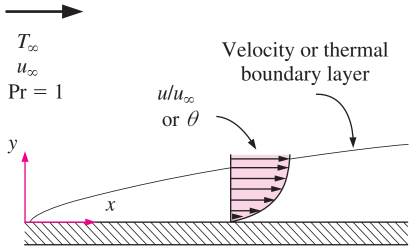


FIGURE 6–27
When $\Pr = 1$, the velocity and thermal boundary layers coincide, and the nondimensional velocity and temperature profiles are identical for steady, incompressible, laminar flow over a flat plate.

and

$$Nu_x = \frac{h_x x}{k} = 0.332\ Pr^{1/3} Re_x^{1/2} \qquad Pr > 0.6 \tag{6-62}$$

The Nu_x values obtained from this relation agree well with measured values.

Solving Eq. 6-58 numerically for the temperature profile for different Prandtl numbers, and using the definition of the thermal boundary layer, it is determined that $\delta/\delta_t \cong Pr^{1/3}$. Then the thermal boundary layer thickness becomes

$$\delta_t = \frac{\delta}{Pr^{1/3}} = \frac{5.0x}{Pr^{1/3}\sqrt{Re_x}} \tag{6-63}$$

Note that these relations are valid only for laminar flow over an isothermal flat plate. Also, the effect of variable properties can be accounted for by evaluating all such properties at the film temperature defined as $T_f = (T_s + T_\infty)/2$.

The Blasius solution gives important insights, but its value is largely historical because of the limitations it involves. Nowadays both laminar and turbulent flows over surfaces are routinely analyzed using numerical methods.

6–9 ▪ NONDIMENSIONALIZED CONVECTION EQUATIONS AND SIMILARITY

When viscous dissipation is negligible, the continuity, momentum, and energy equations for steady, incompressible, laminar flow of a fluid with constant properties are given by Eqs. 6-21, 6-28, and 6-35.

These equations and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by relevant and meaningful constant quantities: all lengths by a characteristic length L (which is the length for a plate), all velocities by a reference velocity $\mathcal{V}$ (which is the free stream velocity for a plate), pressure by $\rho\mathcal{V}^2$ (which is twice the free stream dynamic pressure for a plate), and temperature by a suitable temperature difference (which is $T_\infty - T_s$ for a plate). We get

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{\mathcal{V}}, \quad v^* = \frac{v}{\mathcal{V}}, \quad P^* = \frac{P}{\rho\mathcal{V}^2}, \quad \text{and} \quad T^* = \frac{T - T_s}{T_\infty - T_s}$$

where the asterisks are used to denote nondimensional variables. Introducing these variables into Eqs. 6-21, 6-28, and 6-35 and simplifying give

Continuity:
$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{6-64}$$

Momentum:
$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{dP^*}{dx^*} \tag{6-65}$$

Energy:
$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L\,Pr}\frac{\partial^2 T^*}{\partial y^{*2}} \tag{6-66}$$

with the boundary conditions

$$u^*(0, y^*) = 1, \quad u^*(x^*, 0) = 0, \quad u^*(x^*, \infty) = 1, \quad v^*(x^*, 0) = 0, \tag{6-67}$$
$$T^*(0, y^*) = 1, \quad T^*(x^*, 0) = 0, \quad T^*(x^*, \infty) = 1$$

where $\text{Re}_L = \mathcal{V}L/\nu$ is the dimensionless Reynolds number and $\text{Pr} = \nu/\alpha$ is the Prandtl number. For a given type of geometry, the solutions of problems with the same Re and Nu numbers are similar, and thus Re and Nu numbers serve as *similarity parameters.* Two physical phenomena are *similar* if they have the same dimensionless forms of governing differential equations and boundary conditions (Fig. 6–28).

A major advantage of nondimensionalizing is the significant reduction in the number of parameters. The original problem involves 6 parameters (L, $\mathcal{V}$, T_∞, T_s, ν, α), but the nondimensionalized problem involves just 2 parameters (Re_L and Pr). For a given geometry, problems that have the same values for the similarity parameters have identical solutions. For example, determining the convection heat transfer coefficient for flow over a given surface will require numerical solutions or experimental investigations for several fluids, with several sets of velocities, surface lengths, wall temperatures, and free stream temperatures. The same information can be obtained with far fewer investigations by grouping data into the dimensionless Re and Pr numbers. Another advantage of similarity parameters is that they enable us to group the results of a large number of experiments and to report them conveniently in terms of such parameters (Fig. 6–29).

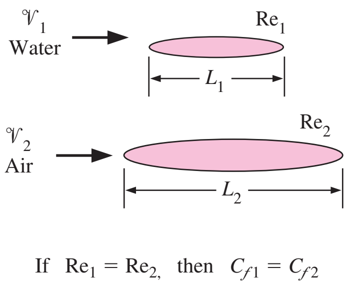


FIGURE 6–28
Two geometrically similar bodies have the same value of friction coefficient at the same Reynolds number.

