

Chapter 17

COMPRESSIBLE FLOW

For the most part, we have limited our consideration so far to flows for which density variations and thus compressibility effects are negligible. In this chapter we lift this limitation and consider flows that involve significant changes in density. Such flows are called *compressible flows*, and they are frequently encountered in devices that involve the flow of gases at very high velocities. Compressible flow combines fluid dynamics and thermodynamics in that both are necessary to the development of the required theoretical background. In this chapter, we develop the general relations associated with one-dimensional compressible flows for an ideal gas with constant specific heats.

We start this chapter by introducing the concepts of *stagnation state*, *speed of sound*, and *Mach number* for compressible flows. The relationships between the static and stagnation fluid properties are developed for isentropic flows of ideal gases, and they are expressed as functions of specific-heat ratios and the Mach number. The effects of area changes for one-dimensional isentropic subsonic and supersonic flows are discussed. These effects are illustrated by considering the isentropic flow through *converging* and *converging-diverging nozzles*. The concept of *shock waves* and the variation of flow properties across normal and oblique shocks are discussed. Finally, we consider the effects of heat transfer on compressible flows and examine steam nozzles.

Objectives

The objectives of Chapter 17 are to:

- Develop the general relations for compressible flows encountered when gases flow at high speeds.
- Introduce the concepts of *stagnation state*, *speed of sound*, and *Mach number* for a compressible fluid.
- Develop the relationships between the static and stagnation fluid properties for isentropic flows of ideal gases.
- Derive the relationships between the static and stagnation fluid properties as functions of specific-heat ratios and Mach number.
- Derive the effects of area changes for one-dimensional isentropic subsonic and supersonic flows.
- Solve problems of isentropic flow through converging and converging-diverging nozzles.
- Discuss the shock wave and the variation of flow properties across the shock wave.
- Develop the concept of duct flow with heat transfer and negligible friction known as *Rayleigh flow*.
- Examine the operation of steam nozzles commonly used in steam turbines.

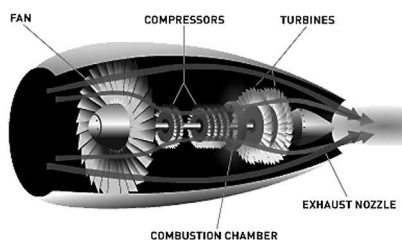


FIGURE 17-1

Aircraft and jet engines involve high speeds, and thus the kinetic energy term should always be considered when analyzing them.

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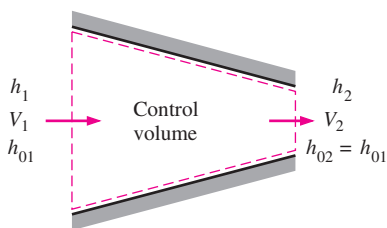


FIGURE 17-2

Steady flow of a fluid through an adiabatic duct.

17-1 ■ STAGNATION PROPERTIES

When analyzing control volumes, we find it very convenient to combine the *internal energy* and the *flow energy* of a fluid into a single term, *enthalpy*, defined per unit mass as $h = u + Pv$. Whenever the kinetic and potential energies of the fluid are negligible, as is often the case, the enthalpy represents the *total energy* of a fluid. For high-speed flows, such as those encountered in jet engines (Fig. 17-1), the potential energy of the fluid is still negligible, but the kinetic energy is not. In such cases, it is convenient to combine the enthalpy and the kinetic energy of the fluid into a single term called **stagnation** (or **total**) **enthalpy** h_0 , defined per unit mass as

$$h_0 = h + \frac{V^2}{2} \quad (\text{kJ/kg}) \quad (17-1)$$

When the potential energy of the fluid is negligible, the stagnation enthalpy represents the *total energy of a flowing fluid stream* per unit mass. Thus it simplifies the thermodynamic analysis of high-speed flows.

Throughout this chapter the ordinary enthalpy h is referred to as the **static enthalpy**, whenever necessary, to distinguish it from the stagnation enthalpy. Notice that the stagnation enthalpy is a combination property of a fluid, just like the static enthalpy, and these two enthalpies become identical when the kinetic energy of the fluid is negligible.

Consider the steady flow of a fluid through a duct such as a nozzle, diffuser, or some other flow passage where the flow takes place adiabatically and with no shaft or electrical work, as shown in Fig. 17-2. Assuming the fluid experiences little or no change in its elevation and its potential energy, the energy balance relation ($\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$) for this single-stream steady-flow system reduces to

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad (17-2)$$

or

$$h_{01} = h_{02} \quad (17-3)$$

That is, in the absence of any heat and work interactions and any changes in potential energy, the stagnation enthalpy of a fluid remains constant during a steady-flow process. Flows through nozzles and diffusers usually satisfy these conditions, and any increase in fluid velocity in these devices creates an equivalent decrease in the static enthalpy of the fluid.

If the fluid were brought to a complete stop, then the velocity at state 2 would be zero and Eq. 17-2 would become

$$h_1 + \frac{V_1^2}{2} = h_2 = h_{02}$$

Thus the *stagnation enthalpy* represents the *enthalpy of a fluid when it is brought to rest adiabatically*.

During a stagnation process, the kinetic energy of a fluid is converted to enthalpy (internal energy + flow energy), which results in an increase in the fluid temperature and pressure (Fig. 17-3). The properties of a fluid at the stagnation state are called **stagnation properties** (stagnation temperature,

stagnation pressure, stagnation density, etc.). The stagnation state and the stagnation properties are indicated by the subscript 0.

The stagnation state is called the **isentropic stagnation state** when the stagnation process is reversible as well as adiabatic (i.e., isentropic). The entropy of a fluid remains constant during an isentropic stagnation process. The actual (irreversible) and isentropic stagnation processes are shown on the h - s diagram in Fig. 17-4. Notice that the stagnation enthalpy of the fluid (and the stagnation temperature if the fluid is an ideal gas) is the same for both cases. However, the actual stagnation pressure is lower than the isentropic stagnation pressure since entropy increases during the actual stagnation process as a result of fluid friction. The stagnation processes are often approximated to be isentropic, and the isentropic stagnation properties are simply referred to as stagnation properties.

When the fluid is approximated as an *ideal gas* with constant specific heats, its enthalpy can be replaced by $c_p T$ and Eq. 17-1 can be expressed as

$$c_p T_0 = c_p T + \frac{V^2}{2}$$

or

$$T_0 = T + \frac{V^2}{2c_p} \quad (17-4)$$

Here T_0 is called the **stagnation** (or **total**) **temperature**, and it represents *the temperature an ideal gas attains when it is brought to rest adiabatically*. The term $V^2/2c_p$ corresponds to the temperature rise during such a process and is called the **dynamic temperature**. For example, the dynamic temperature of air flowing at 100 m/s is $(100 \text{ m/s})^2 / (2 \times 1.005 \text{ kJ/kg} \cdot \text{K}) = 5.0 \text{ K}$. Therefore, when air at 300 K and 100 m/s is brought to rest adiabatically (at the tip of a temperature probe, for example), its temperature rises to the stagnation value of 305 K (Fig. 17-5). Note that for low-speed flows, the stagnation and static (or ordinary) temperatures are practically the same. But for high-speed flows, the temperature measured by a stationary probe placed in the fluid (the stagnation temperature) may be significantly higher than the static temperature of the fluid.

