CHAPTER

8

INTERNAL FORCED CONVECTION

Liquid or gas flow through pipes or ducts is commonly used in heating and cooling applications. The fluid in such applications is forced to flow by a fan or pump through a tube that is sufficiently long to accomplish the desired heat transfer. In this chapter we will pay particular attention to the determination of the *friction factor* and *convection coefficient* since they are directly related to the *pressure drop* and *heat transfer rate,* respectively. These quantities are then used to determine the pumping power requirement and the required tube length.

There is a fundamental difference between external and internal flows. In *external flow,* considered in Chapter 7, the fluid has a free surface, and thus the boundary layer over the surface is free to grow indefinitely. In *internal flow,* however, the fluid is completely confined by the inner surfaces of the tube, and thus there is a limit on how much the boundary layer can grow.

We start this chapter with a general physical description of internal flow, and the *mean velocity* and *mean temperature.* We continue with the discussion of the *hydrodynamic* and *thermal entry lengths, developing flow,* and *fully developed flow.* We then obtain the velocity and temperature profiles for fully developed laminar flow, and develop relations for the friction factor and Nusselt number. Finally we present empirical relations for developing and fully developed flows, and demonstrate their use.

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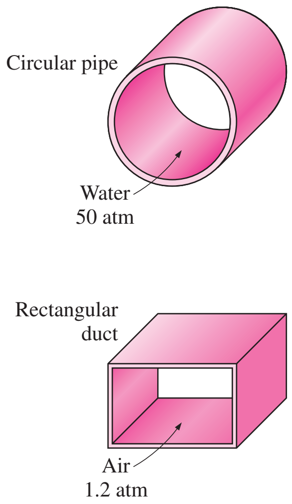


FIGURE 8–1
Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any distortion, but the noncircular pipes cannot.

8–1 ▪ INTRODUCTION

You have probably noticed that most fluids, especially liquids, are transported in circular pipes. This is because pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing any distortion. Noncircular pipes are usually used in applications such as the heating and cooling systems of buildings where the pressure difference is relatively small and the manufacturing and installation costs are lower (Fig. 8–1). For a fixed surface area, the circular tube gives the most heat transfer for the least pressure drop, which explains the overwhelming popularity of circular tubes in heat transfer equipment.

The terms *pipe, duct, tube,* and *conduit* are usually used interchangeably for flow sections. In general, flow sections of circular cross section are referred to as *pipes* (especially when the fluid is a liquid), and the flow sections of noncircular cross section as *ducts* (especially when the fluid is a gas). Small diameter pipes are usually referred to as *tubes.* Given this uncertainty, we will use more descriptive phrases (such as *a circular pipe* or *a rectangular duct*) whenever necessary to avoid any misunderstandings.

Although the theory of fluid flow is reasonably well understood, theoretical solutions are obtained only for a few simple cases such as the fully developed laminar flow in a circular pipe. Therefore, we must rely on the experimental results and the empirical relations obtained for most fluid flow problems rather than closed form analytical solutions. Noting that the experimental results are obtained under carefully controlled laboratory conditions, and that no two systems are exactly alike, we must not be so naive as to view the results obtained as "exact." An error of 10 percent (or more) in friction or convection coefficient calculated using the relations in this chapter is the "norm" rather than the "exception."

Perhaps we should mention that the friction between the fluid layers in a tube may cause a slight rise in fluid temperature as a result of mechanical energy being converted to thermal energy. But this *frictional heating* is too small to warrant any consideration in calculations, and thus is disregarded. For example, in the absence of any heat transfer, no noticeable difference will be detected between the inlet and exit temperatures of a fluid flowing in a tube. The primary consequence of friction in fluid flow is pressure drop. Thus, it is reasonable to assume that any temperature change in the fluid is due to heat transfer. But frictional heating must be considered for flows that involve highly viscous fluids with large velocity gradients.

In most practical applications, the flow of a fluid through a pipe or duct can be approximated to be *one-dimensional,* and thus the properties can be assumed to vary in *one* direction only (the direction of flow). As a result, all properties are *uniform* at any cross section normal to the flow direction, and the properties are assumed to have *bulk average values* over the cross section. But the values of the properties at a cross section *may* change with time unless the flow is steady.

8–2 ▪ MEAN VELOCITY AND MEAN TEMPERATURE

In external flow, the free-stream velocity served as a convenient reference velocity for use in the evaluation of the Reynolds number and the friction coefficient. In internal flow, there is no free stream and thus we need an alternative. The fluid velocity in a tube changes from zero at the surface because of the no-slip condition, to a maximum at the tube center. Therefore, it is convenient to work with an **average** or **mean velocity** $\mathcal{V}_m$, which remains constant for incompressible flow when the cross sectional area of the tube is constant.

The mean velocity in actual heating and cooling applications may change somewhat because of the changes in density with temperature. But, in practice, we evaluate the fluid properties at some average temperature and treat them as constants. The convenience in working with constant properties usually more than justifies the slight loss in accuracy.

The value of the mean velocity $\mathcal{V}_m$ in a tube is determined from the requirement that the *conservation of mass principle* be satisfied (Fig. 8–2). That is,

$$\dot{m} = \rho \mathcal{V}_m A_c = \int_{A_c} \rho \mathcal{V}(r, x) dA_c \tag{8-1}$$

where $\dot{m}$ is the mass flow rate, ρ is the density, A_c is the cross sectional area, and $\mathcal{V}(r, x)$ is the velocity profile. Then the mean velocity for incompressible flow in a circular tube of radius R can be expressed as

$$\mathcal{V}_m = \frac{\int_{A_c} \rho \mathcal{V}(r, x) dA_c}{\rho A_c} = \frac{\int_0^R \rho \mathcal{V}(r, x) 2\pi r dr}{\rho \pi R^2} = \frac{2}{R^2}\int_0^R \mathcal{V}(r, x) r dr \tag{8-2}$$

Therefore, when we know the mass flow rate or the velocity profile, the mean velocity can be determined easily.

When a fluid is heated or cooled as it flows through a tube, the temperature of the fluid at any cross section changes from T_s at the surface of the wall to some maximum (or minimum in the case of heating) at the tube center. In fluid flow it is convenient to work with an **average** or **mean temperature** T_m that remains uniform at a cross section. Unlike the mean velocity, the mean temperature T_m *will change* in the flow direction whenever the fluid is heated or cooled.

The value of the mean temperature T_m is determined from the requirement that the *conservation of energy* principle be satisfied. That is, the energy transported by the fluid through a cross section in actual flow must be equal to the energy that would be transported through the same cross section if the fluid were at a constant temperature T_m. This can be expressed mathematically as (Fig. 8–3)

$$\dot{E}_{\text{fluid}} = \dot{m} C_p T_m = \int_{\dot{m}} C_p T \delta \dot{m} = \int_{A_c} \rho C_p T \mathcal{V} dA_c \tag{8-3}$$

where C_p is the specific heat of the fluid. Note that the product $\dot{m} C_p T_m$ at any cross section along the tube represents the *energy flow* with the fluid at that cross section. Then the mean temperature of a fluid with constant density and specific heat flowing in a circular pipe of radius R can be expressed as

$$T_m = \frac{\int_{\dot{m}} C_p T \delta \dot{m}}{\dot{m} C_p} = \frac{\int_0^R C_p T(\rho \mathcal{V} 2\pi r dr)}{\rho \mathcal{V}_m (\pi R^2) C_p} = \frac{2}{\mathcal{V}_m R^2}\int_0^R T(r, x)\, \mathcal{V}(r, x)\, r dr \tag{8-4}$$

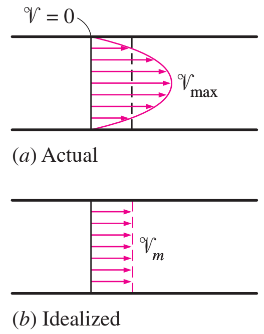


FIGURE 8–2
Actual and idealized velocity profiles for flow in a tube (the mass flow rate of the fluid is the same for both cases).

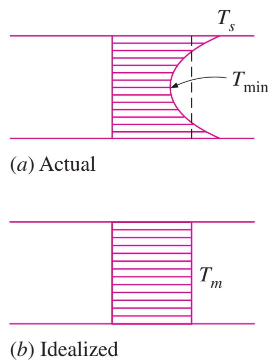


FIGURE 8–3
Actual and idealized temperature profiles for flow in a tube (the rate at which energy is transported with the fluid is the same for both cases).

Note that the mean temperature T_m of a fluid changes during heating or cooling. Also, the fluid properties in internal flow are usually evaluated at the *bulk mean fluid temperature,* which is the arithmetic average of the mean temperatures at the inlet and the exit. That is, $T_b = (T_{m,\,i} + T_{m,\,e})/2$.

Laminar and Turbulent Flow In Tubes

Flow in a tube can be laminar or turbulent, depending on the flow conditions. Fluid flow is streamlined and thus laminar at low velocities, but turns turbulent as the velocity is increased beyond a critical value. Transition from laminar to turbulent flow does not occur suddenly; rather, it occurs over some range of velocity where the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent. Most pipe flows encountered in practice are turbulent. Laminar flow is encountered when highly viscous fluids such as oils flow in small diameter tubes or narrow passages.

For flow in a circular tube, the Reynolds number is defined as

$$\text{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{\mathcal{V}_m D}{\nu} \tag{8-5}$$

where $\mathcal{V}_m$ is the mean fluid velocity, D is the diameter of the tube, and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid.

For flow through noncircular tubes, the Reynolds number as well as the Nusselt number and the friction factor are based on the **hydraulic diameter** D_h defined as (Fig. 8–4)

$$D_h = \frac{4A_c}{p} \tag{8-6}$$

where A_c is the cross sectional area of the tube and p is its perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular tubes since

Circular tubes: $$D_h = \frac{4A_c}{p} = \frac{4\pi D^2/4}{\pi D} = D$$

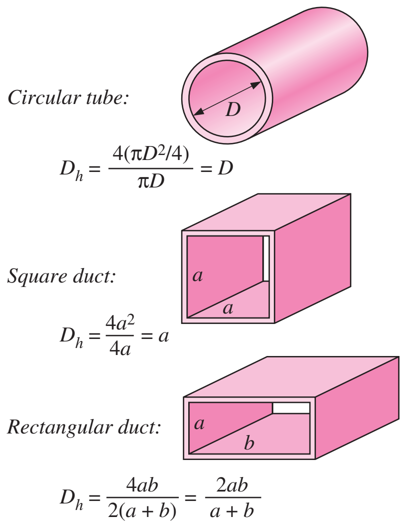


FIGURE 8–4
The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.

It certainly is desirable to have precise values of Reynolds numbers for laminar, transitional, and turbulent flows, but this is not the case in practice. This is because the transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by *surface roughness, pipe vibrations,* and the *fluctuations in the flow.* Under most practical conditions, the flow in a tube is laminar for Re < 2300, turbulent for Re > 10,000, and transitional in between. That is,

$$\begin{array}{rl} \text{Re} < 2300 & \text{laminar flow} \\ 2300 \le \text{Re} \le 10{,}000 & \text{transitional flow} \\ \text{Re} > 10{,}000 & \text{turbulent flow} \end{array}$$

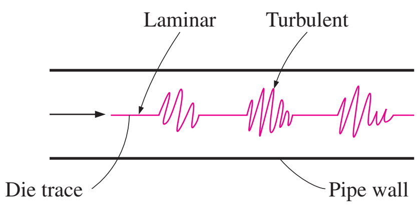


FIGURE 8–5
In the transitional flow region of $2300 \le \text{Re} \le 4000$, the flow switches between laminar and turbulent randomly.

In transitional flow, the flow switches between laminar and turbulent randomly (Fig. 8–5). It should be kept in mind that laminar flow can be maintained at much higher Reynolds numbers in very smooth pipes by avoiding flow disturbances and tube vibrations. In such carefully controlled experiments, laminar flow has been maintained at Reynolds numbers of up to 100,000.

8–3 ▪ THE ENTRANCE REGION

Consider a fluid entering a circular tube at a uniform velocity. As in external flow, the fluid particles in the layer in contact with the surface of the tube will come to a complete stop. This layer will also cause the fluid particles in the adjacent layers to slow down gradually as a result of friction. To make up for this velocity reduction, the velocity of the fluid at the midsection of the tube will have to increase to keep the mass flow rate through the tube constant. As a result, a *velocity boundary layer* develops along the tube. The thickness of this boundary layer increases in the flow direction until the boundary layer reaches the tube center and thus fills the entire tube, as shown in Figure 8–6.

The region from the tube inlet to the point at which the boundary layer merges at the centerline is called the **hydrodynamic entrance region,** and the length of this region is called the **hydrodynamic entry length** L_h. Flow in the entrance region is called *hydrodynamically developing flow* since this is the region where the velocity profile develops. The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged is called the **hydrodynamically fully developed region.** The velocity profile in the fully developed region is *parabolic* in laminar flow and somewhat *flatter* in turbulent flow due to eddy motion in radial direction.

Now consider a fluid at a uniform temperature entering a circular tube whose surface is maintained at a different temperature. This time, the fluid particles in the layer in contact with the surface of the tube will assume the surface temperature. This will initiate convection heat transfer in the tube and the development of a *thermal boundary layer* along the tube. The thickness of this boundary layer also increases in the flow direction until the boundary layer reaches the tube center and thus fills the entire tube, as shown in Figure 8–7.

The region of flow over which the thermal boundary layer develops and reaches the tube center is called the **thermal entrance region,** and the length of this region is called the **thermal entry length** L_t. Flow in the thermal entrance region is called *thermally developing flow* since this is the region where the temperature profile develops. The region beyond the thermal entrance region in which the dimensionless temperature profile expressed as $(T_s - T)/(T_s - T_m)$ remains unchanged is called the **thermally fully developed region.** The region in which the flow is both hydrodynamically and thermally developed and thus both the velocity and dimensionless temperature profiles remain unchanged is called *fully developed flow.* That is,

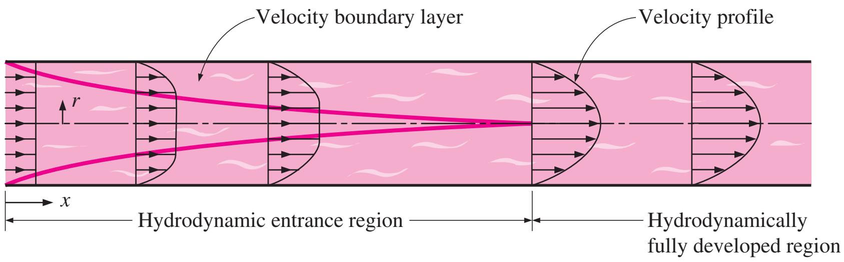


FIGURE 8–6
The development of the velocity boundary layer in a tube. (The developed mean velocity profile will be parabolic in laminar flow, as shown, but somewhat blunt in turbulent flow.)