Parameters before nondimensionalizing

L, $\mathcal{V}$, T_∞, T_s, ν, α

Parameters after nondimensionalizing:

Re, Pr

FIGURE 6–29
The number of parameters is reduced greatly by nondimensionalizing the convection equations.

6–10 ▪ FUNCTIONAL FORMS OF FRICTION AND CONVECTION COEFFICIENTS

The three nondimensionalized boundary layer equations (Eqs. 6-64, 6-65, and 6-66) involve three unknown functions u^*, v^*, and T^*, two independent variables x^* and y^*, and two parameters Re_L and Pr. The pressure $P^*(x^*)$ depends on the geometry involved (it is constant for a flat plate), and it has the same value inside and outside the boundary layer at a specified x^*. Therefore, it can be determined separately from the free stream conditions, and dP^*/dx^* in Eq. 6-65 can be treated as a known function of x^*. Note that the boundary conditions do not introduce any new parameters.

For a given geometry, the solution for u^* can be expressed as

$$u^* = f_1(x^*, y^*, \text{Re}_L) \tag{6-68}$$

Then the shear stress at the surface becomes

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{\mu \mathcal{V}}{L} \frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0} = \frac{\mu \mathcal{V}}{L} f_2(x^*, \text{Re}_L) \tag{6-69}$$

Substituting into its definition gives the local friction coefficient,

$$C_{f,x} = \frac{\tau_s}{\rho \mathcal{V}^2/2} = \frac{\mu \mathcal{V}/L}{\rho \mathcal{V}^2/2} f_2(x^*, \text{Re}_L) = \frac{2}{\text{Re}_L} f_2(x^*, \text{Re}_l) = f_3(x^*, \text{Re}_L) \tag{6-70}$$

Thus we conclude that the friction coefficient for a given geometry can be expressed in terms of the Reynolds number Re and the dimensionless space variable x^* alone (instead of being expressed in terms of x, L, $\mathcal{V}$, ρ, and μ). This is a very significant finding, and shows the value of nondimensionalized equations.

Similarly, the solution of Eq. 6-66 for the dimensionless temperature T^* for a given geometry can be expressed as

$$T^* = g_1(x^*, y^*, \text{Re}_L, \text{Pr}) \tag{6-71}$$

Using the definition of T^*, the convection heat transfer coefficient becomes

$$h = \frac{-k(\partial T/\partial y)|_{y=0}}{T_s - T_\infty} = \frac{-k(T_\infty - T_s)}{L(T_s - T_\infty)} \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = \frac{k}{L}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} \tag{6-72}$$

Substituting this into the Nusselt number relation gives [or alternately, we can rearrange the relation above in dimensionless form as $hL/k = (\partial T^*/\partial y^*)|_{y^*=0}$ and define the dimensionless group hL/k as the Nusselt number]

$$\text{Nu}_x = \frac{hL}{k} = \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = g_2(x^*, \text{Re}_L, \text{Pr}) \tag{6-73}$$

Note that the Nusselt number is equivalent to the *dimensionless temperature gradient at the surface,* and thus it is properly referred to as the dimensionless heat transfer coefficient (Fig. 6–30). Also, the Nusselt number for a given geometry can be expressed in terms of the Reynolds number Re, the Prandtl number Pr, and the space variable x^*, and such a relation can be used for different fluids flowing at different velocities over similar geometries of different lengths.

The average friction and heat transfer coefficients are determined by integrating $C_{f,x}$ and Nu_x over the surface of the given body with respect to x^* from 0 to 1. Integration will remove the dependence on x^*, and the average friction coefficient and Nusselt number can be expressed as

$$C_f = f_4(\text{Re}_L) \quad \text{and} \quad \text{Nu} = g_3(\text{Re}_L, \text{Pr}) \tag{6-74}$$

These relations are extremely valuable as they state that for a given geometry, the friction coefficient can be expressed as a function of Reynolds number alone, and the Nusselt number as a function of Reynolds and Prandtl numbers alone (Fig. 6–31). Therefore, experimentalists can study a problem with a minimum number of experiments, and report their friction and heat transfer coefficient measurements conveniently in terms of Reynolds and Prandtl numbers. For example, a friction coefficient relation obtained with air for a given surface can also be used for water at the same Reynolds number. But it should be kept in mind that the validity of these relations is limited by the limitations on the boundary layer equations used in the analysis.

The experimental data for heat transfer is often represented with reasonable accuracy by a simple power-law relation of the form

$$\text{Nu} = C\,\text{Re}_L^m\,\text{Pr}^n \tag{6-75}$$

where m and n are constant exponents (usually between 0 and 1), and the value of the constant C depends on geometry. Sometimes more complex relations are used for better accuracy.