The pressure a fluid attains when brought to rest isentropically is called the **stagnation pressure** P_0 . For ideal gases with constant specific heats, P_0 is related to the static pressure of the fluid by

$$\frac{P_0}{P} = \left(\frac{T_0}{T} \right)^{k/(k-1)} \quad (17-5)$$

By noting that $\rho = 1/\nu$ and using the isentropic relation $P\nu^k = P_0\nu_0^k$, the ratio of the stagnation density to static density can be expressed as

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T} \right)^{1/(k-1)} \quad (17-6)$$

When stagnation enthalpies are used, there is no need to refer explicitly to kinetic energy. Then the energy balance $\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$ for a single-stream, steady-flow device can be expressed as

$$q_{\text{in}} + w_{\text{in}} + (h_{01} + gz_1) = q_{\text{out}} + w_{\text{out}} + (h_{02} + gz_2) \quad (17-7)$$

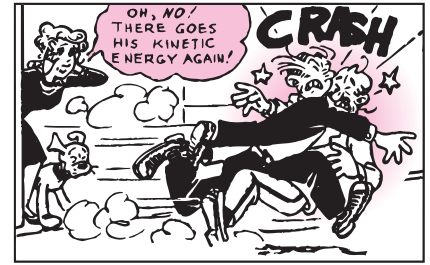


FIGURE 17-3

Kinetic energy is converted to enthalpy during a stagnation process.

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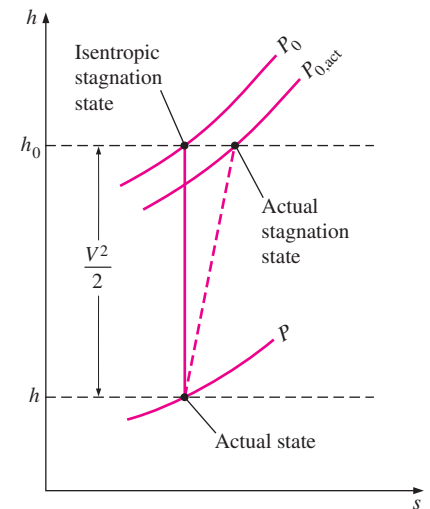


FIGURE 17-4

The actual state, actual stagnation state, and isentropic stagnation state of a fluid on an h - s diagram.

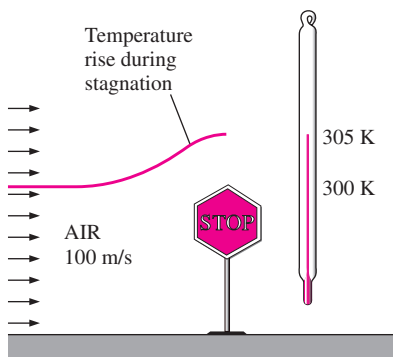


FIGURE 17-5

The temperature of an ideal gas flowing at a velocity V rises by $V^2/2c_p$ when it is brought to a complete stop.

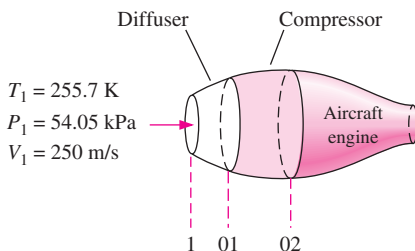


FIGURE 17-6

Schematic for Example 17-1.

where h_{01} and h_{02} are the stagnation enthalpies at states 1 and 2, respectively. When the fluid is an ideal gas with constant specific heats, Eq. 17-7 becomes

$$(q_{\text{in}} - q_{\text{out}}) + (w_{\text{in}} - w_{\text{out}}) = c_p(T_{02} - T_{01}) + g(z_2 - z_1) \quad (17-8)$$

where T_{01} and T_{02} are the stagnation temperatures.

Notice that kinetic energy terms do not explicitly appear in Eqs. 17-7 and 17-8, but the stagnation enthalpy terms account for their contribution.

EXAMPLE 17-1 Compression of High-Speed Air in an Aircraft

An aircraft is flying at a cruising speed of 250 m/s at an altitude of 5000 m where the atmospheric pressure is 54.05 kPa and the ambient air temperature is 255.7 K. The ambient air is first decelerated in a diffuser before it enters the compressor (Fig. 17-6). Assuming both the diffuser and the compressor to be isentropic, determine (a) the stagnation pressure at the compressor inlet and (b) the required compressor work per unit mass if the stagnation pressure ratio of the compressor is 8.

Solution High-speed air enters the diffuser and the compressor of an aircraft. The stagnation pressure of air and the compressor work input are to be determined.

Assumptions 1 Both the diffuser and the compressor are isentropic. 2 Air is an ideal gas with constant specific heats at room temperature.

Properties The constant-pressure specific heat c_p and the specific heat ratio k of air at room temperature are (Table A-2a)

$$c_p = 1.005 \text{ kJ/kg} \cdot \text{K} \quad \text{and} \quad k = 1.4$$

Analysis (a) Under isentropic conditions, the stagnation pressure at the compressor inlet (diffuser exit) can be determined from Eq. 17-5. However, first we need to find the stagnation temperature T_{01} at the compressor inlet. Under the stated assumptions, T_{01} can be determined from Eq. 17-4 to be

$$\begin{aligned} T_{01} &= T_1 + \frac{V_1^2}{2c_p} = 255.7 \text{ K} + \frac{(250 \text{ m/s})^2}{(2)(1.005 \text{ kJ/kg} \cdot \text{K})} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \\ &= 286.8 \text{ K} \end{aligned}$$

Then from Eq. 17-5,

$$\begin{aligned} P_{01} &= P_1 \left(\frac{T_{01}}{T_1} \right)^{k/(k-1)} = (54.05 \text{ kPa}) \left(\frac{286.8 \text{ K}}{255.7 \text{ K}} \right)^{1.4/(1.4-1)} \\ &= \mathbf{80.77 \text{ kPa}} \end{aligned}$$

That is, the temperature of air would increase by 31.1°C and the pressure by 26.72 kPa as air is decelerated from 250 m/s to zero velocity. These increases in the temperature and pressure of air are due to the conversion of the kinetic energy into enthalpy.