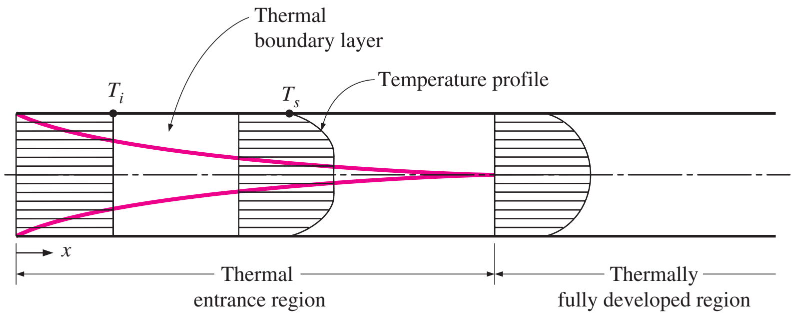


FIGURE 8–7
The development of the thermal boundary layer in a tube. (The fluid in the tube is being cooled.)

$$\textit{Hydrodynamically fully developed:}\qquad \frac{\partial \mathcal{V}(r, x)}{\partial x} = 0 \quad \longrightarrow \quad \mathcal{V} = \mathcal{V}(r) \tag{8-7}$$

$$\textit{Thermally fully developed:}\qquad \frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right] = 0 \tag{8-8}$$

The friction factor is related to the shear stress at the surface, which is related to the slope of the velocity profile at the surface. Noting that the velocity profile remains unchanged in the hydrodynamically fully developed region, the friction factor also remains constant in that region. A similar argument can be given for the heat transfer coefficient in the thermally fully developed region.

In a thermally fully developed region, the derivative of $(T_s - T)/(T_s - T_m)$ with respect to x is zero by definition, and thus $(T_s - T)/(T_s - T_m)$ is independent of x. Then the derivative of $(T_s - T)/(T_s - T_m)$ with respect r must also be independent of x. That is,

$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\bigg|_{r=R} = \frac{-(\partial T/\partial r)\big|_{r=R}}{T_s - T_m} \neq f(x) \tag{8-9}$$

Surface heat flux can be expressed as

$$\dot{q}_s = h_x(T_s - T_m) = k\,\frac{\partial T}{\partial r}\bigg|_{r=R} \quad \longrightarrow \quad h_x = \frac{k(\partial T/\partial r)\big|_{r=R}}{T_s - T_m} \tag{8-10}$$

which, from Eq. 8–9, is independent of x. Thus we conclude that *in the thermally fully developed region of a tube, the local convection coefficient is constant* (does not vary with x). Therefore, both *the friction and convection coefficients remain constant in the fully developed region of a tube.*

Note that the *temperature profile* in the thermally fully developed region may vary with x in the flow direction. That is, unlike the velocity profile, the temperature profile can be different at different cross sections of the tube in the developed region, and it usually is. However, the dimensionless temperature profile defined above remains unchanged in the thermally developed region when the temperature or heat flux at the tube surface remains constant.

During laminar flow in a tube, the magnitude of the dimensionless Prandtl number Pr is a measure of the relative growth of the velocity and thermal boundary layers. For fluids with Pr $\approx$ 1, such as gases, the two boundary layers essentially coincide with each other. For fluids with Pr $\gg$ 1, such as oils, the velocity boundary layer outgrows the thermal boundary layer. As a result, the hydrodynamic entry length is smaller than the thermal entry length. The opposite is true for fluids with $Pr \ll 1$ such as liquid metals.

Consider a fluid that is being heated (or cooled) in a tube as it flows through it. The friction factor and the heat transfer coefficient are *highest* at the tube inlet where the thickness of the boundary layers is zero, and decrease gradually to the fully developed values, as shown in Figure 8–8. Therefore, the pressure drop and heat flux are *higher* in the entrance regions of a tube, and the effect of the entrance region is always to *enhance* the average friction and heat transfer coefficients for the entire tube. This enhancement can be significant for short tubes but negligible for long ones.

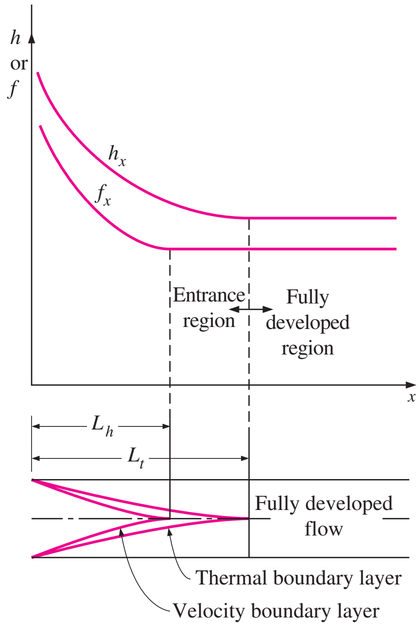


FIGURE 8–8
Variation of the friction factor and the convection heat transfer coefficient in the flow direction for flow in a tube ($Pr > 1$).

Entry Lengths

The hydrodynamic entry length is usually taken to be the distance from the tube entrance where the friction coefficient reaches within about 2 percent of the fully developed value. In *laminar flow,* the hydrodynamic and thermal entry lengths are given approximately as [see Kays and Crawford (1993), Ref. 13, and Shah and Bhatti (1987), Ref. 25]

$$L_{h,\text{ laminar}} \approx 0.05 \text{ Re } D \quad \textbf{(8-11)}$$

$$L_{t,\text{ laminar}} \approx 0.05 \text{ Re Pr } D = \text{Pr } L_{h,\text{ laminar}} \quad \textbf{(8-12)}$$

For Re = 20, the hydrodynamic entry length is about the size of the diameter, but increases linearly with the velocity. In the limiting case of Re = 2300, the hydrodynamic entry length is 115*D*.

In *turbulent flow,* the intense mixing during random fluctuations usually overshadows the effects of momentum and heat diffusion, and therefore the hydrodynamic and thermal entry lengths are of about the same size and independent of the Prandtl number. Also, *the friction factor and the heat transfer coefficient remain constant in fully developed laminar or turbulent flow* since the velocity and normalized temperature profiles do not vary in the flow direction. The hydrodynamic entry length for turbulent flow can be determined from [see Bhatti and Shah (1987), Ref. 1, and Zhi-qing (1982), Ref. 31]

$$L_{h,\text{ turbulent}} = 1.359 \text{ Re}^{1/4} \quad \textbf{(8-13)}$$

The hydrodynamic entry length is much shorter in turbulent flow, as expected, and its dependence on the Reynolds number is weaker. It is 11*D* at Re = 10,000, and increases to 43*D* at Re = 10^5. In practice, it is generally agreed that the entrance effects are confined within a tube length of 10 diameters, and the hydrodynamic and thermal entry lengths are approximately taken to be

$$L_{h,\text{ turbulent}} \approx L_{t,\text{ turbulent}} \approx 10D \quad \textbf{(8-14)}$$

The variation of local Nusselt number along a tube in turbulent flow for both uniform surface temperature and uniform surface heat flux is given in Figure 8–9 for the range of Reynolds numbers encountered in heat transfer equipment. We make these important observations from this figure:

- The Nusselt numbers and thus the convection heat transfer coefficients are much higher in the entrance region.

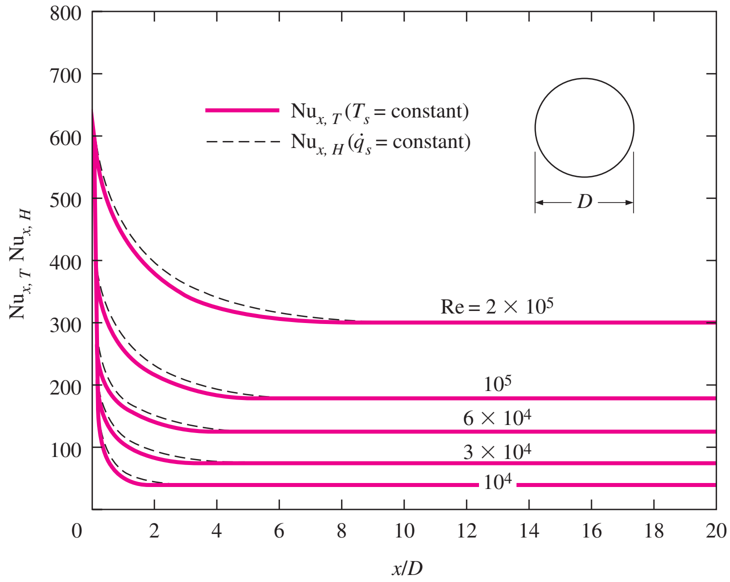


FIGURE 8–9
Variation of local Nusselt number along a tube in turbulent flow for both uniform surface temperature and uniform surface heat flux [Deissler (1953), Ref. 4].

- The Nusselt number reaches a constant value at a distance of less than 10 diameters, and thus the flow can be assumed to be fully developed for $x > 10D$.
- The Nusselt numbers for the uniform surface temperature and uniform surface heat flux conditions are identical in the fully developed regions, and nearly identical in the entrance regions. Therefore, Nusselt number is insensitive to the type of thermal boundary condition, and the turbulent flow correlations can be used for either type of boundary condition.

Precise correlations for the friction and heat transfer coefficients for the entrance regions are available in the literature. However, the tubes used in practice in forced convection are usually several times the length of either entrance region, and thus the flow through the tubes is often assumed to be fully developed for the entire length of the tube. This simplistic approach gives *reasonable* results for long tubes and *conservative* results for short ones.

8–4 ▪ GENERAL THERMAL ANALYSIS

You will recall that in the absence of any work interactions (such as electric resistance heating), the conservation of energy equation for the steady flow of a fluid in a tube can be expressed as (Fig. 8–10)

$$\dot{Q} = \dot{m}C_p(T_e - T_i) \qquad \text{(W)} \tag{8-15}$$

where T_i and T_e are the mean fluid temperatures at the inlet and exit of the tube, respectively, and $\dot{Q}$ is the rate of heat transfer to or from the fluid. Note that the temperature of a fluid flowing in a tube remains constant in the absence of any energy interactions through the wall of the tube.

The thermal conditions at the surface can usually be approximated with reasonable accuracy to be *constant surface temperature* (T_s = constant) or *constant surface heat flux* ($\dot{q}_s$ = constant). For example, the constant surface

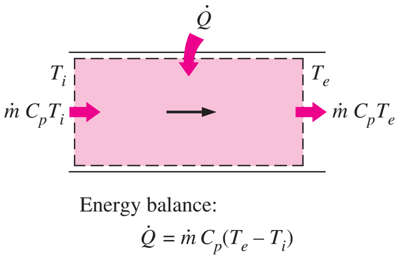


FIGURE 8–10
The heat transfer to a fluid flowing in a tube is equal to the increase in the energy of the fluid.

temperature condition is realized when a phase change process such as boiling or condensation occurs at the outer surface of a tube. The constant surface heat flux condition is realized when the tube is subjected to radiation or electric resistance heating uniformly from all directions.

Surface heat flux is expressed as

$$\dot{q}_s = h_x(T_s - T_m) \qquad \text{(W/m}^2\text{)} \tag{8-16}$$

where h_x is the *local* heat transfer coefficient and T_s and T_m are the surface and the mean fluid temperatures at that location. Note that the mean fluid temperature T_m of a fluid flowing in a tube must change during heating or cooling. Therefore, when $h_x = h =$ constant, the surface temperature T_s must change when $\dot{q}_s =$ constant, and the surface heat flux $\dot{q}_s$ must change when $T_s =$ constant. Thus we may have either $T_s =$ constant or $\dot{q}_s =$ constant at the surface of a tube, but not both. Next we consider convection heat transfer for these two common cases.

Constant Surface Heat Flux ($\dot{q}_s$ = constant)

In the case of $\dot{q}_s =$ constant, the rate of heat transfer can also be expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m} C_p(T_e - T_i) \qquad \text{(W)} \tag{8-17}$$

Then the mean fluid temperature at the tube exit becomes

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} C_p} \tag{8-18}$$

Note that the mean fluid temperature increases *linearly* in the flow direction in the case of constant surface heat flux, since the surface area increases linearly in the flow direction (A_s is equal to the perimeter, which is constant, times the tube length).

The surface temperature in the case of constant surface heat flux $\dot{q}_s$ can be determined from

$$\dot{q}_s = h(T_s - T_m) \longrightarrow T_s = T_m + \frac{\dot{q}_s}{h} \tag{8-19}$$

In the fully developed region, the surface temperature T_s will also increase linearly in the flow direction since h is constant and thus $T_s - T_m =$ constant (Fig. 8–11). Of course this is true when the fluid properties remain constant during flow.

The slope of the mean fluid temperature T_m on a T-x diagram can be determined by applying the steady-flow energy balance to a tube slice of thickness dx shown in Figure 8–12. It gives

$$\dot{m} C_p\, dT_m = \dot{q}_s(p\,dx) \longrightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} C_p} = \text{constant} \tag{8-20}$$

where p is the perimeter of the tube.

Noting that both $\dot{q}_s$ and h are constants, the differentiation of Eq. 8–19 with respect to x gives

$$\frac{dT_m}{dx} = \frac{dT_s}{dx} \tag{8-21}$$

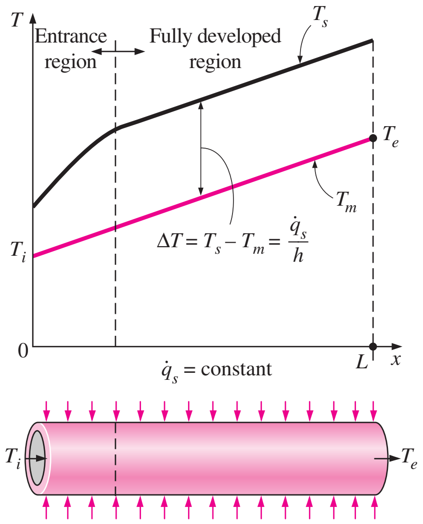


FIGURE 8–11
Variation of the *tube surface* and the *mean fluid* temperatures along the tube for the case of constant surface heat flux.

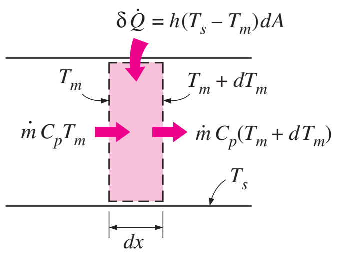


FIGURE 8–12
Energy interactions for a differential control volume in a tube.

Also, the requirement that the dimensionless temperature profile remains unchanged in the fully developed region gives

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0 \longrightarrow \frac{1}{T_s - T_m}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}\right) = 0 \longrightarrow \frac{\partial T}{\partial x} = \frac{dT_s}{dx} \tag{8-22}$$

since $T_s - T_m$ = constant. Combining Eqs. 8–20, 8–21, and 8–22 gives

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} C_p} = \text{constant} \tag{8-23}$$

Then we conclude that *in fully developed flow in a tube subjected to constant surface heat flux, the temperature gradient is independent of x and thus the shape of the temperature profile does not change along the tube* (Fig. 8–13).

For a circular tube, $p = 2\pi R$ and $\dot{m} = \rho \mathcal{V}_m A_c = \rho \mathcal{V}_m(\pi R^2)$, and Eq. 8–23 becomes

Circular tube:
$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho \mathcal{V}_m C_p R} = \text{constant} \tag{8-24}$$

where $\mathcal{V}_m$ is the mean velocity of the fluid.

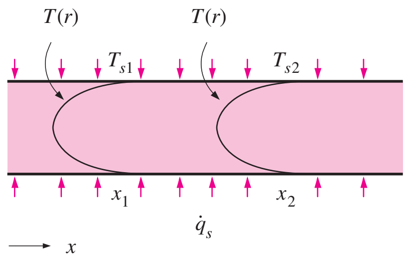


FIGURE 8–13
The shape of the temperature profile remains unchanged in the fully developed region of a tube subjected to constant surface heat flux.

Constant Surface Temperature (T_s = constant)

From Newton's law of cooling, the rate of heat transfer to or from a fluid flowing in a tube can be expressed as

$$\dot{Q} = hA_s\Delta T_{\text{ave}} = hA_s(T_s - T_m)_{\text{ave}} \qquad \text{(W)} \tag{8-25}$$

where h is the average convection heat transfer coefficient, A_s is the heat transfer surface area (it is equal to πDL for a circular pipe of length L), and ΔT_{ave} is some appropriate *average* temperature difference between the fluid and the surface. Below we discuss two suitable ways of expressing ΔT_{ave}.