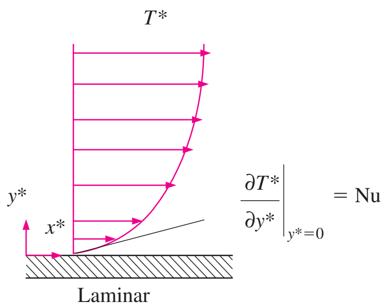


FIGURE 6–30
The Nusselt number is equivalent to the dimensionless temperature gradient at the surface.

Local Nusselt number:

$\text{Nu}_x = \text{function}\,(x^*, \text{Re}_L, \text{Pr})$

Average Nusselt number:

$\text{Nu} = \text{function}\,(\text{Re}_L, \text{Pr})$

A common form of Nusselt number:

$Nu = C\,\text{Re}_L^m\,\text{Pr}^n$

FIGURE 6–31
For a given geometry, the average Nusselt number is a function of Reynolds and Prandtl numbers.

6–11 ▪ ANALOGIES BETWEEN MOMENTUM AND HEAT TRANSFER

In forced convection analysis, we are primarily interested in the determination of the quantities C_f (to calculate shear stress at the wall) and Nu (to calculate heat transfer rates). Therefore, it is very desirable to have a relation between

C_f and Nu so that we can calculate one when the other is available. Such relations are developed on the basis of the similarity between momentum and heat transfers in boundary layers, and are known as *Reynolds analogy* and *Chilton–Colburn analogy.*

Reconsider the nondimensionalized momentum and energy equations for steady, incompressible, laminar flow of a fluid with constant properties and negligible viscous dissipation (Eqs. 6-65 and 6-66). When Pr $= 1$ (which is approximately the case for gases) and $\partial P^*/\partial x^* = 0$ (which is the case when, $u = u_\infty = \mathcal{V} = constant$ in the free stream, as in flow over a flat plate), these equations simplify to

Momentum:
$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\text{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} \tag{6-76}$$

Energy:
$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{\text{Re}_L}\frac{\partial^2 T^*}{\partial y^{*2}} \tag{6-77}$$

which are exactly of the same form for the dimensionless velocity u^* and temperature T^*. The boundary conditions for u^* and T^* are also identical. Therefore, the functions u^* and T^* must be identical, and thus the first derivatives of u^* and T^* at the surface must be equal to each other,

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} \tag{6-78}$$

Then from Eqs. 6-69, 6-70, and 6-73 we have

$$C_{f,x}\frac{\text{Re}_L}{2} = \text{Nu}_x \qquad (\text{Pr} = 1) \tag{6-79}$$

which is known as the **Reynolds analogy** (Fig. 6–32). This is an important analogy since it allows us to determine the heat transfer coefficient for fluids with Pr ≈ 1 from a knowledge of friction coefficient which is easier to measure. Reynolds analogy is also expressed alternately as

$$\frac{C_{f,x}}{2} = \text{St}_x \qquad (\text{Pr} = 1) \tag{6-80}$$

where

$$\text{St} = \frac{h}{\rho C_p \mathcal{V}} = \frac{\text{Nu}}{\text{Re}_L\text{Pr}} \tag{6-81}$$

is the **Stanton number,** which is also a dimensionless heat transfer coefficient.

Reynolds analogy is of limited use because of the restrictions Pr $= 1$ and $\partial P^*/\partial x^* = 0$ on it, and it is desirable to have an analogy that is applicable over a wide range of Pr. This is done by adding a Prandtl number correction. The friction coefficient and Nusselt number for a flat plate are determined in Section 6-8 to be

$$C_{f,x} = 0.664\ \text{Re}_x^{-1/2} \qquad \text{and} \qquad \text{Nu}_x = 0.332\ \text{Pr}^{1/3}\ \text{Re}_x^{1/2} \tag{6-82}$$

Taking their ratio and rearranging give the desired relation, known as the **modified Reynolds analogy** or **Chilton–Colburn analogy,**

Profiles: $u^* = T$

Gradients: $\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$

Analogy: $C_{f,x}\frac{\text{Re}_L}{2} = \text{Nu}_x$

FIGURE 6–32
When Pr $= 1$ and $\partial P^*/\partial x^* \approx 0$, the nondimensional velocity and temperature profiles become identical, and Nu is related to C_f by Reynolds analogy.

$$C_{f,x}\frac{Re_L}{2} = \text{Nu}_x\,\text{Pr}^{-1/3} \quad \text{or} \quad \frac{C_{f,x}}{2} = \frac{h_x}{\rho C_p \mathcal{V}}\text{Pr}^{2/3} \equiv j_H \tag{6-83}$$

for $0.6 < \text{Pr} < 60$. Here j_H is called the *Colburn j-factor.* Although this relation is developed using relations for laminar flow over a flat plate (for which $\partial P^*/\partial x^* = 0$), experimental studies show that it is also applicable approximately for turbulent flow over a surface, even in the presence of pressure gradients. For laminar flow, however, the analogy is not applicable unless $\partial P^*/\partial x^* \approx 0$. Therefore, it does not apply to laminar flow in a pipe. Analogies between C_f and Nu that are more accurate are also developed, but they are more complex and beyond the scope of this book. The analogies given above can be used for both local and average quantities.