(b) To determine the compressor work, we need to know the stagnation temperature of air at the compressor exit T_{02} . The stagnation pressure ratio across the compressor P_{02}/P_{01} is specified to be 8. Since the compression process is assumed to be isentropic, T_{02} can be determined from the ideal-gas isentropic relation (Eq. 17-5):

$$T_{02} = T_{01} \left(\frac{P_{02}}{P_{01}} \right)^{(k-1)/k} = (286.8 \text{ K})(8)^{(1.4-1)/1.4} = 519.5 \text{ K}$$

Disregarding potential energy changes and heat transfer, the compressor work per unit mass of air is determined from Eq. 17–8:

$$\begin{aligned} w_{\text{in}} &= c_p(T_{02} - T_{01}) \\ &= (1.005 \text{ kJ/kg} \cdot \text{K})(519.5 \text{ K} - 286.8 \text{ K}) \\ &= \mathbf{233.9 \text{ kJ/kg}} \end{aligned}$$

Thus the work supplied to the compressor is 233.9 kJ/kg.

Discussion Notice that using stagnation properties automatically accounts for any changes in the kinetic energy of a fluid stream.

17-2 ■ SPEED OF SOUND AND MACH NUMBER

An important parameter in the study of compressible flow is the **speed of sound** (or the **sonic speed**), which is the speed at which an infinitesimally small pressure wave travels through a medium. The pressure wave may be caused by a small disturbance, which creates a slight rise in local pressure.

To obtain a relation for the speed of sound in a medium, consider a pipe that is filled with a fluid at rest, as shown in Fig. 17–7. A piston fitted in the pipe is now moved to the right with a constant incremental velocity dV , creating a sonic wave. The wave front moves to the right through the fluid at the speed of sound c and separates the moving fluid adjacent to the piston from the fluid still at rest. The fluid to the left of the wave front experiences an incremental change in its thermodynamic properties, while the fluid on the right of the wave front maintains its original thermodynamic properties, as shown in Fig. 17–7.

To simplify the analysis, consider a control volume that encloses the wave front and moves with it, as shown in Fig. 17–8. To an observer traveling with the wave front, the fluid to the right will appear to be moving toward the wave front with a speed of c and the fluid to the left to be moving away from the wave front with a speed of $c - dV$. Of course, the observer will think the control volume that encloses the wave front (and herself or himself) is stationary, and the observer will be witnessing a steady-flow process. The mass balance for this single-stream, steady-flow process can be expressed as

$$\dot{m}_{\text{right}} = \dot{m}_{\text{left}}$$

or

$$\rho A c = (\rho + d\rho)A(c - dV)$$

By canceling the cross-sectional (or flow) area A and neglecting the higher-order terms, this equation reduces to

$$c \, d\rho - \rho \, dV = 0$$

No heat or work crosses the boundaries of the control volume during this steady-flow process, and the potential energy change, if any, can be neglected. Then the steady-flow energy balance $e_{\text{in}} = e_{\text{out}}$ becomes

$$h + \frac{c^2}{2} = h + dh + \frac{(c - dV)^2}{2}$$

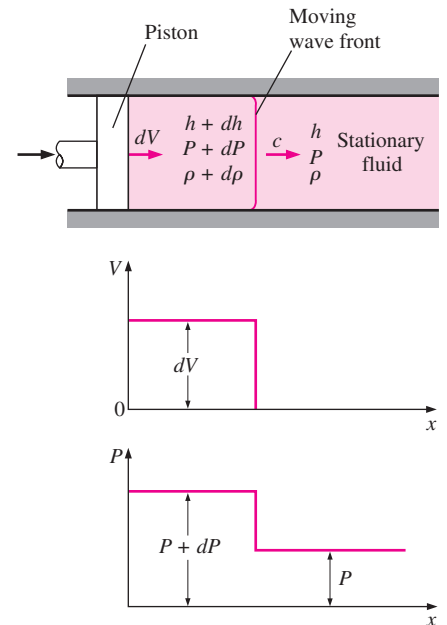


FIGURE 17-7

Propagation of a small pressure wave along a duct.

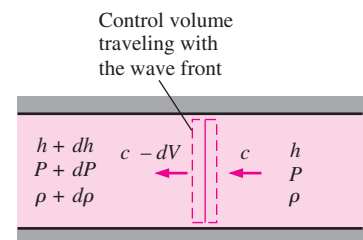


FIGURE 17-8

Control volume moving with the small pressure wave along a duct.

which yields

$$dh - c dV = 0 \quad (\text{b})$$

where we have neglected the second-order term dV^2 . The amplitude of the ordinary sonic wave is very small and does not cause any appreciable change in the pressure and temperature of the fluid. Therefore, the propagation of a sonic wave is not only adiabatic but also very nearly isentropic. Then the second $T ds$ relation developed in Chapter 7 reduces to

$$T ds \xrightarrow{0} = dh - \frac{dP}{\rho}$$

or

$$dh = \frac{dP}{\rho} \quad (\text{c})$$

Combining Eqs. a, b, and c yields the desired expression for the speed of sound as

$$c^2 = \frac{dP}{d\rho} \quad \text{at } s = \text{constant}$$

or

$$c^2 = \left(\frac{\partial P}{\partial \rho} \right)_s \quad (\text{17-9})$$

It is left as an exercise for the reader to show, by using thermodynamic property relations (see Chap. 12) that Eq. 17-9 can also be written as

$$c^2 = k \left(\frac{\partial P}{\partial \rho} \right)_T \quad (\text{17-10})$$

where k is the specific heat ratio of the fluid. Note that the speed of sound in a fluid is a function of the thermodynamic properties of that fluid.

When the fluid is an ideal gas ($P = \rho RT$), the differentiation in Eq. 17-10 can easily be performed to yield

$$c^2 = k \left(\frac{\partial P}{\partial \rho} \right)_T = k \left[\frac{\partial (\rho RT)}{\partial \rho} \right]_T = kRT$$

or

$$c = \sqrt{kRT} \quad (\text{17-11})$$

Noting that the gas constant R has a fixed value for a specified ideal gas and the specific heat ratio k of an ideal gas is, at most, a function of temperature, we see that the speed of sound in a specified ideal gas is a function of temperature alone (Fig. 17-9).

A second important parameter in the analysis of compressible fluid flow is the **Mach number** Ma , named after the Austrian physicist Ernst Mach (1838–1916). It is the ratio of the actual velocity of the fluid (or an object in still air) to the speed of sound in the same fluid at the same state:

$$Ma = \frac{V}{c} \quad (\text{17-12})$$

Note that the Mach number depends on the speed of sound, which depends on the state of the fluid. Therefore, the Mach number of an aircraft cruising

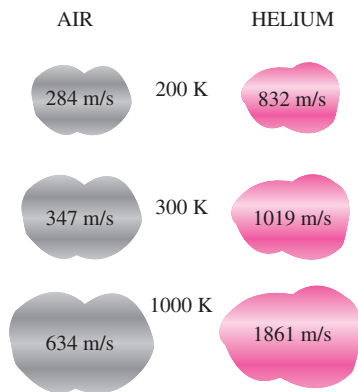


FIGURE 17-9

The speed of sound changes with temperature and varies with the fluid.

at constant velocity in still air may be different at different locations (Fig. 17–10).