In the constant surface temperature (T_s = constant) case, ΔT_{ave} can be expressed *approximately* by the **arithmetic mean temperature difference** ΔT_{am} as

$$\begin{aligned}\Delta T_{\text{ave}} \approx \Delta T_{\text{am}} &= \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2} = T_s - \frac{T_i + T_e}{2} \\ &= T_s - T_b\end{aligned} \tag{8-26}$$

where $T_b = (T_i + T_e)/2$ is the *bulk mean fluid temperature,* which is the *arithmetic average* of the mean fluid temperatures at the inlet and the exit of the tube.

Note that the *arithmetic mean temperature difference* ΔT_{am} is simply the *average* of the *temperature differences* between the surface and the fluid at the inlet and the exit of the tube. Inherent in this definition is the assumption that the mean fluid temperature varies linearly along the tube, which is hardly ever the case when T_s = constant. This simple approximation often gives acceptable results, but not always. Therefore, we need a better way to evaluate ΔT_{ave}.

Consider the heating of a fluid in a tube of constant cross section whose inner surface is maintained at a constant temperature of T_s. We know that the mean temperature of the fluid T_m will increase in the flow direction as a result of heat transfer. The energy balance on a differential control volume shown in Figure 8–12 gives

$$\dot{m}C_p\,dT_m = h(T_s - T_m)dA_s \tag{8-27}$$

That is, the increase in the energy of the fluid (represented by an increase in its mean temperature by dT_m) is equal to the heat transferred to the fluid from the tube surface by convection. Noting that the differential surface area is $dA_s = pdx$, where p is the perimeter of the tube, and that $dT_m = -d(T_s - T_m)$, since T_s is constant, the relation above can be rearranged as

$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hp}{\dot{m}C_p}\,dx \tag{8-28}$$

Integrating from $x = 0$ (tube inlet where $T_m = T_i$) to $x = L$ (tube exit where $T_m = T_e$) gives

$$\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}C_p} \tag{8-29}$$

where $A_s = pL$ is the surface area of the tube and h is the constant *average* convection heat transfer coefficient. Taking the exponential of both sides and solving for T_e gives the following relation which is very useful for the determination of the *mean fluid temperature at the tube exit:*

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p) \tag{8-30}$$

This relation can also be used to determine the mean fluid temperature $T_m(x)$ at any x by replacing $A_s = pL$ by px.

Note that the temperature difference between the fluid and the surface *decays exponentially* in the flow direction, and the rate of decay depends on the magnitude of the exponent $hA_x/\dot{m}C_p$, as shown in Figure 8–14. This dimensionless parameter is called the *number of transfer units,* denoted by NTU, and is a measure of the effectiveness of the heat transfer systems. For NUT $>$ 5, the exit temperature of the fluid becomes almost equal to the surface temperature, $T_e \approx T_s$ (Fig. 8–15). Noting that the fluid temperature can approach the surface temperature but cannot cross it, an NTU of about 5 indicates that the limit is reached for heat transfer, and the heat transfer will not increase no matter how much we extend the length of the tube. A small value of NTU, on the other hand, indicates more opportunities for heat transfer, and the heat transfer will continue increasing as the tube length is increased. A large NTU and thus a large heat transfer surface area (which means a large tube) may be desirable from a heat transfer point of view, but it may be unacceptable from an economic point of view. The selection of heat transfer equipment usually reflects a compromise between heat transfer performance and cost.

Solving Eq. 8–29 for $\dot{m}C_p$ gives

$$\dot{m}C_p = -\frac{hA_s}{\ln[(T_s - T_e)/(T_s - T_i)]} \tag{8-31}$$

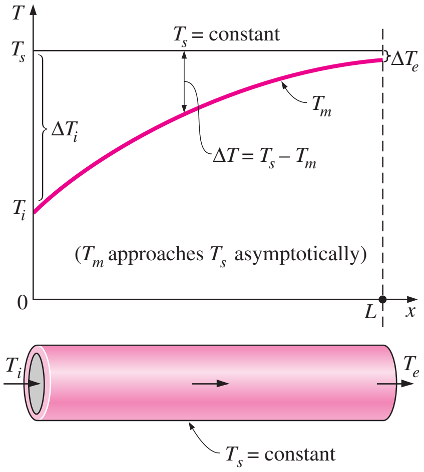
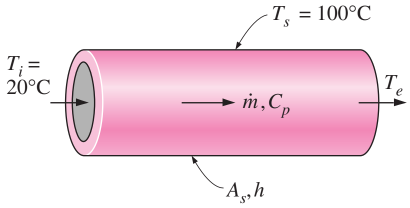


FIGURE 8–14
The variation of the *mean fluid* temperature along the tube for the case of constant temperature.



NTU = $hA_s/\dot{m}C_p$	T_e, °C
0.01	20.8
0.05	23.9
0.10	27.6
0.50	51.5
1.00	70.6
5.00	99.5
10.00	100.0

FIGURE 8–15
An NTU greater than 5 indicates that the fluid flowing in a tube will reach the surface temperature at the exit regardless of the inlet temperature.

Substituting this into Eq. 8–17, we obtain

$$\dot{Q} = hA_s\Delta T_{\ln} \tag{8-32}$$

where

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} \tag{8-33}$$

is the **logarithmic mean temperature difference.** Note that $\Delta T_i = T_s - T_i$ and $\Delta T_e = T_s - T_e$ are the temperature differences between the surface and the fluid at the inlet and the exit of the tube, respectively. This $\Delta T_{\ln}$ relation appears to be prone to misuse, but it is practically fail-safe, since using T_i in place of T_e and vice versa in the numerator and/or the denominator will, at most, affect the sign, not the magnitude. Also, it can be used for both heating ($T_s > T_i$ and T_e) and cooling ($T_s < T_i$ and T_e) of a fluid in a tube.

The logarithmic mean temperature difference $\Delta T_{\ln}$ is obtained by tracing the actual temperature profile of the fluid along the tube, and is an *exact* representation of the *average temperature difference* between the fluid and the surface. It truly reflects the exponential decay of the local temperature difference. When ΔT_e differs from ΔT_i by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_e differs from ΔT_i by greater amounts. Therefore, we should always use the logarithmic mean temperature difference when determining the convection heat transfer in a tube whose surface is maintained at a constant temperature T_s.

EXAMPLE 8–1 Heating of Water in a Tube by Steam

Water enters a 2.5-cm-internal-diameter thin copper tube of a heat exchanger at 15°C at a rate of 0.3 kg/s, and is heated by steam condensing outside at 120°C. If the average heat transfer coefficient is 800 W/m^2 · C, determine the length of the tube required in order to heat the water to 115°C (Fig. 8–16).

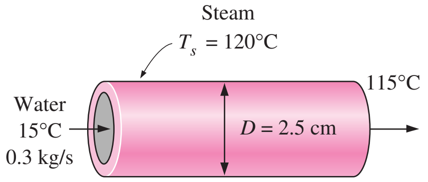


FIGURE 8–16
Schematic for Example 8–1.

SOLUTION Water is heated by steam in a circular tube. The tube length required to heat the water to a specified temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Fluid properties are constant. **3** The convection heat transfer coefficient is constant. **4** The conduction resistance of copper tube is negligible so that the inner surface temperature of the tube is equal to the condensation temperature of steam.

Properties The specific heat of water at the bulk mean temperature of (15 + 115)/2 = 65°C is 4187 J/kg · °C. The heat of condensation of steam at 120°C is 2203 kJ/kg (Table A-9).

Analysis Knowing the inlet and exit temperatures of water, the rate of heat transfer is determined to be

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = (0.3 \text{ kg/s})(4.187 \text{ kJ/kg} \cdot °\text{C})(115°\text{C} - 15°\text{C}) = 125.6 \text{ kW}$$

The logarithmic mean temperature difference is

$$\Delta T_e = T_s - T_e = 120°\text{C} - 115°\text{C} = 5°\text{C}$$
$$\Delta T_i = T_s - T_i = 120°\text{C} - 15°\text{C} = 105°\text{C}$$
$$\Delta T_{\ln} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} = \frac{5 - 105}{\ln(5/105)} = 32.85°\text{C}$$

The heat transfer surface area is

$$\dot{Q} = hA_s\Delta T_{\ln} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\ln}} = \frac{125.6\text{ kW}}{(0.8\text{ kW/m}^2\cdot°\text{C})(32.85°\text{C})} = 4.78\text{ m}^2$$

Then the required length of tube becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{4.78\text{ m}^2}{\pi(0.025\text{ m})} = \mathbf{61\text{ m}}$$

Discussion The bulk mean temperature of water during this heating process is 65°C, and thus the *arithmetic* mean temperature difference is $\Delta T_{am} = 120 - 65 = 55°\text{C}$. Using ΔT_{am} instead of $\Delta T_{\ln}$ would give $L = 36$ m, which is grossly in error. This shows the importance of using the logarithmic mean temperature in calculations.

8–5 ▪ LAMINAR FLOW IN TUBES

We mentioned earlier that flow in tubes is laminar for Re < 2300, and that the flow is fully developed if the tube is sufficiently long (relative to the entry length) so that the entrance effects are negligible. In this section we consider the *steady laminar flow* of an *incompressible fluid* with *constant properties* in the *fully developed region* of a *straight circular tube.* We obtain the momentum and energy equations by applying momentum and energy balances to a differential volume element, and obtain the velocity and temperature profiles by solving them. Then we will use them to obtain relations for the friction factor and the Nusselt number. An important aspect of the analysis below is that it is one of the few available for viscous flow and forced convection.

In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile $\mathcal{V}(r)$ remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component v in the direction normal to flow is everywhere zero. There is no acceleration since the flow is steady.

Now consider a ring-shaped differential volume element of radius r, thickness dr, and length dx oriented coaxially with the tube, as shown in Figure 8–17. The pressure force acting on a submerged plane surface is the product of the pressure at the centroid of the surface and the surface area.

The volume element involves only pressure and viscous effects, and thus the pressure and shear forces must balance each other. A force balance on the volume element in the flow direction gives

$$(2\pi rdrP)_x - (2\pi rdrP)_{x+dx} + (2\pi rdx\tau)_r - (2\pi rdx\tau)_{r+dr} = 0 \quad \textbf{(8-34)}$$

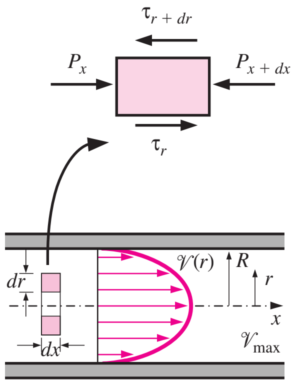


FIGURE 8–17
Free body diagram of a cylindrical fluid element of radius r, thickness dr, and length dx oriented coaxially with a horizontal tube in fully developed steady flow.

which indicates that in fully developed flow in a tube, the viscous and pressure forces balance each other. Dividing by $2\pi dr dx$ and rearranging,

$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{x+dr} - (r\tau)_r}{dr} = 0 \tag{8-35}$$

Taking the limit as $dr, dx \to 0$ gives

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0 \tag{8-36}$$

Substituting $\tau = -\mu(d\mathcal{V}/dr)$ and rearranging gives the desired equation,

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{d\mathcal{V}}{dr}\right) = \frac{dP}{dx} \tag{8-37}$$

The quantity $d\mathcal{V}/dr$ is negative in tube flow, and the negative sign is included to obtain positive values for τ. (Or, $d\mathcal{V}/dr = -d\mathcal{V}/dy$ since $y = R - r$.) The left side of this equation is a function of r and the right side is a function of x. The equality must hold for any value of r and x, and an equality of the form $f(r) = g(x)$ can happen only if both $f(r)$ and $g(x)$ are equal to constants. Thus we conclude that dP/dx = constant. This can be verified by writing a force balance on a volume element of radius R and thickness dx (a slice of the tube), which gives $dP/dx = -2\tau_s/R$. Here τ_s is constant since the viscosity and the velocity profile are constants in the fully developed region. Therefore, dP/dx = constant.

Equation 8–37 can be solved by rearranging and integrating it twice to give

$$\mathcal{V}(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2 \tag{8-38}$$

The velocity profile $\mathcal{V}(r)$ is obtained by applying the boundary conditions $\partial\mathcal{V}/\partial r = 0$ at $r = 0$ (because of symmetry about the centerline) and $\mathcal{V} = 0$ at $r = R$ (the no-slip condition at the tube surface). We get

$$\mathcal{V}(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) \tag{8-39}$$

Therefore, the velocity profile in fully developed laminar flow in a tube is *parabolic* with a maximum at the centerline and minimum at the tube surface. Also, the axial velocity $\mathcal{V}$ is positive for any r, and thus the axial pressure gradient dP/dx must be negative (i.e., pressure must decrease in the flow direction because of viscous effects).

The mean velocity is determined from its definition by substituting Eq. 8–39 into Eq. 8–2, and performing the integration. It gives

$$\mathcal{V}_m = \frac{2}{R^2}\int_0^R \mathcal{V} r dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right) \tag{8-40}$$

Combining the last two equations, the velocity profile is obtained to be

$$\mathcal{V}(r) = 2\mathcal{V}_m\left(1 - \frac{r^2}{R^2}\right) \tag{8-41}$$

This is a convenient form for the velocity profile since $\mathcal{V}_m$ can be determined easily from the flow rate information.

The maximum velocity occurs at the centerline, and is determined from Eq. 8–39 by substituting $r = 0$,

$$\mathcal{V}_{max} = 2\mathcal{V}_m \tag{8-42}$$

Therefore, *the mean velocity is one-half of the maximum velocity.*

Pressure Drop

A quantity of interest in the analysis of tube flow is the *pressure drop* ΔP since it is directly related to the power requirements of the fan or pump to maintain flow. We note that dP/dx = constant, and integrate it from $x = 0$ where the pressure is P_1 to $x = L$ where the pressure is P_2. We get

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} = -\frac{\Delta P}{L} \tag{8-43}$$

Note that in fluid mechanics, the pressure drop ΔP is a positive quantity, and is defined as $\Delta P = P_1 - P_2$. Substituting Eq. 8–43 into the $\mathcal{V}_m$ expression in Eq. 8–40, the *pressure drop* can be expressed as

Laminar flow:
$$\Delta P = \frac{8\mu L\,\mathcal{V}_m}{R^2} = \frac{32\mu L\,\mathcal{V}_m}{D^2} \tag{8-44}$$

In practice, it is found convenient to express the pressure drop for all types of internal flows (laminar or turbulent flows, circular or noncircular tubes, smooth or rough surfaces) as (Fig. 8–18)

$$\Delta P = f\frac{L}{D}\frac{\mu\mathcal{V}_m^2}{2} \tag{8-45}$$

where the dimensionless quantity f is the **friction factor** (also called the *Darcy friction factor* after French engineer Henry Darcy, 1803–1858, who first studied experimentally the effects of roughness on tube resistance). It should not be confused with the *friction coefficient* C_f (also called the *Fanning friction factor*), which is defined as $C_f = \tau_s(\rho\mathcal{V}_m^2/2) = f/4$.

Equation 8–45 gives the pressure drop for a flow section of length L provided that (1) the flow section is horizontal so that there are no hydrostatic or gravity effects, (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure, and (3) the cross sectional area of the flow section is constant and thus the mean flow velocity is constant.