EXAMPLE 6–2 Finding Convection Coefficient from Drag Measurement

A 2-m × 3-m flat plate is suspended in a room, and is subjected to air flow parallel to its surfaces along its 3-m-long side. The free stream temperature and velocity of air are 20°C and 7 m/s. The total drag force acting on the plate is measured to be 0.86 N. Determine the average convection heat transfer coefficient for the plate (Fig. 6–33).

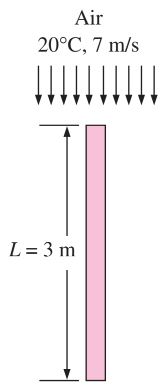


FIGURE 6–33
Schematic for Example 6–2.

SOLUTION A flat plate is subjected to air flow, and the drag force acting on it is measured. The average convection coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 20°C and 1 atm are (Table A-15):

$$\rho = 1.204 \text{ kg/m}^3, \quad C_p = 1.007 \text{ kJ/kg} \cdot \text{K}, \quad \text{Pr} = 0.7309$$

Analysis The flow is along the 3-m side of the plate, and thus the characteristic length is $L = 3$ m. Both sides of the plate are exposed to air flow, and thus the total surface area is

$$A_s = 2WL = 2(2 \text{ m})(3 \text{ m}) = 12 \text{ m}^2$$

For flat plates, the drag force is equivalent to friction force. The average friction coefficient C_f can be determined from Eq. 6-11,

$$F_f = C_f A_s \frac{\rho \mathcal{V}^2}{2}$$

Solving for C_f and substituting,

$$C_f = \frac{F_f}{\rho A_s \mathcal{V}^2/2} = \frac{0.86 \text{ N}}{(1.204 \text{ kg/m}^3)(12 \text{ m}^2)(7 \text{ m/s})^2/2}\left(\frac{1\text{kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) = 0.00243$$

Then the average heat transfer coefficient can be determined from the modified Reynolds analogy (Eq. 6-83) to be

$$h = \frac{C_f}{2}\frac{\rho \mathcal{V} C_p}{\text{Pr}^{2/3}} = \frac{0.00243}{2}\frac{(1.204 \text{ kg/m}^3)(7 \text{ m/s})(1007 \text{ J/kg} \cdot \text{°C})}{0.7309^{2/3}} = 12.7 \text{ W/m}^2 \cdot \text{°C}$$

Discussion This example shows the great utility of momentum-heat transfer analogies in that the convection heat transfer coefficient can be determined from a knowledge of friction coefficient, which is easier to determine.

SUMMARY

Convection heat transfer is expressed by *Newton's law of cooling* as

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

where h is the convection heat transfer coefficient, T_s is the surface temperature, and T_∞ is the free-stream temperature. The convection coefficient is also expressed as

$$h = \frac{-k_{fluid}(\partial T/\partial y)_{y=0}}{T_s - T_\infty}$$

The *Nusselt number,* which is the dimensionless heat transfer coefficient, is defined as

$$\text{Nu} = \frac{hL_c}{k}$$

where k is the thermal conductivity of the fluid and L_c is the characteristic length.

The highly ordered fluid motion characterized by smooth streamlines is called *laminar.* The highly disordered fluid motion that typically occurs at high velocities is characterized by velocity fluctuations is called *turbulent.* The random and rapid fluctuations of groups of fluid particles, called *eddies,* provide an additional mechanism for momentum and heat transfer.

The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the *velocity boundary layer.* The *boundary layer thickness,* δ, is defined as the distance from the surface at which $u = 0.99u_\infty$. The hypothetical line of $u = 0.99u_\infty$ divides the flow over a plate into the *boundary layer region* in which the viscous effects and the velocity changes are significant, and the *inviscid flow region,* in which the frictional effects are negligible.