Fluid flow regimes are often described in terms of the flow Mach number. The flow is called **sonic** when $Ma = 1$, **subsonic** when $Ma < 1$, **supersonic** when $Ma > 1$, **hypersonic** when $Ma \gg 1$, and **transonic** when $Ma \cong 1$.

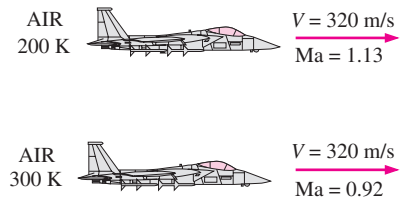


FIGURE 17–10

The Mach number can be different at different temperatures even if the velocity is the same.

EXAMPLE 17–2 Mach Number of Air Entering a Diffuser

Air enters a diffuser shown in Fig. 17–11 with a velocity of 200 m/s. Determine (a) the speed of sound and (b) the Mach number at the diffuser inlet when the air temperature is 30°C.

Solution Air enters a diffuser with a high velocity. The speed of sound and the Mach number are to be determined at the diffuser inlet.

Assumptions Air at specified conditions behaves as an ideal gas.

Properties The gas constant of air is $R = 0.287 \text{ kJ/kg} \cdot \text{K}$, and its specific heat ratio at 30°C is 1.4 (Table A–2a).

Analysis We note that the speed of sound in a gas varies with temperature, which is given to be 30°C.

(a) The speed of sound in air at 30°C is determined from Eq. 17–11 to be

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(303 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 349 \text{ m/s}$$

(b) Then the Mach number becomes

$$Ma = \frac{V}{c} = \frac{200 \text{ m/s}}{349 \text{ m/s}} = 0.573$$

Discussion The flow at the diffuser inlet is subsonic since $Ma < 1$.

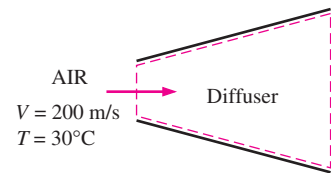


FIGURE 17–11

Schematic for Example 17–2.

17–3 ■ ONE-DIMENSIONAL ISENTROPIC FLOW

During fluid flow through many devices such as nozzles, diffusers, and turbine blade passages, flow quantities vary primarily in the flow direction only, and the flow can be approximated as one-dimensional isentropic flow with good accuracy. Therefore, it merits special consideration. Before presenting a formal discussion of one-dimensional isentropic flow, we illustrate some important aspects of it with an example.

EXAMPLE 17–3 Gas Flow through a Converging–Diverging Duct

Carbon dioxide flows steadily through a varying cross-sectional-area duct such as a nozzle shown in Fig. 17–12 at a mass flow rate of 3 kg/s. The carbon dioxide enters the duct at a pressure of 1400 kPa and 200°C with a low velocity, and it expands in the nozzle to a pressure of 200 kPa. The duct is designed so that the flow can be approximated as isentropic. Determine the density, velocity, flow area, and Mach number at each location along the duct that corresponds to a pressure drop of 200 kPa.

Solution Carbon dioxide enters a varying cross-sectional-area duct at specified conditions. The flow properties are to be determined along the duct.

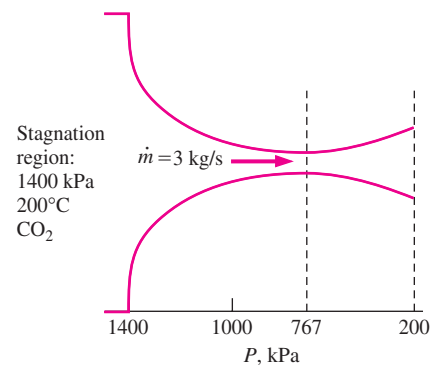


FIGURE 17–12

Schematic for Example 17–3.

Assumptions 1 Carbon dioxide is an ideal gas with constant specific heats at room temperature. 2 Flow through the duct is steady, one-dimensional, and isentropic.

Properties For simplicity we use $c_p = 0.846 \text{ kJ/kg} \cdot \text{K}$ and $k = 1.289$ throughout the calculations, which are the constant-pressure specific heat and specific heat ratio values of carbon dioxide at room temperature. The gas constant of carbon dioxide is $R = 0.1889 \text{ kJ/kg} \cdot \text{K}$ (Table A-2a).

Analysis We note that the inlet temperature is nearly equal to the stagnation temperature since the inlet velocity is small. The flow is isentropic, and thus the stagnation temperature and pressure throughout the duct remain constant. Therefore,

$$T_0 \cong T_1 = 200^\circ\text{C} = 473 \text{ K}$$

and

$$P_0 \cong P_1 = 1400 \text{ kPa}$$

To illustrate the solution procedure, we calculate the desired properties at the location where the pressure is 1200 kPa, the first location that corresponds to a pressure drop of 200 kPa.

From Eq. 17-5,

$$T = T_0 \left(\frac{P}{P_0} \right)^{(k-1)/k} = (473 \text{ K}) \left(\frac{1200 \text{ kPa}}{1400 \text{ kPa}} \right)^{(1.289-1)/1.289} = 457 \text{ K}$$

From Eq. 17-4,

$$\begin{aligned} V &= \sqrt{2c_p(T_0 - T)} \\ &= \sqrt{2(0.846 \text{ kJ/kg} \cdot \text{K})(473 \text{ K} - 457 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} \\ &= \mathbf{164.5 \text{ m/s}} \end{aligned}$$

From the ideal-gas relation,

$$\rho = \frac{P}{RT} = \frac{1200 \text{ kPa}}{(0.1889 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(457 \text{ K})} = \mathbf{13.9 \text{ kg/m}^3}$$

From the mass flow rate relation,

$$A = \frac{\dot{m}}{\rho V} = \frac{3 \text{ kg/s}}{(13.9 \text{ kg/m}^3)(164.5 \text{ m/s})} = 13.1 \times 10^{-4} \text{ m}^2 = \mathbf{13.1 \text{ cm}^2}$$

From Eqs. 17-11 and 17-12,

$$c = \sqrt{kRT} = \sqrt{(1.289)(0.1889 \text{ kJ/kg} \cdot \text{K})(457 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 333.6 \text{ m/s}$$

$$\text{Ma} = \frac{V}{c} = \frac{164.5 \text{ m/s}}{333.6 \text{ m/s}} = \mathbf{0.493}$$

The results for the other pressure steps are summarized in Table 17-1 and are plotted in Fig. 17-13.