Setting Eqs. 8–44 and 8–45 equal to each other and solving for f gives the friction factor for the *fully developed laminar flow in a circular tube* to be

Circular tube, laminar:
$$f = \frac{64\mu}{\rho D\mathcal{V}_m} = \frac{64}{\text{Re}} \tag{8-46}$$

This equation shows that *in laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the tube*

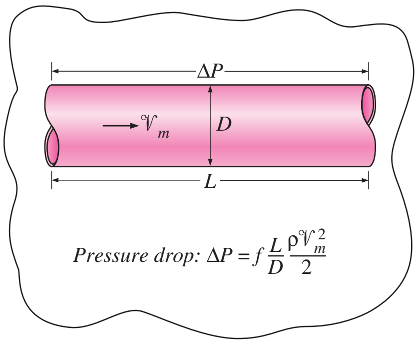


FIGURE 8–18
The relation for pressure drop is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular pipes, and smooth or rough surfaces.

surface. Once the pressure drop is available, the required pumping power is determined from

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P \tag{8-47}$$

where $\dot{V}$ is the volume flow rate of flow, which is expressed as

$$\dot{V} = \mathcal{V}_{\text{ave}} A_c = \frac{\Delta P R^2}{8\mu L}\pi R^2 = \frac{\pi R^4 \Delta P}{8\mu L} = \frac{\pi D^4 \Delta P}{128\mu L} \tag{8-48}$$

This equation is known as the **Poiseuille's Law,** and this flow is called the *Hagen–Poiseuille flow* in honor of the works of G. Hagen (1797–1839) and J. Poiseuille (1799–1869) on the subject. Note from Eq. 8–48 that *for a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the tube and the viscosity of the fluid, but it is inversely proportional to the fourth power of the radius (or diameter) of the tube.* Therefore, the pumping power requirement for a piping system can be reduced by a factor of 16 by doubling the tube diameter (Fig. 8–19). Of course the benefits of the reduction in the energy costs must be weighed against the increased cost of construction due to using a larger diameter tube.

The pressure drop is caused by viscosity, and it is directly related to the wall shear stress. For the ideal inviscid flow, the pressure drop is zero since there are no viscous effects. Again, Eq. 8–47 is valid for both laminar and turbulent flows in circular and noncircular tubes.

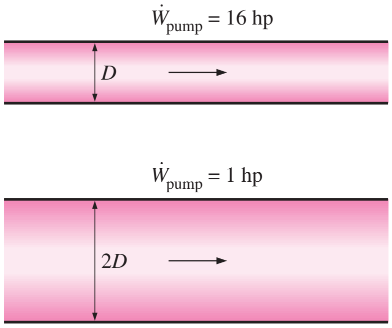


FIGURE 8–19
The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.

Temperature Profile and the Nusselt Number

In the analysis above, we have obtained the velocity profile for fully developed flow in a circular tube from a momentum balance applied on a volume element, determined the friction factor and the pressure drop. Below we obtain the energy equation by applying the energy balance to a differential volume element, and solve it to obtain the temperature profile for the constant surface temperature and the constant surface heat flux cases.

Reconsider steady laminar flow of a fluid in a circular tube of radius R. The fluid properties ρ, k, and C_p are constant, and the work done by viscous stresses is negligible. The fluid flows along the x-axis with velocity $\mathcal{V}$. The flow is fully developed so that $\mathcal{V}$ is independent of x and thus $\mathcal{V} = \mathcal{V}(r)$. Noting that energy is transferred by mass in the x-direction, and by conduction in the r-direction (heat conduction in the x-direction is assumed to be negligible), the steady-flow energy balance for a cylindrical shell element of thickness dr and length dx can be expressed as (Fig. 8–20)

$$\dot{m}C_pT_x - \dot{m}C_pT_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0 \tag{8-49}$$

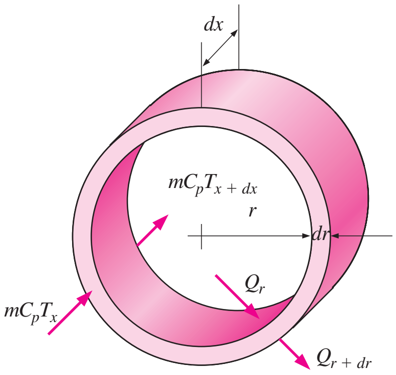


FIGURE 8–20
The differential volume element used in the derivation of energy balance relation.

where $\dot{m} = \rho\mathcal{V}A_c = \rho\mathcal{V}(2\pi r dr)$. Substituting and dividing by $2\pi r dr dx$ gives, after rearranging,

$$\rho C_p \mathcal{V}\frac{T_{x+dx} - T_x}{dx} = -\frac{1}{2\pi r dx}\frac{\dot{Q}_{r+dr} - \dot{Q}_r}{dr} \tag{8-50}$$

or

$$\mathcal{V}\frac{\partial T}{\partial x} = -\frac{1}{2\rho C_p \pi r dx}\frac{\partial \dot{Q}}{\partial r} \tag{8-51}$$

where we used the definition of derivative. But

$$\frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r}\left(-k2\pi r dx \frac{\partial T}{\partial r}\right) = -2\pi k dx \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{8-52}$$

Substituting and using $\alpha = k/\rho C_p$ gives

$$\mathcal{V}\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{dr}\left(r\frac{\partial T}{\partial r}\right) \tag{8-53}$$

which states that *the rate of net energy transfer to the control volume by mass flow is equal to the net rate of heat conduction in the radial direction.*

Constant Surface Heat Flux

For fully developed flow in a circular pipe subjected to constant surface heat flux, we have, from Eq. 8–24,

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho \mathcal{V}_m C_p R} = \text{constant} \tag{8-54}$$

If heat conduction in the x-direction were considered in the derivation of Eq. 8–53, it would give an additional term $\alpha\partial^2T/\partial x^2$, which would be equal to zero since $\partial T/\partial x$ = constant and thus $T = T(r)$. Therefore, the assumption that there is no axial heat conduction is satisfied exactly in this case.

Substituting Eq. 8–54 and the relation for velocity profile (Eq. 8–41) into Eq. 8–53 gives

$$\frac{4\dot{q}_s}{kR}\left(1 - \frac{r^2}{R^2}\right) = \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \tag{8-55}$$

which is a second-order ordinary differential equation. Its general solution is obtained by separating the variables and integrating twice to be

$$T = \frac{\dot{q}_s}{kR}\left(r^2 - \frac{r^2}{4R^2}\right) + C_1 r + C_2 \tag{8-56}$$

The desired solution to the problem is obtained by applying the boundary conditions $\partial T/\partial x = 0$ at $r = 0$ (because of symmetry) and $T = T_s$ at $r = R$. We get

$$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right) \tag{8-57}$$

The bulk mean temperature T_m is determined by substituting the velocity and temperature profile relations (Eqs. 8–41 and 8–57) into Eq. 8–4 and performing the integration. It gives

$$T_m = T_s - \frac{11}{24}\frac{\dot{q}_s R}{k} \tag{8-58}$$

Combining this relation with $\dot{q}_s = h(T_s - T_m)$ gives

$$h = \frac{24}{11}\frac{k}{R} = \frac{48}{11}\frac{k}{D} = 4.36\frac{k}{D} \tag{8-59}$$

or

Circular tube, laminar ($\dot{q}_x$ = constant): $$\text{Nu} = \frac{hD}{k} = 4.36 \tag{8-60}$$

Therefore, for fully developed laminar flow in a circular tube subjected to constant surface heat flux, the Nusselt number is a constant. There is no dependence on the Reynolds or the Prandtl numbers.

Constant Surface Temperature

A similar analysis can be performed for fully developed laminar flow in a circular tube for the case of constant surface temperature T_s. The solution procedure in this case is more complex as it requires iterations, but the Nusselt number relation obtained is equally simple (Fig. 8–21):

Circular tube, laminar (T_s = constant): $$\text{Nu} = \frac{hD}{k} = 3.66 \tag{8-61}$$

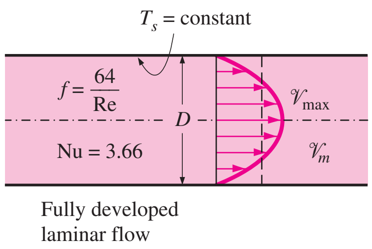


FIGURE 8–21
In laminar flow in a tube with constant surface temperature, both the *friction factor* and the *heat transfer coefficient* remain constant in the fully developed region.

The thermal conductivity k for use in the Nu relations above should be evaluated at the bulk mean fluid temperature, which is the arithmetic average of the mean fluid temperatures at the inlet and the exit of the tube. For laminar flow, the effect of *surface roughness* on the friction factor and the heat transfer coefficient is negligible.

Laminar Flow in Noncircular Tubes

The friction factor f and the Nusselt number relations are given in Table 8–1 for *fully developed laminar flow* in tubes of various cross sections. The Reynolds and Nusselt numbers for flow in these tubes are based on the hydraulic diameter $D_h = 4A_c/p$, where A_c is the cross sectional area of the tube and p is its perimeter. Once the Nusselt number is available, the convection heat transfer coefficient is determined from $h = k\text{Nu}/D_h$.

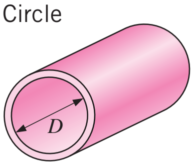
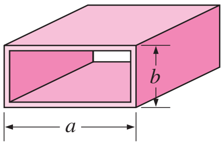
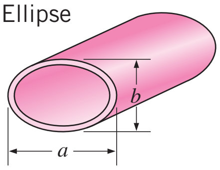
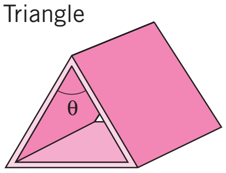
Developing Laminar Flow in the Entrance Region

For a circular tube of length L subjected to constant surface temperature, the average Nusselt number for the *thermal entrance region* can be determined from (Edwards et al., 1979)

Entry region, laminar: $$\text{Nu} = 3.66 + \frac{0.065\,(D/L)\,\text{Re}\,\text{Pr}}{1 + 0.04[(D/L)\,\text{Re}\,\text{Pr}]^{2/3}} \tag{8-62}$$

TABLE 8–1

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $\text{Re} = \mathcal{V}_m D_h/\nu$, and $\text{Nu} = hD_h/k$)

Tube Geometry	a/b or $\theta°$	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Note that the average Nusselt number is larger at the entrance region, as expected, and it approaches asymptotically to the fully developed value of 3.66 as $L \to \infty$. This relation assumes that the flow is hydrodynamically developed when the fluid enters the heating section, but it can also be used approximately for flow developing hydrodynamically.

When the difference between the surface and the fluid temperatures is large, it may be necessary to account for the variation of viscosity with temperature. The average Nusselt number for developing laminar flow in a circular tube in that case can be determined from [Sieder and Tate (1936), Ref. 26]

$$\text{Nu} = 1.86\left(\frac{\text{Re Pr } D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \tag{8-63}$$

All properties are evaluated at the bulk mean fluid temperature, except for μ_s, which is evaluated at the surface temperature.

The average Nusselt number for the thermal entrance region of flow between *isothermal parallel plates* of length L is expressed as (Edwards et al., 1979)

Entry region, laminar:
$$\text{Nu} = 7.54 + \frac{0.03\,(D_h/L)\,\text{Re}\,\text{Pr}}{1 + 0.016[(D_h/L)\,\text{Re}\,\text{Pr}]^{2/3}} \tag{8-64}$$

where D_h is the hydraulic diameter, which is twice the spacing of the plates. This relation can be used for $\text{Re} \leq 2800$.

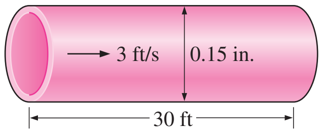


FIGURE 8–22
Schematic for Example 8–2.

EXAMPLE 8–2 Pressure Drop in a Pipe

Water at 40°F (ρ = 62.42 lbm/ft^3 and μ = 3.74 lbm/ft · h) is flowing in a 0.15-in.-diameter 30-ft-long pipe steadily at an average velocity of 3 ft/s (Fig. 8–22). Determine the pressure drop and the pumping power requirement to overcome this pressure drop.

SOLUTION The average flow velocity in a pipe is given. The pressure drop and the required pumping power are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors.

Properties The density and dynamic viscosity of water are given to be ρ = 62.42 lbm/ft^3 and μ = 3.74 lbm/ft · h = 0.00104 lbm/ft · s.

Analysis First we need to determine the flow regime. The Reynolds number is

$$\text{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{(62.42 \text{ lbm/ft}^3)(3 \text{ ft/s})(0.12/12 \text{ ft})}{3.74 \text{ lbm/ft} \cdot \text{h}}\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 1803$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the pressure drop become

$$f = \frac{64}{\text{Re}} = \frac{64}{1803} = 0.0355$$

$$\Delta P = f\frac{L}{D}\frac{\rho \mathcal{V}_m^2}{2} = 0.0355\,\frac{30 \text{ ft}}{0.12/12 \text{ ft}}\,\frac{(62.42 \text{ lbm/ft}^3)(3 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.174 \text{ lbm} \cdot \text{ft/s}^2}\right)$$
$$= \mathbf{930 \text{ lbf/ft}^2 = 6.46 \text{ psi}}$$

The volume flow rate and the pumping power requirements are

$$\dot{V} = \mathcal{V}_m A_c = \mathcal{V}_m(\pi D^2/4) = (3 \text{ ft/s})[\pi(0.12/12 \text{ ft})^2/4] = 0.000236 \text{ ft}^3/\text{s}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.000236 \text{ ft}^3/\text{s})(930 \text{ lbf/ft}^2)\left(\frac{1 \text{ W}}{0.737 \text{ lbf} \cdot \text{ft/s}}\right) = \mathbf{0.30 \text{ W}}$$

Therefore, mechanical power input in the amount of 0.30 W is needed to overcome the frictional losses in the flow due to viscosity.

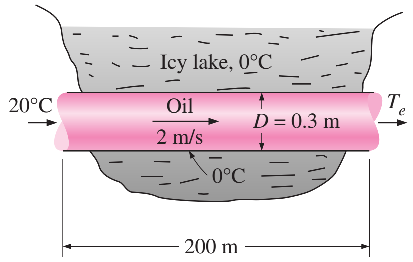


FIGURE 8–23
Schematic for Example 8–3.

EXAMPLE 8–3 Flow of Oil in a Pipeline through a Lake

Consider the flow of oil at 20°C in a 30-cm-diameter pipeline at an average velocity of 2 m/s (Fig. 8–23). A 200-m-long section of the pipeline passes through icy waters of a lake at 0°C. Measurements indicate that the surface temperature of the pipe is very nearly 0°C. Disregarding the thermal resistance of the pipe material, determine (*a*) the temperature of the oil when the pipe leaves the lake, (*b*) the rate of heat transfer from the oil, and (*c*) the pumping power required to overcome the pressure losses and to maintain the flow of the oil in the pipe.

SOLUTION Oil flows in a pipeline that passes through icy waters of a lake at 0°C. The exit temperature of the oil, the rate of heat loss, and the pumping power needed to overcome pressure losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is very nearly 0°C. **3** The thermal resistance of the pipe is negligible. **4** The inner surfaces of the pipeline are smooth. **5** The flow is hydrodynamically developed when the pipeline reaches the lake.