The friction force per unit area is called *shear stress,* and the shear stress at the wall surface is expressed as

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad \text{or} \qquad \tau_s = C_f \frac{\rho \mathcal{V}^2}{2}$$

where μ is the dynamic viscosity, $\mathcal{V}$ is the upstream velocity, and C_f is the dimensionless *friction coefficient.* The property $\nu = \mu/\rho$ is the *kinematic viscosity.* The friction force over the entire surface is determined from

$$F_f = C_f A_s \frac{\rho \mathcal{V}^2}{2}$$

The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is the *thermal boundary layer.* The *thickness* of the thermal boundary layer δ_t at any location along the surface is the distance from the surface at which the temperature difference $T - T_s$ equals $0.99(T_\infty - T_s)$. The relative thickness of the velocity and the thermal boundary layers is best described by the dimensionless *Prandtl number,* defined as

$$\text{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

For external flow, the dimensionless *Reynolds number* is expressed as

$$\text{Re} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{\mathcal{V}L_c}{\nu} = \frac{\rho \mathcal{V} L_c}{\mu}$$

For a flat plate, the characteristic length is the distance x from the leading edge. The Reynolds number at which the flow becomes turbulent is called the *critical Reynolds number.* For flow over a flat plate, its value is taken to be $\text{Re}_{cr} = \mathcal{V}x_{cr}/\nu = 5 \times 10^5$.

The continuity, momentum, and energy equations for steady two-dimensional incompressible flow with constant properties are determined from mass, momentum, and energy balances to be

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$

Energy: $$\rho C_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi$$

where the *viscous dissipation function* Φ is

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2$$

Using the boundary layer approximations and a similarity variable, these equations can be solved for parallel steady incompressible flow over a flat plate, with the following results:

Velocity boundary layer thickness: $$\delta = \frac{5.0}{\sqrt{\mathcal{V}/\nu x}} = \frac{5.0x}{\sqrt{\text{Re}_x}}$$

Local friction coefficient: $$C_{f,x} = \frac{\tau_w}{\rho\mathcal{V}^2/2} = 0.664\,\text{Re}_x^{-1/2}$$

Local Nusselt number: $$\text{Nu}_x = \frac{h_x x}{k} = 0.332\,\text{Pr}^{1/3}\,\text{Re}_x^{1/2}$$

Thermal boundary layer thickness: $$\delta_t = \frac{\delta}{\text{Pr}^{1/3}} = \frac{5.0x}{\text{Pr}^{1/3}\sqrt{\text{Re}_x}}$$

The average friction coefficient and Nusselt number are expressed in functional form as

$$C_f = f_4(\text{Re}_L) \qquad \text{and} \qquad \text{Nu} = g_3(\text{Re}_L, \text{Pr})$$

The Nusselt number can be expressed by a simple power-law relation of the form

$$\text{Nu} = C\,\text{Re}_L^m\,\text{Pr}^n$$

where m and n are constant exponents, and the value of the constant C depends on geometry. The *Reynolds analogy* relates the convection coefficient to the friction coefficient for fluids with $\text{Pr} \approx 1$, and is expressed as

$$C_{f,x}\frac{\text{Re}_L}{2} = \text{Nu}_x \qquad \text{or} \qquad \frac{C_{f,x}}{2} = \text{St}_x$$

where

$$\text{St} = \frac{h}{\rho C_p \mathcal{V}} = \frac{\text{Nu}}{\text{Re}_L\,\text{Pr}}$$

is the *Stanton number*. The analogy is extended to other Prandtl numbers by the *modified Reynolds analogy* or *Chilton–Colburn analogy*, expressed as

$$C_{f,x}\frac{\text{Re}_L}{2} = \text{Nu}_x\text{Pr}^{-1/3}$$

or

$$\frac{C_{f,x}}{2} = \frac{h_x}{\rho C_p \mathcal{V}}\text{Pr}^{2/3} \equiv j_H \quad (0.6 < \text{Pr} < 60)$$

These analogies are also applicable approximately for turbulent flow over a surface, even in the presence of pressure gradients.

REFERENCES AND SUGGESTED READING

1. H. Blasius. "The Boundary Layers in Fluids with Little Friction (in German)." *Z. Math. Phys.,* 56, 1 (1908); pp. 1–37; English translation in National Advisory Committee for Aeronautics Technical Memo No. 1256, February 1950.
2. R. W. Fox and A. T. McDonald. *Introduction to Fluid Mechanics.* 5th. ed. New York, Wiley, 1999.
3. J. P. Holman. *Heat Transfer.* 9th ed. New York: McGraw-Hill, 2002.
4. F. P. Incropera and D. P. DeWitt. *Introduction to Heat Transfer.* 4th ed. New York: John Wiley & Sons, 2002.
5. W. M. Kays and M. E. Crawford. *Convective Heat and Mass Transfer.* 3rd ed. New York: McGraw-Hill, 1993.
6. F. Kreith and M. S. Bohn. *Principles of Heat Transfer.* 6th ed. Pacific Grove, CA: Brooks/Cole, 2001.
7. M. N. Özisik. *Heat Transfer—A Basic Approach.* New York: McGraw-Hill, 1985.
8. O. Reynolds. "On the Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and the Law of Resistance in Parallel Channels." *Philosophical Transactions of the Royal Society of London* 174 (1883), pp. 935–82.
9. H. Schlichting. *Boundary Layer Theory.* 7th ed. New York: McGraw-Hill, 1979.
10. G. G. Stokes. "On the Effect of the Internal Friction of Fluids on the Motion of Pendulums." *Cambridge Philosophical Transactions,* IX, 8, 1851.
11. N. V. Suryanarayana. *Engineering Heat Transfer.* St. Paul, MN: West, 1995.
12. F. M. White. *Heat and Mass Transfer.* Reading, MA: Addison-Wesley, 1988.