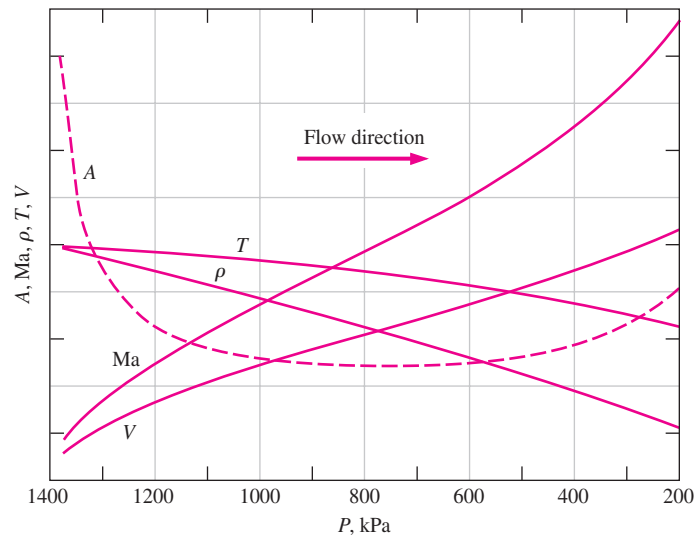
Discussion Note that as the pressure decreases, the temperature and speed of sound decrease while the fluid velocity and Mach number increase in the flow direction. The density decreases slowly at first and rapidly later as the fluid velocity increases.

TABLE 17–1

Variation of fluid properties in flow direction in duct described in Example 17–3 for $\dot{m} = 3 \text{ kg/s} = \text{constant}$

P , kPa	T , K	V , m/s	ρ , kg/m ³	c , m/s	A , cm ²	Ma
1400	473	0	15.7	339.4	∞	0
1200	457	164.5	13.9	333.6	13.1	0.493
1000	439	240.7	12.1	326.9	10.3	0.736
800	417	306.6	10.1	318.8	9.64	0.962
767*	413	317.2	9.82	317.2	9.63	1.000
600	391	371.4	8.12	308.7	10.0	1.203
400	357	441.9	5.93	295.0	11.5	1.498
200	306	530.9	3.46	272.9	16.3	1.946

* 767 kPa is the critical pressure where the local Mach number is unity.

**FIGURE 17–13**

Variation of normalized fluid properties and cross-sectional area along a duct as the pressure drops from 1400 to 200 kPa.

We note from Example 17–3 that the flow area decreases with decreasing pressure up to a critical-pressure value where the Mach number is unity, and then it begins to increase with further reductions in pressure. The Mach number is unity at the location of smallest flow area, called the **throat** (Fig. 17–14). Note that the velocity of the fluid keeps increasing after passing the throat although the flow area increases rapidly in that region. This increase in velocity past the throat is due to the rapid decrease in the fluid density. The flow area of the duct considered in this example first decreases and then increases. Such ducts are called **converging–diverging nozzles**. These nozzles are used to accelerate gases to supersonic speeds and should not be confused with *Venturi nozzles*, which are used strictly for incompressible flow. The first use of such a nozzle occurred in 1893 in a steam turbine

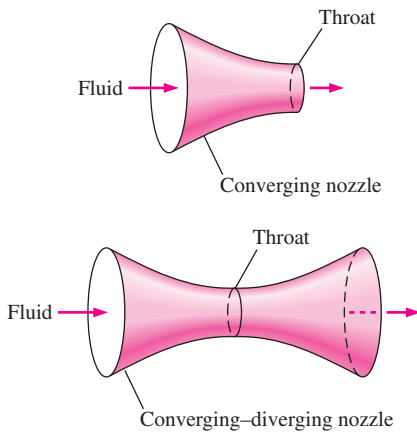


FIGURE 17-14

The cross section of a nozzle at the smallest flow area is called the *throat*.

designed by a Swedish engineer, Carl G. B. de Laval (1845–1913), and therefore converging–diverging nozzles are often called *Laval nozzles*.

Variation of Fluid Velocity with Flow Area

It is clear from Example 17–3 that the couplings among the velocity, density, and flow areas for isentropic duct flow are rather complex. In the remainder of this section we investigate these couplings more thoroughly, and we develop relations for the variation of static-to-stagnation property ratios with the Mach number for pressure, temperature, and density.

We begin our investigation by seeking relationships among the pressure, temperature, density, velocity, flow area, and Mach number for one-dimensional isentropic flow. Consider the mass balance for a steady-flow process:

$$\dot{m} = \rho AV = \text{constant}$$

Differentiating and dividing the resultant equation by the mass flow rate, we obtain

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \quad (17-13)$$

Neglecting the potential energy, the energy balance for an isentropic flow with no work interactions can be expressed in the differential form as (Fig. 17–15)

$$\frac{dP}{\rho} + V dV = 0 \quad (17-14)$$

This relation is also the differential form of Bernoulli’s equation when changes in potential energy are negligible, which is a form of the conservation of momentum principle for steady-flow control volumes. Combining Eqs. 17–13 and 17–14 gives

$$\frac{dA}{A} = \frac{dP}{\rho} \left(\frac{1}{V^2} - \frac{d\rho}{dP} \right) \quad (17-15)$$

Rearranging Eq. 17–9 as $(\partial\rho/\partial P)_s = 1/c^2$ and substituting into Eq. 17–15 yield

$$\frac{dA}{A} = \frac{dP}{\rho V^2} (1 - \text{Ma}^2) \quad (17-16)$$

This is an important relation for isentropic flow in ducts since it describes the variation of pressure with flow area. We note that A , ρ , and V are positive quantities. For *subsonic* flow ($\text{Ma} < 1$), the term $1 - \text{Ma}^2$ is positive; and thus dA and dP must have the same sign. That is, the pressure of the fluid must increase as the flow area of the duct increases and must decrease as the flow area of the duct decreases. Thus, at subsonic velocities, the pressure decreases in converging ducts (subsonic nozzles) and increases in diverging ducts (subsonic diffusers).

In *supersonic* flow ($\text{Ma} > 1$), the term $1 - \text{Ma}^2$ is negative, and thus dA and dP must have opposite signs. That is, the pressure of the fluid must

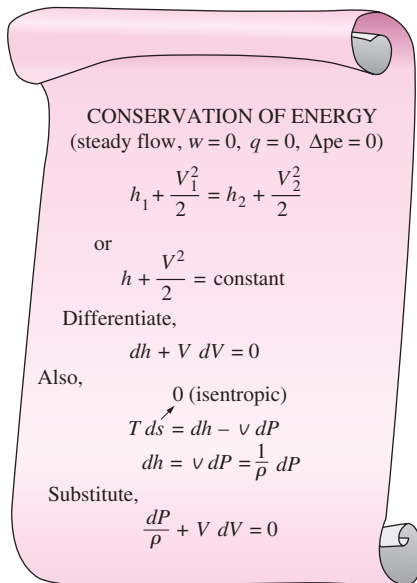


FIGURE 17-15

Derivation of the differential form of the energy equation for steady isentropic flow.

increase as the flow area of the duct decreases and must decrease as the flow area of the duct increases. Thus, at supersonic velocities, the pressure decreases in diverging ducts (supersonic nozzles) and increases in converging ducts (supersonic diffusers).