Properties We do not know the exit temperature of the oil, and thus we cannot determine the bulk mean temperature, which is the temperature at which the properties of oil are to be evaluated. The mean temperature of the oil at the inlet is 20°C, and we expect this temperature to drop somewhat as a result of heat loss to the icy waters of the lake. We evaluate the properties of the oil at the inlet temperature, but we will repeat the calculations, if necessary, using properties at the evaluated bulk mean temperature. At 20°C we read (Table A-14)

$$\rho = 888 \text{ kg/m}^3 \qquad \nu = 901 \times 10^{-6} \text{ m}^2\text{/s}$$
$$k = 0.145 \text{ W/m} \cdot {}^\circ\text{C} \qquad C_p = 1880 \text{ J/kg} \cdot {}^\circ\text{C}$$
$$\text{Pr} = 10{,}400$$

Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(2 \text{ m/s})(0.3 \text{ m})}{901 \times 10^{-6} \text{ m}^2\text{/s}} = 666$$

which is less than the critical Reynolds number of 2300. Therefore, the flow is laminar, and the thermal entry length in this case is roughly

$$L_t \approx 0.05 \text{ Re Pr } D = 0.05 \times 666 \times 10{,}400 \times (0.3 \text{ m}) \approx 104{,}000 \text{ m}$$

which is much greater than the total length of the pipe. This is typical of fluids with high Prandtl numbers. Therefore, we assume thermally developing flow and determine the Nusselt number from

$$\begin{aligned} \text{Nu} = \frac{hD}{k} &= 3.66 + \frac{0.065\,(D/L)\text{ Re Pr}}{1 + 0.04\,[(D/L)\text{ Re Pr}]^{2/3}} \\ &= 3.66 + \frac{0.065(0.3/200) \times 666 \times 10{,}400}{1 + 0.04[(0.3/200) \times 666 \times 10{,}400]^{2/3}} \\ &= 37.3 \end{aligned}$$

Note that this Nusselt number is considerably higher than the fully developed value of 3.66. Then,

$$h = \frac{k}{D}\text{Nu} = \frac{0.145 \text{ W/m}}{0.3 \text{ m}}(37.3) = 18.0 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

Also,

$$A_s = pL = \pi DL = \pi(0.3 \text{ m})(200 \text{ m}) = 188.5 \text{ m}^2$$
$$\dot{m} = \rho A_c \mathcal{V}_m = (888 \text{ kg/m}^3)[\tfrac{1}{4}\pi(0.3 \text{ m})^2](2 \text{ m/s}) = 125.5 \text{ kg/s}$$

Next we determine the exit temperature of oil from

$$\begin{aligned} T_e &= T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p) \\ &= 0^\circ\text{C} - [(0 - 20)^\circ\text{C}]\exp\left[-\frac{(18.0 \text{ W/m}^2 \cdot {}^\circ\text{C})(188.5 \text{ m}^2)}{(125.5 \text{ kg/s})(1880 \text{ J/kg} \cdot {}^\circ\text{C})}\right] \\ &= \mathbf{19.71^\circ C} \end{aligned}$$

Thus, the mean temperature of oil drops by a mere 0.29°C as it crosses the lake. This makes the bulk mean oil temperature 19.86°C, which is practically identical to the inlet temperature of 20°C. Therefore, we do not need to re-evaluate the properties.

(*b*) The logarithmic mean temperature difference and the rate of heat loss from the oil are

$$\Delta T_{\text{ln}} = \frac{T_i - T_e}{\ln \dfrac{T_s - T_e}{T_s - T_i}} = \frac{20 - 19.71}{\ln \dfrac{0 - 19.71}{0 - 20}} = -19.85^\circ\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\text{ln}} = (18.0 \text{ W/m}^2 \cdot {}^\circ\text{C})(188.5 \text{ m}^2)(-19.85^\circ\text{C}) = \mathbf{-6.74 \times 10^4}$$

Therefore, the oil will lose heat at a rate of 67.4 kW as it flows through the pipe in the icy waters of the lake. Note that ΔT_{ln} is identical to the arithmetic mean temperature in this case, since $\Delta T_i \approx \Delta T_e$.

(*c*) The laminar flow of oil is hydrodynamically developed. Therefore, the friction factor can be determined from

$$f = \frac{64}{\text{Re}} = \frac{64}{666} = 0.0961$$

Then the pressure drop in the pipe and the required pumping power become

$$\Delta P = f\frac{L}{D}\frac{\rho\mathcal{V}_m^2}{2} = 0.0961\frac{200 \text{ m}}{0.3 \text{ m}}\frac{(888 \text{ kg/m}^3)(2 \text{ m/s})^2}{2} = 1.14 \times 10^5 \text{ N/m}^2$$

$$\dot{W}_{\text{pump}} = \frac{\dot{m}\Delta P}{\rho} = \frac{(125.5 \text{ kg/s})(1.14 \times 10^5 \text{ N/m}^2)}{888 \text{ kg/m}^3} = \mathbf{16.1 \text{ kW}}$$

Discussion We will need a 16.1-kW pump just to overcome the friction in the pipe as the oil flows in the 200-m-long pipe through the lake.

8–6 ▪ TURBULENT FLOW IN TUBES

We mentioned earlier that flow in smooth tubes is fully turbulent for $\mathrm{Re} > 10{,}000$. Turbulent flow is commonly utilized in practice because of the higher heat transfer coefficients associated with it. Most correlations for the friction and heat transfer coefficients in turbulent flow are based on experimental studies because of the difficulty in dealing with turbulent flow theoretically.

For *smooth* tubes, the friction factor in turbulent flow can be determined from the explicit *first Petukhov equation* [Petukhov (1970), Ref. 21] given as

Smooth tubes: $$f = (0.790 \ln \mathrm{Re} - 1.64)^{-2} \qquad 10^4 < \mathrm{Re} < 10^6 \tag{8-65}$$

The Nusselt number in turbulent flow is related to the friction factor through the *Chilton–Colburn analogy* expressed as

$$\mathrm{Nu} = 0.125\, f\, \mathrm{Re}\mathrm{Pr}^{1/3} \tag{8-66}$$

Once the friction factor is available, this equation can be used conveniently to evaluate the Nusselt number for both smooth and rough tubes.

For fully developed turbulent flow in *smooth tubes,* a simple relation for the Nusselt number can be obtained by substituting the simple power law relation $f = 0.184\,\mathrm{Re}^{-0.2}$ for the friction factor into Eq. 8–66. It gives

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{pmatrix} 0.7 \le \mathrm{Pr} \le 160 \\ \mathrm{Re} > 10{,}000 \end{pmatrix} \tag{8-67}$$

which is known as the *Colburn equation.* The accuracy of this equation can be improved by modifying it as

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n} \tag{8-68}$$

where $n = 0.4$ for *heating* and 0.3 for *cooling* of the fluid flowing through the tube. This equation is known as the *Dittus–Boelter equation* [Dittus and Boelter (1930), Ref. 6] and it is preferred to the Colburn equation.

The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$. When the temperature difference between the fluid and the wall is very large, it may be necessary to use a correction factor to account for the different viscosities near the wall and at the tube center.

The Nusselt number relations above are fairly simple, but they may give errors as large as 25 percent. This error can be reduced considerably to less than 10 percent by using more complex but accurate relations such as the *second Petukhov equation* expressed as

$$\mathrm{Nu} = \frac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(f/8)^{0.5}\,(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 10^4 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix} \tag{8-69}$$

The accuracy of this relation at lower Reynolds numbers is improved by modifying it as [Gnielinski (1976), Ref. 8]

$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{0.5}\,(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 3 \times 10^3 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix} \tag{8-70}$$

where the friction factor f can be determined from an appropriate relation such as the first Petukhov equation. Gnielinski's equation should be preferred in calculations. Again properties should be evaluated at the bulk mean fluid temperature.

The relations above are not very sensitive to the *thermal conditions* at the tube surfaces and can be used for both T_s = constant and $\dot{q}_s$ = constant cases. Despite their simplicity, the correlations already presented give sufficiently accurate results for most engineering purposes. They can also be used to obtain rough estimates of the friction factor and the heat transfer coefficients in the transition region $2300 \leq \text{Re} \leq 10{,}000$, especially when the Reynolds number is closer to 10,000 than it is to 2300.

The relations given so far do not apply to liquid metals because of their very low Prandtl numbers. For liquid metals ($0.004 < \text{Pr} < 0.01$), the following relations are recommended by Sleicher and Rouse (1975, Ref. 27) for $10^4 < \text{Re} < 10^6$:

Liquid metals, T_s = constant: $$\text{Nu} = 4.8 + 0.0156\ \text{Re}^{0.85}\ \text{Pr}_s^{0.93} \tag{8-71}$$

Liquid metals, $\dot{q}_s$ = constant: $$\text{Nu} = 6.3 + 0.0167\ \text{Re}^{0.85}\ \text{Pr}_s^{0.93} \tag{8-72}$$

where the subscript s indicates that the Prandtl number is to be evaluated at the surface temperature.

Relative Roughness, ε/L	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

*Smooth surface. All values are for Re = 10^6, and are calculated from Eq. 8–73.

FIGURE 8–24
The friction factor is minimum for a smooth pipe and increases with roughness.

Rough Surfaces

Any irregularity or roughness on the surface disturbs the laminar sublayer, and affects the flow. Therefore, unlike laminar flow, the friction factor and the convection coefficient in turbulent flow are strong functions of surface roughness.

The friction factor in fully developed turbulent flow depends on the Reynolds number and the *relative roughness* ε/D. In 1939, C. F. Colebrook (Ref. 3) combined all the friction factor data for transition and turbulent flow in smooth as well as rough pipes into the following implicit relation known as the **Colebrook equation.**

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad \text{(turbulent flow)} \tag{8-73}$$

In 1944, L. F. Moody (Ref. 17) plotted this formula into the famous **Moody chart** given in the Appendix. It presents the friction factors for pipe flow as a function of the Reynolds number and ε/D over a wide range. For smooth tubes, the agreement between the Petukhov and Colebrook equations is very good. The friction factor is minimum for a smooth pipe (but still not zero because of the no-slip condition), and increases with roughness (Fig. 8–24).

Although the Moody chart is developed for circular pipes, it can also be used for noncircular pipes by replacing the diameter by the hydraulic diameter. At very large Reynolds numbers (to the right of the dashed line on the chart) the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number. In calculations, we should make sure that we use the internal diameter of the pipe, which may be different than the nominal diameter. For example, the internal diameter of a steel pipe whose nominal diameter is 1 in. is 1.049 in. (Table 8–2).

TABLE 8–2
Standard sizes for Schedule 40 steel pipes

Nominal Size, in.	Actual Inside Diameter, in.
⅛	0.269
¼	0.364
⅜	0.493
½	0.622
¾	0.824
1	1.049
1½	1.610
2	2.067
2½	2.469
3	3.068
5	5.047
10	10.02

Commercially available pipes differ from those used in the experiments in that the roughness of pipes in the market is not uniform, and it is difficult to give a precise description of it. Equivalent roughness values for some commercial pipes are given in Table 8–3, as well as on the Moody chart. But it should be kept in mind that these values are for new pipes, and the relative roughness of pipes may increase with use as a result of corrosion, scale buildup, and precipitation. As a result, the friction factor may increase by a factor of 5 to 10. Actual operating conditions must be considered in the design of piping systems. Also, the Moody chart and its equivalent Colebrook equation involve several uncertainties (the roughness size, experimental error, curve fitting of data, etc.), and thus the results obtained should not be treated as "exact." It is usually considered to be accurate to ±15 percent over the entire range in the figure.

The Colebrook equation is implicit in f, and thus the determination of the friction factor requires tedious iteration unless an equation solver is used. An approximate explicit relation for f is given by S. E. Haaland in 1983 (Ref. 9) as

$$\frac{1}{\sqrt{f}} \approx -1.8 \log \left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right] \quad \textbf{(8-74)}$$

The results obtained from this relation are within 2 percent of those obtained from Colebrook equation, and we recommend using this relation rather than the Moody chart to avoid reading errors.

In turbulent flow, wall roughness increases the heat transfer coefficient h by a factor of 2 or more [Dipprey and Sabersky (1963), Ref. 5]. The convection heat transfer coefficient for rough tubes can be calculated approximately from the Nusselt number relations such as Eq. 8–70 by using the friction factor determined from the Moody chart or the Colebrook equation. However, this approach is not very accurate since there is no further increase in h with f for $f > 4f_{\text{smooth}}$ [Norris (1970), Ref. 20] and correlations developed specifically for rough tubes should be used when more accuracy is desired.

TABLE 8–3

Equivalent roughness values for new commercial pipes*

	Roughness, ε	
Material	ft	mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

*The uncertainty in these values can be as much as ±60 percent.

Developing Turbulent Flow in the Entrance Region

The entry lengths for turbulent flow are typically short, often just 10 tube diameters long, and thus the Nusselt number determined for fully developed turbulent flow can be used approximately for the entire tube. This simple approach gives reasonable results for pressure drop and heat transfer for long tubes and conservative results for short ones. Correlations for the friction and heat transfer coefficients for the entrance regions are available in the literature for better accuracy.

Turbulent Flow in Noncircular Tubes

The velocity and temperature profiles in turbulent flow are nearly straight lines in the core region, and any significant velocity and temperature gradients occur in the viscous sublayer (Fig. 8–25). Despite the small thickness of laminar sublayer (usually much less than 1 percent of the pipe diameter), the characteristics of the flow in this layer are very important since they set the stage for flow in the rest of the pipe. Therefore, pressure drop and heat transfer characteristics of turbulent flow in tubes are dominated by the very thin

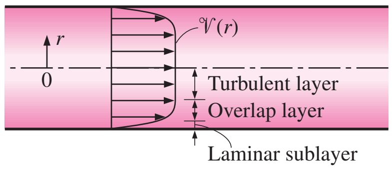


FIGURE 8–25

In turbulent flow, the velocity profile is nearly a straight line in the core region, and any significant velocity gradients occur in the viscous sublayer.

viscous sublayer next to the wall surface, and the shape of the core region is not of much significance. Consequently, the turbulent flow relations given above for circular tubes can also be used for noncircular tubes with reasonable accuracy by replacing the diameter D in the evaluation of the Reynolds number by the hydraulic diameter $D_h = 4A_c/p$.

Flow through Tube Annulus

Some simple heat transfer equipments consist of two concentric tubes, and are properly called *double-tube heat exchangers* (Fig. 8–26). In such devices, one fluid flows through the tube while the other flows through the annular space. The governing differential equations for both flows are identical. Therefore, steady laminar flow through an annulus can be studied analytically by using suitable boundary conditions.

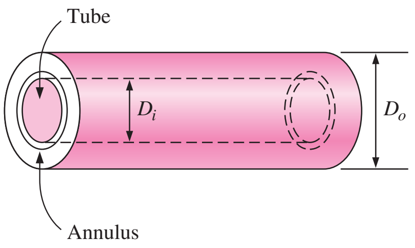


FIGURE 8–26
A double-tube heat exchanger that consists of two concentric tubes.