PROBLEMS*

Mechanism and Types of Convection

6–1C What is forced convection? How does it differ from natural convection? Is convection caused by winds forced or natural convection?

6–2C What is external forced convection? How does it differ from internal forced convection? Can a heat transfer system involve both internal and external convection at the same time? Give an example.

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with an EES-CD icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

6–3C In which mode of heat transfer is the convection heat transfer coefficient usually higher, natural convection or forced convection? Why?

6–4C Consider a hot baked potato. Will the potato cool faster or slower when we blow the warm air coming from our lungs on it instead of letting it cool naturally in the cooler air in the room? Explain.

6–5C What is the physical significance of the Nusselt number? How is it defined?

6–6C When is heat transfer through a fluid conduction and when is it convection? For what case is the rate of heat transfer higher? How does the convection heat transfer coefficient differ from the thermal conductivity of a fluid?

6–7C Define incompressible flow and incompressible fluid. Must the flow of a compressible fluid necessarily be treated as compressible?

6–8 During air cooling of potatoes, the heat transfer coefficient for combined convection, radiation, and evaporation is determined experimentally to be as shown:

Air Velocity, m/s	Heat Transfer Coefficient, W/m² · °C
0.66	14.0
1.00	19.1
1.36	20.2
1.73	24.4

Consider a 10-cm-diameter potato initially at 20°C with a thermal conductivity of 0.49 W/m · °C. Potatoes are cooled by refrigerated air at 5°C at a velocity of 1 m/s. Determine the initial rate of heat transfer from a potato, and the initial value of the temperature gradient in the potato at the surface.
Answers: 9.0 W, −585°C/m

6–9 An average man has a body surface area of 1.8 m^2 and a skin temperature of 33°C. The convection heat transfer coefficient for a clothed person walking in still air is expressed as $h = 8.6\mathcal{V}^{0.53}$ for $0.5 < \mathcal{V} < 2$ m/s, where $\mathcal{V}$ is the walking velocity in m/s. Assuming the average surface temperature of the clothed person to be 30°C, determine the rate of heat loss from an average man walking in still air at 10°C by convection at a walking velocity of (*a*) 0.5 m/s, (*b*) 1.0 m/s, (*c*) 1.5 m/s, and (*d*) 2.0 m/s.

6–10 The convection heat transfer coefficient for a clothed person standing in moving air is expressed as $h = 14.8\mathcal{V}^{0.69}$ for $0.15 < \mathcal{V} < 1.5$ m/s, where $\mathcal{V}$ is the air velocity. For a person with a body surface area of 1.7 m^2 and an average surface temperature of 29°C, determine the rate of heat loss from the person in windy air at 10°C by convection for air velocities of (*a*) 0.5 m/s, (*b*) 1.0 m/s, and (*c*) 1.5 m/s.

6–11 During air cooling of oranges, grapefruit, and tangelos, the heat transfer coefficient for combined convection, radiation, and evaporation for air velocities of $0.11 < \mathcal{V} < 0.33$ m/s is determined experimentally and is expressed as $h = 5.05\, k_{air} Re^{1/3}/D$, where the diameter D is the characteristic length. Oranges are cooled by refrigerated air at 5°C and 1 atm at a velocity of 0.5 m/s. Determine (*a*) the initial rate of heat transfer from a 7-cm-diameter orange initially at 15°C with a thermal conductivity of 0.50 W/m · °C, (*b*) the value of the initial temperature gradient inside the orange at the surface, and (*c*) the value of the Nusselt number.

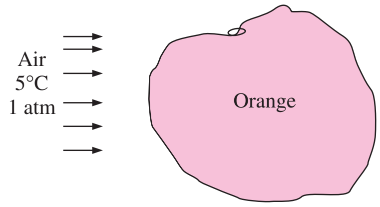


FIGURE P6–11

Velocity and Thermal Boundary Layers

6–12C What is viscosity? What causes viscosity in liquids and in gases? Is dynamic viscosity typically higher for a liquid or for a gas?

6–13C What is Newtonian fluid? Is water a Newtonian fluid?

6–14C What is the no-slip condition? What causes it?