Another important relation for the isentropic flow of a fluid is obtained by substituting $\rho V = -dP/dV$ from Eq. 17-14 into Eq. 17-16:

$$\frac{dA}{A} = -\frac{dV}{V}(1 - \text{Ma}^2) \quad (17-17)$$

This equation governs the shape of a nozzle or a diffuser in subsonic or supersonic isentropic flow. Noting that A and V are positive quantities, we conclude the following:

$$\begin{aligned} \text{For subsonic flow (Ma} < 1), \quad & \frac{dA}{dV} < 0 \\ \text{For supersonic flow (Ma} > 1), \quad & \frac{dA}{dV} > 0 \\ \text{For sonic flow (Ma} = 1), \quad & \frac{dA}{dV} = 0 \end{aligned}$$

Thus the proper shape of a nozzle depends on the highest velocity desired relative to the sonic velocity. To accelerate a fluid, we must use a converging nozzle at subsonic velocities and a diverging nozzle at supersonic velocities. The velocities encountered in most familiar applications are well below the sonic velocity, and thus it is natural that we visualize a nozzle as a converging duct. However, the highest velocity we can achieve by a converging nozzle is the sonic velocity, which occurs at the exit of the nozzle. If we extend the converging nozzle by further decreasing the flow area, in hopes of accelerating the fluid to supersonic velocities, as shown in Fig. 17-16, we are up for disappointment. Now the sonic velocity will occur at the exit of the converging extension, instead of the exit of the original nozzle, and the mass flow rate through the nozzle will decrease because of the reduced exit area.

Based on Eq. 17-16, which is an expression of the conservation of mass and energy principles, we must add a diverging section to a converging nozzle to accelerate a fluid to supersonic velocities. The result is a converging–diverging nozzle. The fluid first passes through a subsonic (converging) section, where the Mach number increases as the flow area of the nozzle decreases, and then reaches the value of unity at the nozzle throat. The fluid continues to accelerate as it passes through a supersonic (diverging) section. Noting that $\dot{m} = \rho AV$ for steady flow, we see that the large decrease in density makes acceleration in the diverging section possible. An example of this type of flow is the flow of hot combustion gases through a nozzle in a gas turbine.

The opposite process occurs in the engine inlet of a supersonic aircraft. The fluid is decelerated by passing it first through a supersonic diffuser, which has a flow area that decreases in the flow direction. Ideally, the flow reaches a Mach number of unity at the diffuser throat. The fluid is further

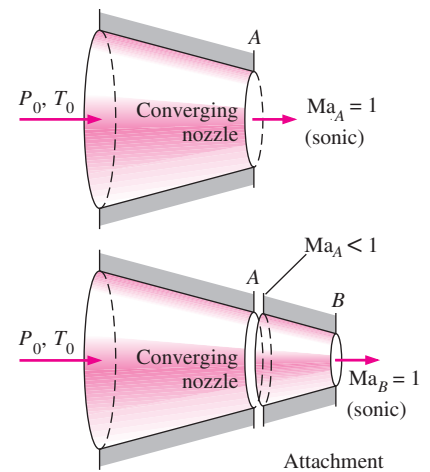
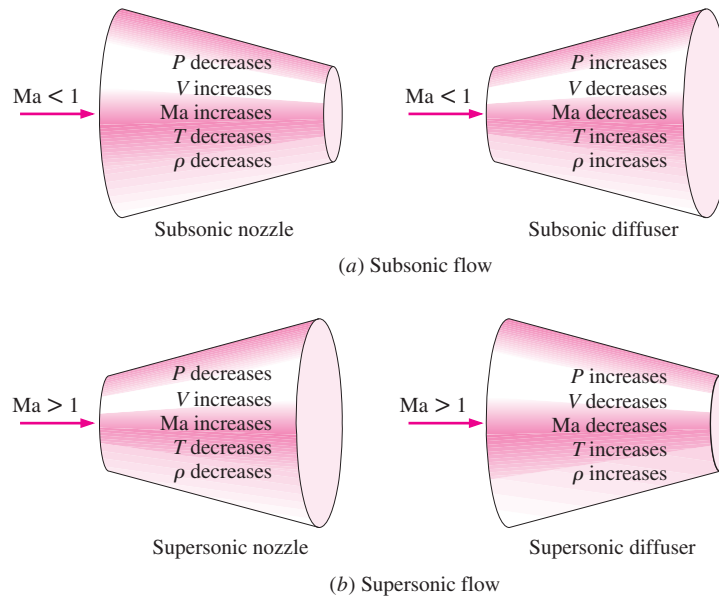


FIGURE 17-16

We cannot obtain supersonic velocities by attaching a converging section to a converging nozzle. Doing so will only move the sonic cross section farther downstream and decrease the mass flow rate.

**FIGURE 17-17**

Variation of flow properties in subsonic and supersonic nozzles and diffusers.

decelerated in a subsonic diffuser, which has a flow area that increases in the flow direction, as shown in Fig. 17-17.

Property Relations for Isentropic Flow of Ideal Gases

Next we develop relations between the static properties and stagnation properties of an ideal gas in terms of the specific heat ratio k and the Mach number Ma . We assume the flow is isentropic and the gas has constant specific heats.

The temperature T of an ideal gas anywhere in the flow is related to the stagnation temperature T_0 through Eq. 17-4:

$$T_0 = T + \frac{V^2}{2c_p}$$

or

$$\frac{T_0}{T} = 1 + \frac{V^2}{2c_p T}$$

Noting that $c_p = kR/(k-1)$, $c^2 = kRT$, and $Ma = V/c$, we see that

$$\frac{V^2}{2c_p T} = \frac{V^2}{2[kR/(k-1)]T} = \left(\frac{k-1}{2}\right) \frac{V^2}{c^2} = \left(\frac{k-1}{2}\right) Ma^2$$

Substituting yields

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right) Ma^2 \quad (17-18)$$

which is the desired relation between T_0 and T .