Consider a concentric annulus of inner diameter D_i and outer diameter D_o. The hydraulic diameter of annulus is

$$D_h = \frac{4A_c}{p} = \frac{4\pi(D_o^2 - D_i^2)/4}{\pi(D_o + D_i)} = D_o - D_i \tag{8-75}$$

Annular flow is associated with two Nusselt numbers—Nu_i on the inner tube surface and Nu_o on the outer tube surface—since it may involve heat transfer on both surfaces. The Nusselt numbers for fully developed laminar flow with one surface isothermal and the other adiabatic are given in Table 8–4. When Nusselt numbers are known, the convection coefficients for the inner and the outer surfaces are determined from

TABLE 8–4
Nusselt number for fully developed laminar flow in an annulus with one surface isothermal and the other adiabatic (Kays and Perkins, Ref. 14)

D_i/D_o	Nu_i	Nu_o
0	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

$$\mathrm{Nu}_i = \frac{h_i D_h}{k} \quad \text{and} \quad \mathrm{Nu}_o = \frac{h_o D_h}{k} \tag{8-76}$$

For fully developed turbulent flow, the inner and outer convection coefficients are approximately equal to each other, and the tube annulus can be treated as a noncircular duct with a hydraulic diameter of $D_h = D_o - D_i$. The Nusselt number in this case can be determined from a suitable turbulent flow relation such as the Gnielinski equation. To improve the accuracy of Nusselt numbers obtained from these relations for annular flow, Petukhov and Roizen (1964, Ref. 22) recommend multiplying them by the following correction factors when one of the tube walls is adiabatic and heat transfer is through the other wall:

$$F_i = 0.86\left(\frac{D_i}{D_o}\right)^{-0.16} \quad \text{(outer wall adiabatic)} \tag{8-77}$$

$$F_o = 0.86\left(\frac{D_i}{D_o}\right)^{-0.16} \quad \text{(inner wall adiabatic)} \tag{8-78}$$

Heat Transfer Enhancement

Tubes with rough surfaces have much higher heat transfer coefficients than tubes with smooth surfaces. Therefore, tube surfaces are often intentionally *roughened, corrugated,* or *finned* in order to *enhance* the convection heat transfer coefficient and thus the convection heat transfer rate (Fig. 8–27). Heat transfer in turbulent flow in a tube has been increased by as much as

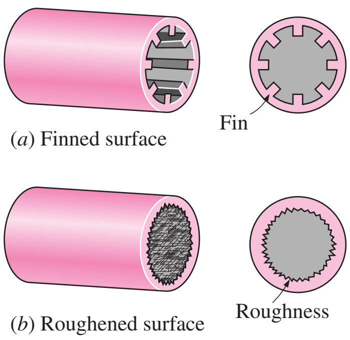


FIGURE 8–27
Tube surfaces are often *roughened, corrugated,* or *finned* in order to *enhance* convection heat transfer.

400 percent by roughening the surface. Roughening the surface, of course, also increases the friction factor and thus the power requirement for the pump or the fan.

The convection heat transfer coefficient can also be increased by inducing pulsating flow by pulse generators, by inducing swirl by inserting a twisted tape into the tube, or by inducing secondary flows by coiling the tube.

EXAMPLE 8–4 Pressure Drop in a Water Pipe

Water at 60°F (ρ = 62.36 lbm/ft^3 and μ = 2.713 lbm/ft · h) is flowing steadily in a 2-in.-diameter horizontal pipe made of stainless steel at a rate of 0.2 ft^3/s (Fig. 8–28). Determine the pressure drop and the required pumping power input for flow through a 200-ft-long section of the pipe.

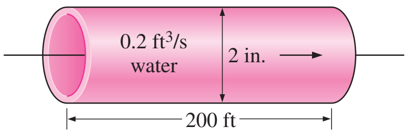


FIGURE 8–28
Schematic for Example 8–4.

SOLUTION The flow rate through a specified water pipe is given. The pressure drop and the pumping power requirements are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as a pump or a turbine.

Properties The density and dynamic viscosity of water are given by ρ = 62.36 lbm/ft^3 and μ = 2.713 lbm/ft · h = 0.0007536 lbm/ft · s, respectively.

Analysis First we calculate the mean velocity and the Reynolds number to determine the flow regime:

$$\mathcal{V} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.2\ \text{ft}^3/\text{s}}{\pi(2/12\ \text{ft})^2/4} = 9.17\ \text{ft/s}$$

$$\text{Re} = \frac{\rho\mathcal{V}D}{\mu} = \frac{(62.36\ \text{lbm/ft}^3)(9.17\ \text{ft/s})(2/12\ \text{ft})}{2.713\ \text{lbm/ft}\cdot\text{h}}\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 126{,}400$$

which is greater than 10,000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{0.000007\ \text{ft}}{2/12\ \text{ft}} = 0.000042$$

The friction factor corresponding to this relative roughness and the Reynolds number can simply be determined from the Moody chart. To avoid the reading error, we determine it from the Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.000042}{3.7} + \frac{2.51}{126{,}400\sqrt{f}}\right)$$

Using an equation solver or an iterative scheme, the friction factor is determined to be f = 0.0174. Then the pressure drop and the required power input become

$$\Delta P = f\frac{L}{D}\frac{\rho\mathcal{V}^2}{2} = 0.0174\,\frac{200\ \text{ft}}{2/12\ \text{ft}}\,\frac{(62.36\ \text{lbm/ft}^3)(9.17\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm}\cdot\text{ft/s}^2}\right)$$
$$= \mathbf{1700\ lbf/ft^2 = 11.8\ psi}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.2\ \text{ft}^3/\text{s})(1700\ \text{lbf/ft}^2)\left(\frac{1\ \text{W}}{0.737\ \text{lbf}\cdot\text{ft/s}}\right) = \mathbf{461\ W}$$

Therefore, power input in the amount of 461 W is needed to overcome the frictional losses in the pipe.

Discussion The friction factor also could be determined easily from the explicit Haaland relation. It would give $f = 0.0172$, which is sufficiently close to 0.0174. Also, the friction factor corresponding to $\varepsilon = 0$ in this case is 0.0171, which indicates that stainless steel pipes can be assumed to be smooth with negligible error.

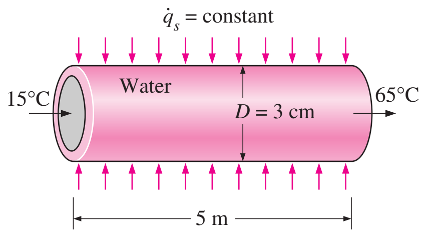


FIGURE 8–29
Schematic for Example 8–5.

EXAMPLE 8–5 Heating of Water by Resistance Heaters in a Tube

Water is to be heated from 15°C to 65°C as it flows through a 3-cm-internal-diameter 5-m-long tube (Fig. 8–29). The tube is equipped with an electric resistance heater that provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 10 L/min, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at the exit.

SOLUTION Water is to be heated in a tube equipped with an electric resistance heater on its surface. The power rating of the heater and the inner surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth.

Properties The properties of water at the bulk mean temperature of $T_b = (T_i + T_e)/2 = (15 + 65)/2 = 40°C$ are (Table A-9).

$$\rho = 992.1 \text{ kg/m}^3 \qquad C_p = 4179 \text{ J/kg}\cdot°\text{C}$$
$$k = 0.631 \text{ W/m}\cdot°\text{C} \qquad \text{Pr} = 4.32$$
$$\nu = \mu/\rho = 0.658 \times 10^{-6} \text{ m}^2/\text{s}$$

Analysis The cross sectional and heat transfer surface areas are

$$A_c = \tfrac{1}{4}\pi D^2 = \tfrac{1}{4}\pi(0.03 \text{ m})^2 = 7.069 \times 10^{-4} \text{ m}^2$$
$$A_s = pL = \pi DL = \pi(0.03 \text{ m})(5 \text{ m}) = 0.471 \text{ m}^2$$

The volume flow rate of water is given as $\dot{V} = 10$ L/min $= 0.01$ m^3/min. Then the mass flow rate becomes

$$\dot{m} = \rho\dot{V} = (992.1 \text{ kg/m}^3)(0.01 \text{ m}^3/\text{min}) = 9.921 \text{ kg/min} = 0.1654 \text{ kg/s}$$

To heat the water at this mass flow rate from 15°C to 65°C, heat must be supplied to the water at a rate of

$$\begin{aligned}\dot{Q} &= \dot{m}C_p(T_e - T_i)\\ &= (0.1654 \text{ kg/s})(4.179 \text{ kJ/kg}\cdot°\text{C})(65 - 15)°\text{C}\\ &= 34.6 \text{ kJ/s} = 34.6 \text{ kW}\end{aligned}$$

All of this energy must come from the resistance heater. Therefore, the power rating of the heater must be **34.6 kW.**

The surface temperature T_s of the tube at any location can be determined from

$$\dot{q}_s = h(T_s - T_m) \quad \rightarrow \quad T_s = T_m + \frac{\dot{q}_s}{h}$$

where h is the heat transfer coefficient and T_m is the mean temperature of the fluid at that location. The surface heat flux is constant in this case, and its value can be determined from

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{34.6 \text{ kW}}{0.471 \text{ m}^2} = 73.46 \text{ kW/m}^2$$

To determine the heat transfer coefficient, we first need to find the mean velocity of water and the Reynolds number:

$$\mathcal{V}_m = \frac{\dot{V}}{A_c} = \frac{0.010 \text{ m}^3/\text{min}}{7.069 \times 10^{-4} \text{ m}^2} = 14.15 \text{ m/min} = 0.236 \text{ m/s}$$

$$\text{Re} = \frac{\mathcal{V}_m D}{\nu} = \frac{(0.236 \text{ m/s})(0.03 \text{ m})}{0.658 \times 10^{-6} \text{ m}^2/\text{s}} = 10{,}760$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry length is roughly

$$L_h \approx L_t \approx 10D = 10 \times 0.03 = 0.3 \text{ m}$$

which is much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe and determine the Nusselt number from

$$\text{Nu} = \frac{hD}{k} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.4} = 0.023(10{,}760)^{0.8} (4.34)^{0.4} = 69.5$$

Then,

$$h = \frac{k}{D}\text{Nu} = \frac{0.631 \text{ W/m} \cdot {}^\circ\text{C}}{0.03 \text{ m}}(69.5) = 1462 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

and the surface temperature of the pipe at the exit becomes

$$T_s = T_m + \frac{\dot{q}_s}{h} = 65^\circ\text{C} + \frac{73{,}460 \text{ W/m}^2}{1462 \text{ W/m}^2 \cdot {}^\circ\text{C}} = \mathbf{115^\circ C}$$

Discussion Note that the inner surface temperature of the pipe will be 50°C higher than the mean water temperature at the pipe exit. This temperature difference of 50°C between the water and the surface will remain constant throughout the fully developed flow region.

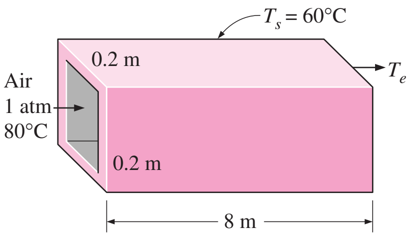


FIGURE 8–30
Schematic for Example 8–6.

EXAMPLE 8–6 Heat Loss from the Ducts of a Heating System

Hot air at atmospheric pressure and 80°C enters an 8-m-long uninsulated square duct of cross section 0.2 m × 0.2 m that passes through the attic of a house at a rate of 0.15 m³/s (Fig. 8–30). The duct is observed to be nearly isothermal at 60°C. Determine the exit temperature of the air and the rate of heat loss from the duct to the attic space.

SOLUTION Heat loss from uninsulated square ducts of a heating system in the attic is considered. The exit temperature and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas.

Properties We do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk mean temperature of air, which is the temperature at which the properties are to be determined. The temperature of air at the inlet is 80°C and we expect this temperature to drop somewhat as a result of heat loss through the duct whose surface is at 60°C. At 80°C and 1 atm we read (Table A-15)

$$\rho = 0.9994 \text{ kg/m}^3 \qquad C_p = 1008 \text{ J/kg} \cdot °\text{C}$$
$$k = 0.02953 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7154$$
$$\nu = 2.097 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis The characteristic length (which is the hydraulic diameter), the mean velocity, and the Reynolds number in this case are

$$D_h = \frac{4A_c}{p} = \frac{4a^2}{4a} = a = 0.2 \text{ m}$$
$$\mathcal{V}_m = \frac{\dot{V}}{A_c} = \frac{0.15 \text{ m}^3/\text{s}}{(0.2 \text{ m})^2} = 3.75 \text{ m/s}$$
$$\text{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(3.75 \text{ m/s})(0.2 \text{ m})}{2.097 \times 10^{-5} \text{ m}^2/\text{s}} = 35{,}765$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10 \times 0.2 \text{ m} = 2 \text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct and determine the Nusselt number from

$$\text{Nu} = \frac{hD_h}{k} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.3} = 0.023(35{,}765)^{0.8} (0.7154)^{0.3} = 91.4$$

Then,

$$h = \frac{k}{D_h}\text{Nu} = \frac{0.02953\ \text{W/m}\cdot{}^\circ\text{C}}{0.2\ \text{m}}(91.4) = 13.5\ \text{W/m}^2\cdot{}^\circ\text{C}$$
$$A_s = pL = 4aL = 4 \times (0.2\ \text{m})(8\ \text{m}) = 6.4\ \text{m}^2$$
$$\dot{m} = \rho\dot{V} = (1.009\ \text{kg/m}^3)(0.15\ \text{m}^3/\text{s}) = 0.151\ \text{kg/s}$$

Next, we determine the exit temperature of air from

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p)$$
$$= 60^\circ\text{C} - [(60 - 80)^\circ\text{C}]\exp\left[-\frac{(13.5\ \text{W/m}^2\cdot{}^\circ\text{C})(6.4\ \text{m}^2)}{(0.151\ \text{kg/s})(1008\ \text{J/kg}\cdot{}^\circ\text{C})}\right]$$
$$= \mathbf{71.3^\circ C}$$

Then the logarithmic mean temperature difference and the rate of heat loss from the air become

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln\dfrac{T_s - T_e}{T_s - T_i}} = \frac{80 - 71.3}{\ln\dfrac{60 - 71.3}{60 - 80}} = -15.2^\circ\text{C}$$
$$\dot{Q} = hA_s\,\Delta T_{\ln} = (13.5\ \text{W/m}^2\cdot{}^\circ\text{C})(6.4\ \text{m}^2)(-15.2^\circ\text{C}) = \mathbf{-1313\ W}$$

Therefore, air will lose heat at a rate of 1313 W as it flows through the duct in the attic.

Discussion The average fluid temperature is (80 + 71.3)/2 = 75.7°C, which is sufficiently close to 80°C at which we evaluated the properties of air. Therefore, it is not necessary to re-evaluate the properties at this temperature and to repeat the calculations.

SUMMARY

Internal flow is characterized by the fluid being completely confined by the inner surfaces of the tube. The mean velocity and mean temperature for a circular tube of radius R are expressed as

$$\mathcal{V}_m = \frac{2}{R^2}\int_0^R \mathcal{V}(r, x)r\,dr \qquad \text{and} \qquad T_m = \frac{2}{\mathcal{V}_m R^2}\int_0^R \mathcal{V}Tr\,dr$$

The Reynolds number for internal flow and the hydraulic diameter are defined as

$$\text{Re} = \frac{\rho\mathcal{V}_m D}{\mu} = \frac{\mathcal{V}_m D}{\nu} \qquad \text{and} \qquad D_h = \frac{4A_c}{p}$$

The flow in a tube is laminar for Re < 2300, turbulent for Re > 10,000, and transitional in between.