6–15C Consider two identical small glass balls dropped into two identical containers, one filled with water and the other with oil. Which ball will reach the bottom of the container first? Why?

6–16C How does the dynamic viscosity of (*a*) liquids and (*b*) gases vary with temperature?

6–17C What fluid property is responsible for the development of the velocity boundary layer? For what kind of fluids will there be no velocity boundary layer on a flat plate?

6–18C What is the physical significance of the Prandtl number? Does the value of the Prandtl number depend on the type of flow or the flow geometry? Does the Prandtl number of air change with pressure? Does it change with temperature?

6–19C Will a thermal boundary layer develop in flow over a surface even if both the fluid and the surface are at the same temperature?

Laminar and Turbulent Flows

6–20C How does turbulent flow differ from laminar flow? For which flow is the heat transfer coefficient higher?

6–21C What is the physical significance of the Reynolds number? How is it defined for external flow over a plate of length *L?*

6–22C What does the friction coefficient represent in flow over a flat plate? How is it related to the drag force acting on the plate?

6–23C What is the physical mechanism that causes the friction factor to be higher in turbulent flow?

6–24C What is turbulent viscosity? What is it caused by?

6–25C What is turbulent thermal conductivity? What is it caused by?

Convection Equations and Similarity Solutions

6–26C Under what conditions can a curved surface be treated as a flat plate in fluid flow and convection analysis?

6–27C Express continuity equation for steady two-dimensional flow with constant properties, and explain what each term represents.

6–28C Is the acceleration of a fluid particle necessarily zero in steady flow? Explain.

6–29C For steady two-dimensional flow, what are the boundary layer approximations?

6–30C For what types of fluids and flows is the viscous dissipation term in the energy equation likely to be significant?

6–31C For steady two-dimensional flow over an isothermal flat plate in the *x*-direction, express the boundary conditions for the velocity components *u* and *v*, and the temperature *T* at the plate surface and at the edge of the boundary layer.

6–32C What is a similarity variable, and what is it used for? For what kinds of functions can we expect a similarity solution for a set of partial differential equations to exist?

6–33C Consider steady, laminar, two-dimensional flow over an isothermal plate. Does the thickness of the velocity boundary layer increase or decrease with (*a*) distance from the leading edge, (*b*) free-stream velocity, and (*c*) kinematic viscosity?

6–34C Consider steady, laminar, two-dimensional flow over an isothermal plate. Does the wall shear stress increase, decrease, or remain constant with distance from the leading edge?

6–35C What are the advantages of nondimensionalizing the convection equations?

6–36C Consider steady, laminar, two-dimensional, incompressible flow with constant properties and a Prandtl number of unity. For a given geometry, is it correct to say that both the average friction and heat transfer coefficients depend on the Reynolds number only?

6–37 Oil flow in a journal bearing can be treated as parallel flow between two large isothermal plates with one plate moving at a constant velocity of 12 m/s and the other stationary. Consider such a flow with a uniform spacing of 0.7 mm between the plates. The temperatures of the upper and lower plates are 40°C and 15°C, respectively. By simplifying and solving the continuity, momentum, and energy equations, determine (*a*) the velocity and temperature distributions in the oil, (*b*) the maximum temperature and where it occurs, and (*c*) the heat flux from the oil to each plate.

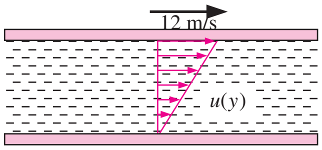


FIGURE P6–37

6–38 Repeat Problem 6–37 for a spacing of 0.4 mm.

6–39 A 6-cm-diameter shaft rotates at 3000 rpm in a 20-cm-long bearing with a uniform clearance of 0.2 mm. At steady operating conditions, both the bearing and the shaft in the vicinity of the oil gap are at 50°C, and the viscosity and thermal conductivity of lubricating oil are 0.05 N · s/m² and 0.17 W/m · K. By simplifying and solving the continuity, momentum, and energy equations, determine (*a*) the maximum temperature of oil, (*b*) the rates of heat transfer to the bearing and the shaft, and (*c*) the mechanical power wasted by the viscous dissipation in the oil. *Answers:* (*a*) 53.3°C, (*b*) 419 W, (*c*) 838 W

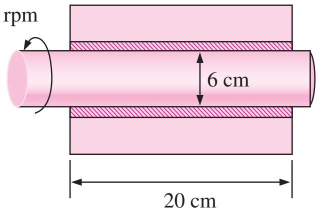


FIGURE P6-39

6–40 Repeat Problem 6–39 by assuming the shaft to have reached peak temperature and thus heat transfer to the shaft to be negligible, and the bearing surface still to be maintained at 50°C.