The ratio of the stagnation to static pressure is obtained by substituting Eq. 17-18 into Eq. 17-5:

$$\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2} \right) \text{Ma}^2 \right]^{k/(k-1)} \quad (17-19)$$

The ratio of the stagnation to static density is obtained by substituting Eq. 17-18 into Eq. 17-6:

$$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{k-1}{2} \right) \text{Ma}^2 \right]^{1/(k-1)} \quad (17-20)$$

Numerical values of T/T_0 , P/P_0 , and ρ/ρ_0 are listed versus the Mach number in Table A-32 for $k = 1.4$, which are very useful for practical compressible flow calculations involving air.

The properties of a fluid at a location where the Mach number is unity (the throat) are called **critical properties**, and the ratios in Eqs. (17-18) through (17-20) are called **critical ratios** (Fig. 17-18). It is common practice in the analysis of compressible flow to let the superscript asterisk (*) represent the critical values. Setting $\text{Ma} = 1$ in Eqs. 17-18 through 17-20 yields

$$\frac{T^*}{T_0} = \frac{2}{k+1} \quad (17-21)$$

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1} \right)^{k/(k-1)} \quad (17-22)$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1} \right)^{1/(k-1)} \quad (17-23)$$

These ratios are evaluated for various values of k and are listed in Table 17-2. The critical properties of compressible flow should not be confused with the properties of substances at the critical point (such as the critical temperature T_c and critical pressure P_c).

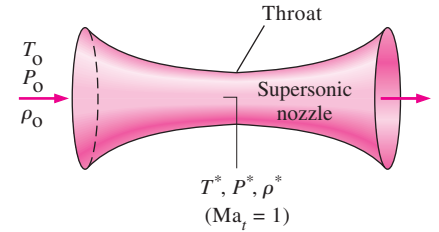
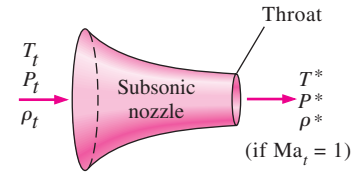


FIGURE 17-18

When $\text{Ma}_t = 1$, the properties at the nozzle throat become the critical properties.

TABLE 17-2

The critical-pressure, critical-temperature, and critical-density ratios for isentropic flow of some ideal gases

	Superheated steam, $k = 1.3$	Hot products of combustion, $k = 1.33$	Air, $k = 1.4$	Monatomic gases, $k = 1.667$
$\frac{P^*}{P_0}$	0.5457	0.5404	0.5283	0.4871
$\frac{T^*}{T_0}$	0.8696	0.8584	0.8333	0.7499
$\frac{\rho^*}{\rho_0}$	0.6276	0.6295	0.6340	0.6495

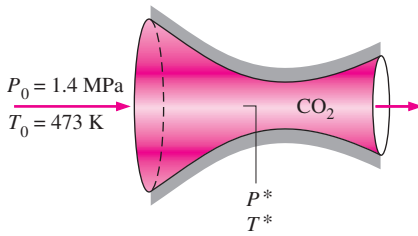


FIGURE 17-19
Schematic for Example 17-4.

EXAMPLE 17-4 Critical Temperature and Pressure in Gas Flow

Calculate the critical pressure and temperature of carbon dioxide for the flow conditions described in Example 17-3 (Fig. 17-19).

Solution For the flow discussed in Example 17-3, the critical pressure and temperature are to be calculated.

Assumptions **1** The flow is steady, adiabatic, and one-dimensional. **2** Carbon dioxide is an ideal gas with constant specific heats.

Properties The specific heat ratio of carbon dioxide at room temperature is $k = 1.289$ (Table A-2a).

Analysis The ratios of critical to stagnation temperature and pressure are determined to be

$$\frac{T^*}{T_0} = \frac{2}{k + 1} = \frac{2}{1.289 + 1} = 0.8737$$

$$\frac{P^*}{P_0} = \left(\frac{2}{k + 1} \right)^{k/(k-1)} = \left(\frac{2}{1.289 + 1} \right)^{1.289/(1.289-1)} = 0.5477$$

Noting that the stagnation temperature and pressure are, from Example 17-3, $T_0 = 473$ K and $P_0 = 1400$ kPa, we see that the critical temperature and pressure in this case are

$$T^* = 0.8737T_0 = (0.8737)(473 \text{ K}) = \mathbf{413 \text{ K}}$$

$$P^* = 0.5477P_0 = (0.5477)(1400 \text{ kPa}) = \mathbf{767 \text{ kPa}}$$

Discussion Note that these values agree with those listed in Table 17-1, as expected. Also, property values other than these at the throat would indicate that the flow is not critical, and the Mach number is not unity.

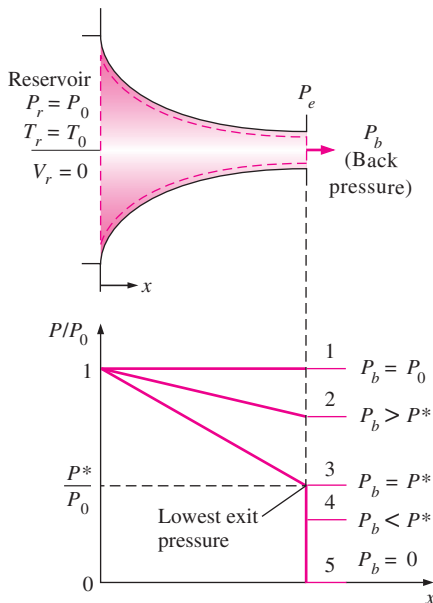


FIGURE 17-20
The effect of back pressure on the pressure distribution along a converging nozzle.

17-4 ■ ISENTROPIC FLOW THROUGH NOZZLES

Converging or converging–diverging nozzles are found in many engineering applications including steam and gas turbines, aircraft and spacecraft propulsion systems, and even industrial blasting nozzles and torch nozzles. In this section we consider the effects of **back pressure** (i.e., the pressure applied at the nozzle discharge region) on the exit velocity, the mass flow rate, and the pressure distribution along the nozzle.

Converging Nozzles

Consider the subsonic flow through a converging nozzle as shown in Fig. 17-20. The nozzle inlet is attached to a reservoir at pressure P_r and temperature T_r . The reservoir is sufficiently large so that the nozzle inlet velocity is negligible. Since the fluid velocity in the reservoir is zero and the flow through the nozzle is approximated as isentropic, the stagnation pressure and stagnation temperature of the fluid at any cross section through the nozzle are equal to the reservoir pressure and temperature, respectively.

Now we begin to reduce the back pressure and observe the resulting effects on the pressure distribution along the length of the nozzle, as shown in Fig. 17–20. If the back pressure P_b is equal to P_1 , which is equal to P_r , there is no flow and the pressure distribution is uniform along the nozzle. When the back pressure is reduced to P_2 , the exit plane pressure P_e also drops to P_2 . This causes the pressure along the nozzle to decrease in the flow direction.