The length of the region from the tube inlet to the point at which the boundary layer merges at the centerline is the *hydrodynamic entry length* L_h. The region beyond the entrance region in which the velocity profile is fully developed is the *hydrodynamically fully developed region.* The length of the region of flow over which the thermal boundary layer develops and reaches the tube center is the *thermal entry length* L_t. The region in which the flow is both hydrodynamically and thermally developed is the *fully developed flow region.* The entry lengths are given by

$$L_{h,\ \text{laminar}} \approx 0.05\ \text{Re}\ D$$
$$L_{t,\ \text{laminar}} \approx 0.05\ \text{Re}\ \text{Pr}\ D = \text{Pr}\ L_{h,\ \text{laminar}}$$
$$L_{h,\ \text{turbulent}} \approx L_{t,\ \text{turbulent}} \approx 10D$$

For $\dot{q}_s$ = constant, the rate of heat transfer is expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m}C_p(T_e - T_i)$$

For T_s = constant, we have

$$\dot{Q} = hA_s\Delta T_{\ln} = \dot{m}C_p(T_e - T_i)$$
$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p)$$
$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

The pressure drop and required pumping power for a volume flow rate of $\dot{V}$ are

$$\Delta P = \frac{L}{D}\frac{\rho \mathcal{V}_m^2}{2} \quad \text{and} \quad \dot{W}_{\text{pump}} = \dot{V}\Delta P$$

For *fully developed laminar flow* in a circular pipe, we have:

$$\mathcal{V}(r) = 2\mathcal{V}_m\left(1 - \frac{r^2}{R^2}\right) = \mathcal{V}_{\max}\left(1 - \frac{r^2}{R^2}\right)$$
$$f = \frac{64\mu}{\rho D\mathcal{V}_m} = \frac{64}{\text{Re}}$$
$$\dot{V} = \mathcal{V}_{\text{ave}}A_c = \frac{\Delta PR^2}{8\mu L}\pi R^2 = \frac{\pi R^4\,\Delta P}{8\mu L} = \frac{\pi R^4\,\Delta P}{128\mu L}$$

Circular tube, laminar ($\dot{q}_s$ = constant): $\quad \text{Nu} = \frac{hD}{k} = 4.36$

Circular tube, laminar (T_s = constant): $\quad \text{Nu} = \frac{hD}{k} = 3.66$

For *developing laminar flow* in the entrance region with constant surface temperature, we have

Circular tube: $\quad \text{Nu} = 3.66 + \frac{0.065(D/L)\,\text{Re Pr}}{1 + 0.04[(D/L)\,\text{Re Pr}]^{2/3}}$

Circular tube: $\quad \text{Nu} = 1.86\left(\frac{\text{Re Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$

Parallel plates: $\quad \text{Nu} = 7.54 + \frac{0.03(D_h/L)\,\text{Re Pr}}{1 + 0.016[(D_h/L)\,\text{Re Pr}]^{2/3}}$

For *fully developed turbulent flow with smooth surfaces,* we have

$$f = (0.790 \ln \text{Re} - 1.64)^{-2} \qquad 10^4 < \text{Re} < 10^6$$
$$\text{Nu} = 0.125 f\,\text{Re}\,\text{Pr}^{1/3}$$
$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{1/3} \qquad \left(\begin{matrix}0.7 \le \text{Pr} \le 160\\ \text{Re} > 10{,}000\end{matrix}\right)$$

$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^n$ with $n = 0.4$ for *heating* and 0.3 for *cooling* of fluid

$$\text{Nu} = \frac{(f/8)(\text{Re} - 1000)\,\text{Pr}}{1 + 12.7(f/8)^{0.5}(\text{Pr}^{2/3} - 1)} \qquad \left(\begin{matrix}0.5 \le \text{Pr} \le 2000\\ 3 \times 10^3 < \text{Re} < 5 \times 10^6\end{matrix}\right)$$

The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$. For liquid metal flow in the range of $10^4 < \text{Re} < 10^6$ we have:

T_s = constant: $\quad \text{Nu} = 4.8 + 0.0156\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93}$

$\dot{q}_s$ = constant: $\quad \text{Nu} = 6.3 + 0.0167\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93}$

For *fully developed turbulent flow with rough surfaces,* the friction factor f is determined from the Moody chart or

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \approx -1.8\log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

For a *concentric annulus,* the hydraulic diameter is $D_h = D_o - D_i$, and the Nusselt numbers are expressed as

$$\text{Nu}_i = \frac{h_i D_h}{k} \quad \text{and} \quad \text{Nu}_o = \frac{h_o D_h}{k}$$

where the values for the Nusselt numbers are given in Table 8–4.

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PROBLEMS*

General Flow Analysis

8–1C Why are liquids usually transported in circular pipes?

8–2C Show that the Reynolds number for flow in a circular tube of diameter D can be expressed as $\text{Re} = 4\dot{m}/(\pi D\mu)$.

8–3C Which fluid at room temperature requires a larger pump to move at a specified velocity in a given tube: water or engine oil? Why?

8–4C What is the generally accepted value of the Reynolds number above which the flow in smooth pipes is turbulent?

8–5C What is hydraulic diameter? How is it defined? What is it equal to for a circular tube of diameter?

8–6C How is the hydrodynamic entry length defined for flow in a tube? Is the entry length longer in laminar or turbulent flow?

8–7C Consider laminar flow in a circular tube. Will the friction factor be higher near the inlet of the tube or near the exit? Why? What would your response be if the flow were turbulent?

8–8C How does surface roughness affect the pressure drop in a tube if the flow is turbulent? What would your response be if the flow were laminar?

8–9C How does the friction factor f vary along the flow direction in the fully developed region in (*a*) laminar flow and (*b*) turbulent flow?

8–10C What fluid property is responsible for the development of the velocity boundary layer? For what kinds of fluids will there be no velocity boundary layer in a pipe?

8–11C What is the physical significance of the number of transfer units $\text{NTU} = hA/\dot{m}C_p$? What do small and large NTU values tell about a heat transfer system?

8–12C What does the logarithmic mean temperature difference represent for flow in a tube whose surface temperature is constant? Why do we use the logarithmic mean temperature instead of the arithmetic mean temperature?

8–13C How is the thermal entry length defined for flow in a tube? In what region is the flow in a tube fully developed?

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with an EES-CD icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

8–14C Consider laminar forced convection in a circular tube. Will the heat flux be higher near the inlet of the tube or near the exit? Why?

8–15C Consider turbulent forced convection in a circular tube. Will the heat flux be higher near the inlet of the tube or near the exit? Why?

8–16C In the fully developed region of flow in a circular tube, will the velocity profile change in the flow direction? How about the temperature profile?

8–17C Consider the flow of oil in a tube. How will the hydrodynamic and thermal entry lengths compare if the flow is laminar? How would they compare if the flow were turbulent?

8–18C Consider the flow of mercury (a liquid metal) in a tube. How will the hydrodynamic and thermal entry lengths compare if the flow is laminar? How would they compare if the flow were turbulent?

8–19C What do the mean velocity $\mathcal{V}_m$ and the mean temperature T_m represent in flow through circular tubes of constant diameter?

8–20C Consider fluid flow in a tube whose surface temperature remains constant. What is the appropriate temperature difference for use in Newton's law of cooling with an average heat transfer coefficient?

8–21 Air enters a 20-cm-diameter 12-m-long underwater duct at 50°C and 1 atm at a mean velocity of 7 m/s, and is cooled by the water outside. If the average heat transfer coefficient is 85 W/m^2 · °C and the tube temperature is nearly equal to the water temperature of 5°C, determine the exit temperature of air and the rate of heat transfer.

8–22 Cooling water available at 10°C is used to condense steam at 30°C in the condenser of a power plant at a rate of 0.15 kg/s by circulating the cooling water through a bank of 5-m-long 1.2-cm-internal-diameter thin copper tubes. Water enters the tubes at a mean velocity of 4 m/s, and leaves at a temperature of 24°C. The tubes are nearly isothermal at 30°C. Determine the average heat transfer coefficient between the water and the tubes, and the number of tubes needed to achieve the indicated heat transfer rate in the condenser.

8–23 Repeat Problem 8–22 for steam condensing at a rate of 0.60 kg/s.

8–24 Combustion gases passing through a 3-cm-internal-diameter circular tube are used to vaporize waste water at atmospheric pressure. Hot gases enter the tube at 115 kPa and 250°C at a mean velocity of 5 m/s, and leave at 150°C. If the average heat transfer coefficient is 120 W/m^2 · °C and the inner surface temperature of the tube is 110°C, determine (*a*) the tube length and (*b*) the rate of evaporation of water.

8–25 Repeat Problem 8–24 for a heat transfer coefficient of 60 W/m^2 · °C.

Laminar and Turbulent Flow in Tubes

8–26C How is the friction factor for flow in a tube related to the pressure drop? How is the pressure drop related to the pumping power requirement for a given mass flow rate?

8–27C Someone claims that the shear stress at the center of a circular pipe during fully developed laminar flow is zero. Do you agree with this claim? Explain.

8–28C Someone claims that in fully developed turbulent flow in a tube, the shear stress is a maximum at the tube surface. Do you agree with this claim? Explain.

8–29C Consider fully developed flow in a circular pipe with negligible entrance effects. If the length of the pipe is doubled, the pressure drop will (*a*) double, (*b*) more than double, (*c*) less than double, (*d*) reduce by half, or (*e*) remain constant.

8–30C Someone claims that the volume flow rate in a circular pipe with laminar flow can be determined by measuring the velocity at the centerline in the fully developed region, multiplying it by the cross sectional area, and dividing the result by 2. Do you agree? Explain.

8–31C Someone claims that the average velocity in a circular pipe in fully developed laminar flow can be determined by simply measuring the velocity at $R/2$ (midway between the wall surface and the centerline). Do you agree? Explain.

8–32C Consider fully developed laminar flow in a circular pipe. If the diameter of the pipe is reduced by half while the flow rate and the pipe length are held constant, the pressure drop will (*a*) double, (*b*) triple, (*c*) quadruple, (*d*) increase by a factor of 8, or (*e*) increase by a factor of 16.

8–33C Consider fully developed laminar flow in a circular pipe. If the viscosity of the fluid is reduced by half by heating while the flow rate is held constant, how will the pressure drop change?

8–34C How does surface roughness affect the heat transfer in a tube if the fluid flow is turbulent? What would your response be if the flow in the tube were laminar?

8–35 Water at 15°C ($\rho = 999.1$ kg/m^3 and $\mu = 1.138 \times 10^{-3}$ kg/m · s) is flowing in a 4-cm-diameter and 30-m long horizontal pipe made of stainless steel steadily at a rate of 5 L/s. Determine (*a*) the pressure drop and (*b*) the pumping power requirement to overcome this pressure drop.

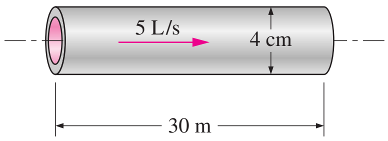

FIGURE P8–35

8–36 In fully developed laminar flow in a circular pipe, the velocity at $R/2$ (midway between the wall surface and the centerline) is measured to be 6 m/s. Determine the velocity at the center of the pipe. *Answer:* 8 m/s

8–37 The velocity profile in fully developed laminar flow in a circular pipe of inner radius $R = 2$ cm, in m/s, is given by $\mathcal{V}(r) = 4(1 - r^2/R^2)$. Determine the mean and maximum velocities in the pipe, and the volume flow rate.

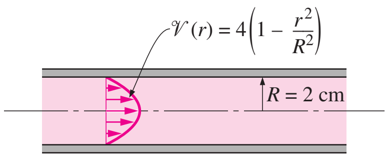

FIGURE P8–37

8–38 Repeat Problem 8–37 for a pipe of inner radius 5 cm.

8–39 Water at 10°C ($\rho = 999.7$ kg/m^3 and $\mu = 1.307 \times 10^{-3}$ kg/m · s) is flowing in a 0.20-cm-diameter 15-m-long pipe steadily at an average velocity of 1.2 m/s. Determine (*a*) the pressure drop and (*b*) the pumping power requirement to overcome this pressure drop.
Answers: (*a*) 188 kPa,(*b*) 0.71 W

8–40 Water is to be heated from 10°C to 80°C as it flows through a 2-cm-internal-diameter, 7-m-long tube. The tube is equipped with an electric resistance heater, which provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 8 L/min, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at the exit.

8–41 Hot air at atmospheric pressure and 85°C enters a 10-m-long uninsulated square duct of cross section 0.15 m × 0.15 m that passes through the attic of a house at a rate of 0.10 m^3/s. The duct is observed to be nearly isothermal at 70°C. Determine the exit temperature of the air and the rate of heat loss from the duct to the air space in the attic.
Answers: 75.7°C, 941 W

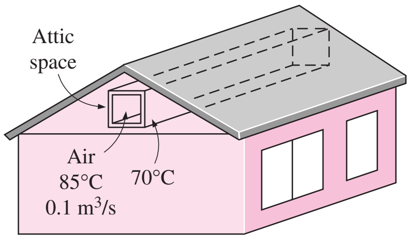

FIGURE P8–41

8–42 Reconsider Problem 8–41. Using EES (or other) software, investigate the effect of the volume flow rate of air on the exit temperature of air and the rate of heat loss. Let the flow rate vary from 0.05 m^3/s to 0.15 m^3/s.

Plot the exit temperature and the rate of heat loss as a function of flow rate, and discuss the results.

8–43 Consider an air solar collector that is 1 m wide and 5 m long and has a constant spacing of 3 cm between the glass cover and the collector plate. Air enters the collector at 30°C at a rate of 0.15 m^3/s through the 1-m-wide edge and flows along the 5-m-long passage way. If the average temperatures of the glass cover and the collector plate are 20°C and 60°C, respectively, determine (*a*) the net rate of heat transfer to the air in the collector and (*b*) the temperature rise of air as it flows through the collector.

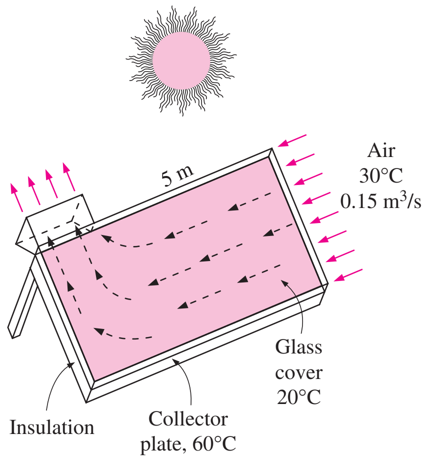

FIGURE P8–43

8–44 Consider the flow of oil at 10°C in a 40-cm-diameter pipeline at an average velocity of 0.5 m/s. A 300-m-long section of the pipeline passes through icy waters of a lake at 0°C. Measurements indicate that the surface temperature of the pipe is very nearly 0°C. Disregarding the thermal resistance of the pipe material, determine (*a*) the temperature of the oil when the pipe leaves the lake, (*b*) the rate of heat transfer from the oil, and (*c*) the pumping power required to overcome the pressure losses and to maintain the flow oil in the pipe.

8–45 Consider laminar flow of a fluid through a square channel maintained at a constant temperature. Now the mean velocity of the fluid is doubled. Determine the change in the pressure drop and the change in the rate of heat transfer between the fluid and the walls of the channel. Assume the flow regime remains unchanged.

8–46 Repeat Problem 8–45 for turbulent flow.

8–47E The hot water needs of a household are to be met by heating water at 55°F to 200°F by a parabolic solar collector at a rate of 4 lbm/s. Water flows through a 1.25-in.-diameter thin aluminum tube whose outer surface is blackanodized in order to maximize its solar absorption ability. The centerline of the tube coincides with the focal line of the collector, and a glass sleeve is placed outside the tube to minimize the heat losses. If solar energy is transferred to water at a net rate of 350 Btu/h per ft length of the tube, determine the required length of the parabolic collector to meet the hot water requirements of this house. Also, determine the surface temperature of the tube at the exit.

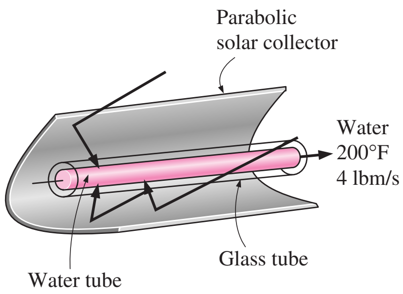

FIGURE P8–47E

8–48 A 15-cm × 20-cm printed circuit board whose components are not allowed to come into direct contact with air for reliability reasons is to be cooled by passing cool air through a 20-cm-long channel of rectangular cross section 0.2 cm × 14 cm drilled into the board. The heat generated by the electronic components is conducted across the thin layer of the board to the channel, where it is removed by air that enters the channel at 15°C. The heat flux at the top surface of the channel can be considered to be uniform, and heat transfer through other surfaces is negligible. If the velocity of the air at the inlet of the channel is not to exceed 4 m/s and the surface temperature of the channel is to remain under 50°C, determine the maximum total power of the electronic components that can safely be mounted on this circuit board.