6–41 Reconsider Problem 6–39. Using EES (or other) software, investigate the effect of shaft velocity on the mechanical power wasted by viscous dissipation. Let the shaft rotation vary from 0 rpm to 5000 rpm. Plot the power wasted versus the shaft rpm, and discuss the results.

6–42 Consider a 5-cm-diameter shaft rotating at 2500 rpm in a 10-cm-long bearing with a clearance of 0.5 mm. Determine the power required to rotate the shaft if the fluid in the gap is (*a*) air, (*b*) water, and (*c*) oil at 40°C and 1 atm.

6–43 Consider the flow of fluid between two large parallel isothermal plates separated by a distance *L*. The upper plate is moving at a constant velocity of $\mathcal{V}$ and maintained at temperature T_0 while the lower plate is stationary and insulated. By simplifying and solving the continuity, momentum, and energy equations, obtain relations for the maximum temperature of fluid, the location where it occurs, and heat flux at the upper plate.

6–44 Reconsider Problem 6–43. Using the results of this problem, obtain a relation for the volumetric heat generation rate $\dot{g}$, in W/m³. Then express the convection problem as an equivalent conduction problem in the oil layer. Verify your model by solving the conduction problem and obtaining a relation for the maximum temperature, which should be identical to the one obtained in the convection analysis.

6–45 A 5-cm-diameter shaft rotates at 4500 rpm in a 15-cm-long, 8-cm-outer-diameter cast iron bearing (k = 70 W/m · K) with a uniform clearance of 0.6 mm filled with lubricating oil (μ = 0.03 N · s/m^2 and k = 0.14 W/m · K). The bearing is cooled externally by a liquid, and its outer surface is maintained at 40°C. Disregarding heat conduction through the shaft and assuming one-dimensional heat transfer, determine (*a*) the rate of heat transfer to the coolant, (*b*) the surface temperature of the shaft, and (*c*) the mechanical power wasted by the viscous dissipation in oil.

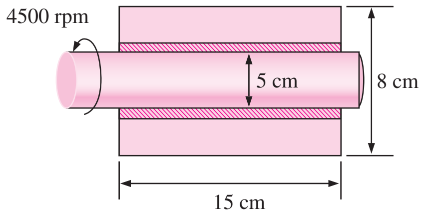


FIGURE P6–45

6–46 Repeat Problem 6–45 for a clearance of 1 mm.

Momentum and Heat Transfer Analogies

6–47C How is Reynolds analogy expressed? What is the value of it? What are its limitations?

6–48C How is the modified Reynolds analogy expressed? What is the value of it? What are its limitations?

6–49 A 4-m × 4-m flat plate maintained at a constant temperature of 80°C is subjected to parallel flow of air at 1 atm, 20°C, and 10 m/s. The total drag force acting on the upper surface of the plate is measured to be 2.4 N. Using momentum-heat transfer analogy, determine the average convection heat transfer coefficient, and the rate of heat transfer between the upper surface of the plate and the air.

6–50 A metallic airfoil of elliptical cross section has a mass of 50 kg, surface area of 12 m^2, and a specific heat of 0.50 kJ/kg · °C). The airfoil is subjected to air flow at 1 atm, 25°C, and 8 m/s along its 3-m-long side. The average temperature of the airfoil is observed to drop from 160°C to 150°C within 2 min of cooling. Assuming the surface temperature of the airfoil to be equal to its average temperature and using momentum-heat transfer analogy, determine the average friction coefficient of the airfoil surface. *Answer:* 0.000227

6–51 Repeat Problem 6–50 for an air-flow velocity of 12 m/s.

6–52 The electrically heated 0.6-m-high and 1.8-m-long windshield of a car is subjected to parallel winds at 1 atm, 0°C, and 80 km/h. The electric power consumption is observed to be 50 W when the exposed surface temperature of the windshield is 4°C. Disregarding radiation and heat transfer from the inner surface and using the momentum-heat transfer analogy, determine drag force the wind exerts on the windshield.

6–53 Consider an airplane cruising at an altitude of 10 km where standard atmospheric conditions are −50°C and 26.5 kPa at a speed of 800 km/h. Each wing of the airplane can be modeled as a 25-m × 3-m flat plate, and the friction coefficient of the wings is 0.0016. Using the momentum-heat transfer analogy, determine the heat transfer coefficient for the wings at cruising conditions. *Answer:* 89.6 W/m^2 · °C

Design and Essay Problems

6–54 Design an experiment to measure the viscosity of liquids using a vertical funnel with a cylindrical reservoir of height h and a narrow flow section of diameter D and length L. Making appropriate assumptions, obtain a relation for viscosity in terms of easily measurable quantities such as density and volume flow rate.

6–55 A facility is equipped with a wind tunnel, and can measure the friction coefficient for flat surfaces and airfoils. Design an experiment to determine the mean heat transfer coefficient for a surface using friction coefficient data.