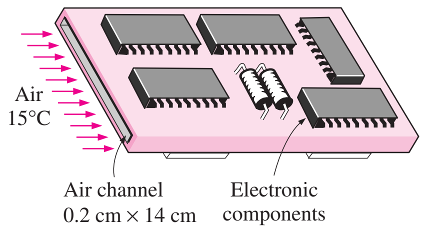

FIGURE P8–48

8–49 Repeat Problem 8–48 by replacing air with helium, which has six times the thermal conductivity of air.

8–50 Reconsider Problem 8–48. Using EES (or other) software, investigate the effects of air velocity at the inlet of the channel and the maximum surface temperature on the maximum total power dissipation of electronic components. Let the air velocity vary from 1 m/s to 10 m/s and the surface temperature from 30°C to 90°C. Plot the power dissipation as functions of air velocity and surface temperature, and discuss the results.

8–51 Air enters a 7-m-long section of a rectangular duct of cross section 15 cm × 20 cm at 50°C at an average velocity of 7 m/s. If the walls of the duct are maintained at 10°C, determine (*a*) the outlet temperature of the air, (*b*) the rate of heat transfer from the air, and (*c*) the fan power needed to overcome the pressure losses in this section of the duct.
Answers: (*a*) 32.8°C, (*b*) 3674 W, (*c*) 4.2 W

8–52 Reconsider Problem 8–51. Using EES (or other) software, investigate the effect of air velocity on the exit temperature of air, the rate of heat transfer, and the fan power. Let the air velocity vary from 1 m/s to 10 m/s. Plot the exit temperature, the rate of heat transfer, and the fan power as a function of the air velocity, and discuss the results.

8–53 Hot air at 60°C leaving the furnace of a house enters a 12-m-long section of a sheet metal duct of rectangular cross section 20 cm × 20 cm at an average velocity of 4 m/s. The thermal resistance of the duct is negligible, and the outer surface of the duct, whose emissivity is 0.3, is exposed to the cold air at 10°C in the basement, with a convection heat transfer coefficient of 10 W/m² · °C. Taking the walls of the basement to be at 10°C also, determine (*a*) the temperature at which the hot air will leave the basement and (*b*) the rate of heat loss from the hot air in the duct to the basement.

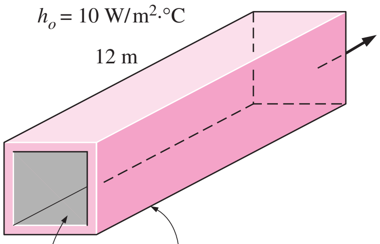

FIGURE P8–53

8–54 Reconsider Problem 8–53. Using EES (or other) software, investigate the effects of air velocity and the surface emissivity on the exit temperature of air and the rate of heat loss. Let the air velocity vary from 1 m/s to 10 m/s and the emissivity from 0.1 to 1.0. Plot the exit temperature and the rate of heat loss as functions of air velocity and emissivity, and discuss the results.

8–55 The components of an electronic system dissipating 90 W are located in a 1-m-long horizontal duct whose cross section is 16 cm × 16 cm. The components in the duct are cooled by forced air, which enters at 32°C at a rate of 0.65 m³/min. Assuming 85 percent of the heat generated inside is transferred to air flowing through the duct and the remaining 15 percent is lost through the outer surfaces of the duct, determine (*a*) the exit temperature of air and (*b*) the highest component surface temperature in the duct.

8–56 Repeat Problem 8–55 for a circular horizontal duct of 15-cm diameter.

8–57 Consider a hollow-core printed circuit board 12 cm high and 18 cm long, dissipating a total of 20 W. The width of the air gap in the middle of the PCB is 0.25 cm. The cooling air enters the 12-cm-wide core at 32°C at a rate of 0.8 L/s. Assuming the heat generated to be uniformly distributed over the two side surfaces of the PCB, determine (*a*) the temperature at which the air leaves the hollow core and (*b*) the highest temperature on the inner surface of the core.
Answers: (*a*) 54.0°C, (*b*) 72.8°C

8–58 Repeat Problem 8–57 for a hollow-core PCB dissipating 35 W.

8–59E Water at 54°F is heated by passing it through 0.75-in.-internal-diameter thin-walled copper tubes. Heat is supplied to the water by steam that condenses outside the copper tubes at 250°F. If water is to be heated to 140°F at a rate of 0.7 lbm/s, determine (*a*) the length of the copper tube that needs to be used and (*b*) the pumping power required to overcome pressure losses. Assume the entire copper tube to be at the steam temperature of 250°F.

8–60 A computer cooled by a fan contains eight PCBs, each dissipating 10 W of power. The height of the PCBs is 12 cm and the length is 18 cm. The clearance between the tips of the components on the PCB and the back surface of the adjacent PCB is 0.3 cm. The cooling air is supplied by a 10-W fan mounted at the inlet. If the temperature rise of air as it flows through the case of the computer is not to exceed 10°C, determine (*a*) the flow rate of the air that the fan needs to deliver, (*b*) the fraction of the temperature rise of air that is due to the heat generated by the fan and its motor, and (*c*) the highest allowable inlet air temperature if the surface temperature of the

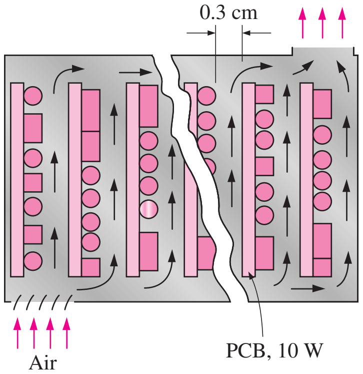

FIGURE P8–60

components is not to exceed 70°C anywhere in the system. Use air properties at 25°C.

Review Problems

8–61 A geothermal district heating system involves the transport of geothermal water at 110°C from a geothermal well to a city at about the same elevation for a distance of 12 km at a rate of 1.5 m^3/s in 60-cm-diameter stainless steel pipes. The fluid pressures at the wellhead and the arrival point in the city are to be the same. The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. (*a*) Assuming the pump-motor efficiency to be 65 percent, determine the electric power consumption of the system for pumping. (*b*) Determine the daily cost of power consumption of the system if the unit cost of electricity is $0.06/kWh. (*c*) The temperature of geothermal water is estimated to drop 0.5°C during this long flow. Determine if the frictional heating during flow can make up for this drop in temperature.

8–62 Repeat Problem 8–61 for cast iron pipes of the same diameter.

8–63 The velocity profile in fully developed laminar flow in a circular pipe, in m/s, is given by $\mathcal{V}(r) = 6(1 - 100r^2)$ where r is the radial distance from the centerline of the pipe in m. Determine (*a*) the radius of the pipe, (*b*) the mean velocity through the pipe, and (*c*) the maximum velocity in the pipe.

8–64E The velocity profile in fully developed laminar flow of water at 40°F in a 80-ft-long horizontal circular pipe, in ft/s, is given by $\mathcal{V}(r) = 0.8(1 - 625r^2)$ where r is the radial distance from the centerline of the pipe in ft. Determine (*a*) the volume flow rate of water through the pipe, (*b*) the pressure drop across the pipe, and (*c*) the useful pumping power required to overcome this pressure drop.

8–65 The compressed air requirements of a manufacturing facility are met by a 150-hp compressor located in a room that is maintained at 20°C. In order to minimize the compressor work, the intake port of the compressor is connected to the outside through an 11-m-long, 20-cm-diameter duct made of thin aluminum sheet. The compressor takes in air at a rate of 0.27 m^3/s at the outdoor conditions of 10°C and 95 kPa. Disregarding the thermal resistance of the duct and taking the heat transfer coefficient on the outer surface of the duct to be 10 $W/m^2 \cdot °C$, determine (*a*) the power used by the compressor to overcome the pressure drop in this duct, (*b*) the rate of heat transfer to the incoming cooler air, and (*c*) the temperature rise of air as it flows through the duct.

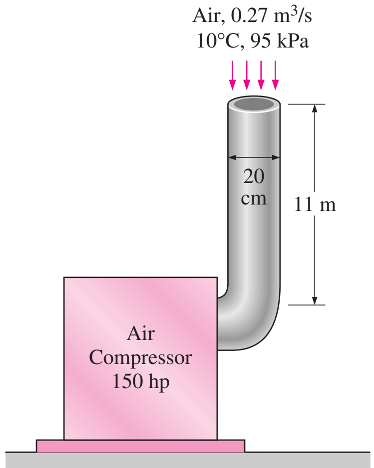

FIGURE P8–65

8–66 A house built on a riverside is to be cooled in summer by utilizing the cool water of the river, which flows at an average temperature of 15°C. A 15-m-long section of a circular duct of 20-cm diameter passes through the water. Air enters the underwater section of the duct at 25°C at a velocity of 3 m/s. Assuming the surface of the duct to be at the temperature of the water, determine the outlet temperature of air as it leaves the underwater portion of the duct. Also, for an overall fan efficiency of 55 percent, determine the fan power input needed to overcome the flow resistance in this section of the duct.

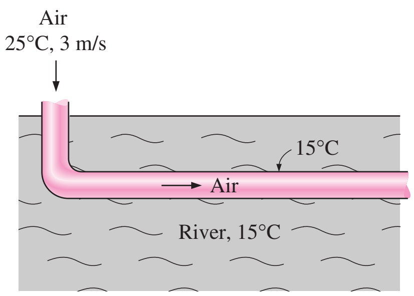

FIGURE P8–66

8–67 Repeat Problem 8–66 assuming that a 0.15-mm-thick layer of mineral deposit ($k = 3$ W/m · °C) formed on the inner surface of the pipe.

8–68E The exhaust gases of an automotive engine leave the combustion chamber and enter a 8-ft-long and 3.5-in.-diameter thin-walled steel exhaust pipe at 800°F and 15.5 psia at a rate of 0.2 lbm/s. The surrounding ambient air is at a temperature of 80°F, and the heat transfer coefficient on the outer surface of the exhaust pipe is 3 $Btu/h \cdot ft^2 \cdot °F$. Assuming the exhaust gases to have the properties of air, determine (*a*) the velocity of the exhaust gases at the inlet of the exhaust pipe and (*b*) the temperature at which the exhaust gases will leave the pipe and enter the air.

8–69 Hot water at 90°C enters a 15-m section of a cast iron pipe ($k = 52$ W/m · °C) whose inner and outer diameters are 4 and 4.6 cm, respectively, at an average velocity of 0.8 m/s. The outer surface of the pipe, whose emissivity is 0.7, is exposed to the cold air at 10°C in a basement, with a convection heat

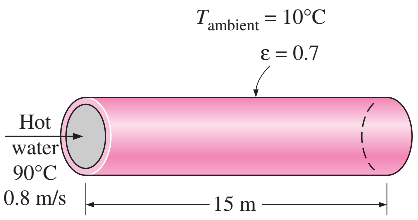

FIGURE P8–69

transfer coefficient of 15 W/m^2 · °C. Taking the walls of the basement to be at 10°C also, determine (*a*) the rate of heat loss from the water and (*b*) the temperature at which the water leaves the basement.

8–70 Repeat Problem 8–69 for a pipe made of copper (k = 386 W/m · °C) instead of cast iron.

8–71 D. B. Tuckerman and R. F. Pease of Stanford University demonstrated in the early 1980s that integrated circuits can be cooled very effectively by fabricating a series of microscopic channels 0.3 mm high and 0.05 mm wide in the back of the substrate and covering them with a plate to confine the fluid flow within the channels. They were able to dissipate 790 W of power generated in a 1-cm^2 silicon chip at a junction-to-ambient temperature difference of 71°C using water as the coolant flowing at a rate of 0.01 L/s through 100 such channels under a 1-cm × 1-cm silicon chip. Heat is transferred primarily through the base area of the channel, and it was found that the increased surface area and thus the fin effect are of lesser importance. Disregarding the entrance effects and ignoring any heat transfer from the side and cover surfaces, determine (*a*) the temperature rise of water as it flows through the microchannels and (*b*) the average surface temperature of the base of the microchannels for a power dissipation of 50 W. Assume the water enters the channels at 20°C.

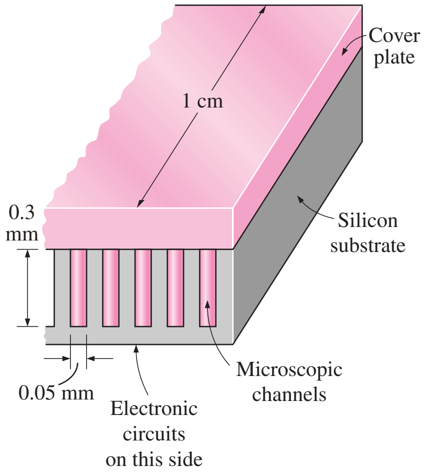

FIGURE P8–71

8–72 Liquid-cooled systems have high heat transfer coefficients associated with them, but they have the inherent disadvantage that they present potential leakage problems. Therefore, air is proposed to be used as the microchannel coolant. Repeat Problem 8–71 using air as the cooling fluid instead of water, entering at a rate of 0.5 L/s.

8–73 Hot exhaust gases leaving a stationary diesel engine at 450°C enter a 15-cm-diameter pipe at an average velocity of 3.6 m/s. The surface temperature of the pipe is 180°C. Determine the pipe length if the exhaust gases are to leave the pipe at 250°C after transferring heat to water in a heat recovery unit. Use properties of air for exhaust gases.

8–74 Geothermal steam at 165°C condenses in the shell side of a heat exchanger over the tubes through which water flows. Water enters the 4-cm-diameter, 14-m-long tubes at 20°C at a rate of 0.8 kg/s. Determine the exit temperature of water and the rate of condensation of geothermal steam.

8–75 Cold air at 5°C enters a 12-cm-diameter 20-m-long isothermal pipe at a velocity of 2.5 m/s and leaves at 19°C. Estimate the surface temperature of the pipe.

8–76 Oil at 10°C is to be heated by saturated steam at 1 atm in a double-pipe heat exchanger to a temperature of 30°C. The inner and outer diameters of the annular space are 3 cm and 5 cm, respectively, and oil enters at with a mean velocity of 0.8 m/s. The inner tube may be assumed to be isothermal at 100°C, and the outer tube is well insulated. Assuming fully developed flow for oil, determine the tube length required to heat the oil to the indicated temperature. In reality, will you need a shorter or longer tube? Explain.

Design and Essay Problems

8–77 Electronic boxes such as computers are commonly cooled by a fan. Write an essay on forced air cooling of electronic boxes and on the selection of the fan for electronic devices.

8–78 Design a heat exchanger to pasteurize milk by steam in a dairy plant. Milk is to flow through a bank of 1.2-cm internal diameter tubes while steam condenses outside the tubes at 1 atm. Milk is to enter the tubes at 4°C, and it is to be heated to 72°C at a rate of 15 L/s. Making reasonable assumptions, you are to specify the tube length and the number of tubes, and the pump for the heat exchanger.

8–79 A desktop computer is to be cooled by a fan. The electronic components of the computer consume 80 W of power under full-load conditions. The computer is to operate in environments at temperatures up to 50°C and at elevations up to 3000 m where the atmospheric pressure is 70.12 kPa. The exit temperature of air is not to exceed 60°C to meet the reliability requirements. Also, the average velocity of air is not to exceed 120 m/min at the exit of the computer case, where the fan is installed to keep the noise level down. Specify the flow rate of the fan that needs to be installed and the diameter of the casing of the fan.