When the back pressure is reduced to P_3 ($= P^*$, which is the pressure required to increase the fluid velocity to the speed of sound at the exit plane or throat), the mass flow reaches a maximum value and the flow is said to be **choked**. Further reduction of the back pressure to level P_4 or below does not result in additional changes in the pressure distribution, or anything else along the nozzle length.

Under steady-flow conditions, the mass flow rate through the nozzle is constant and can be expressed as

$$\dot{m} = \rho AV = \left(\frac{P}{RT} \right) A (\text{Ma} \sqrt{kRT}) = P A \text{Ma} \sqrt{\frac{k}{RT}}$$

Solving for T from Eq. 17–18 and for P from Eq. 17–19 and substituting,

$$\dot{m} = \frac{A \text{Ma} P_0 \sqrt{k/(RT_0)}}{[1 + (k-1)\text{Ma}^2/2]^{(k+1)/[2(k-1)]}} \quad (17-24)$$

Thus the mass flow rate of a particular fluid through a nozzle is a function of the stagnation properties of the fluid, the flow area, and the Mach number. Equation 17–24 is valid at any cross section, and thus \dot{m} can be evaluated at any location along the length of the nozzle.

For a specified flow area A and stagnation properties T_0 and P_0 , the maximum mass flow rate can be determined by differentiating Eq. 17–24 with respect to Ma and setting the result equal to zero. It yields $\text{Ma} = 1$. Since the only location in a nozzle where the Mach number can be unity is the location of minimum flow area (the throat), the mass flow rate through a nozzle is a maximum when $\text{Ma} = 1$ at the throat. Denoting this area by A^* , we obtain an expression for the maximum mass flow rate by substituting $\text{Ma} = 1$ in Eq. 17–24:

$$\dot{m}_{\max} = A^* P_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1} \right)^{(k+1)/[2(k-1)]} \quad (17-25)$$

Thus, for a particular ideal gas, the maximum mass flow rate through a nozzle with a given throat area is fixed by the stagnation pressure and temperature of the inlet flow. The flow rate can be controlled by changing the stagnation pressure or temperature, and thus a converging nozzle can be used as a flowmeter. The flow rate can also be controlled, of course, by varying the throat area. This principle is vitally important for chemical processes, medical devices, flowmeters, and anywhere the mass flux of a gas must be known and controlled.

A plot of \dot{m} versus P_b/P_0 for a converging nozzle is shown in Fig. 17–21. Notice that the mass flow rate increases with decreasing P_b/P_0 , reaches a maximum at $P_b = P^*$, and remains constant for P_b/P_0 values less than this

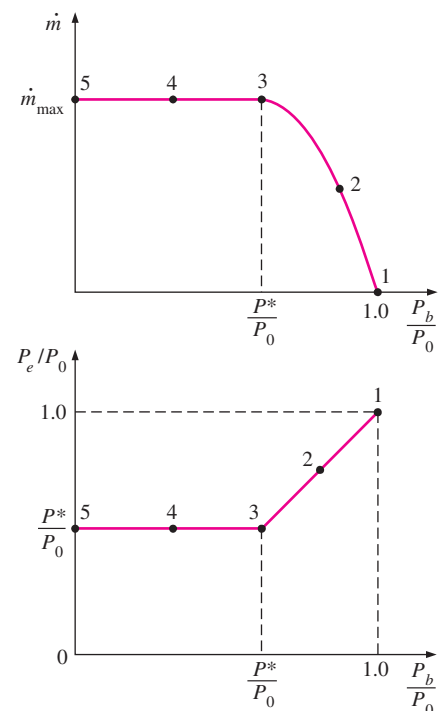


FIGURE 17–21

The effect of back pressure P_b on the mass flow rate \dot{m} and the exit pressure P_e of a converging nozzle.

critical ratio. Also illustrated on this figure is the effect of back pressure on the nozzle exit pressure P_e . We observe that

$$P_e = \begin{cases} P_b & \text{for } P_b \geq P^* \\ P^* & \text{for } P_b < P^* \end{cases}$$

To summarize, for all back pressures lower than the critical pressure P^* , the pressure at the exit plane of the converging nozzle P_e is equal to P^* , the Mach number at the exit plane is unity, and the mass flow rate is the maximum (or choked) flow rate. Because the velocity of the flow is sonic at the throat for the maximum flow rate, a back pressure lower than the critical pressure cannot be sensed in the nozzle upstream flow and does not affect the flow rate.

The effects of the stagnation temperature T_0 and stagnation pressure P_0 on the mass flow rate through a converging nozzle are illustrated in Fig. 17–22 where the mass flow rate is plotted against the static-to-stagnation pressure ratio at the throat P_t/P_0 . An increase in P_0 (or a decrease in T_0) will increase the mass flow rate through the converging nozzle; a decrease in P_0 (or an increase in T_0) will decrease it. We could also conclude this by carefully observing Eqs. 17–24 and 17–25.

A relation for the variation of flow area A through the nozzle relative to throat area A^* can be obtained by combining Eqs. 17–24 and 17–25 for the same mass flow rate and stagnation properties of a particular fluid. This yields

$$\frac{A}{A^*} = \frac{1}{\text{Ma}} \left[\left(\frac{2}{k+1} \right) \left(1 + \frac{k-1}{2} \text{Ma}^2 \right) \right]^{(k+1)/[2(k-1)]} \quad (17-26)$$

Table A–32 gives values of A/A^* as a function of the Mach number for air ($k = 1.4$). There is one value of A/A^* for each value of the Mach number, but there are two possible values of the Mach number for each value of A/A^* —one for subsonic flow and another for supersonic flow.

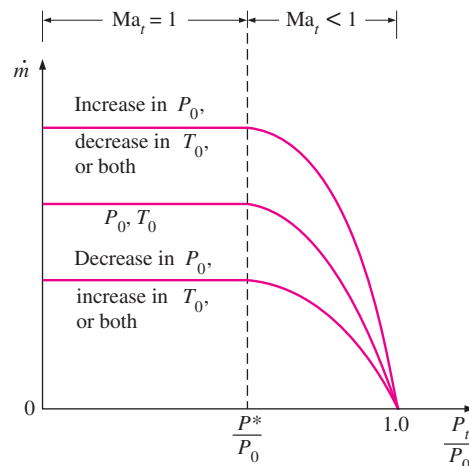


FIGURE 17–22

The variation of the mass flow rate through a nozzle with inlet stagnation properties.

Another parameter sometimes used in the analysis of one-dimensional isentropic flow of ideal gases is Ma^* , which is the ratio of the local velocity to the speed of sound at the throat:

$$Ma^* = \frac{V}{c^*} \quad (17-27)$$

It can also be expressed as

$$Ma^* = \frac{V}{c} \frac{c}{c^*} = \frac{Ma c}{c^*} = \frac{Ma \sqrt{kRT}}{\sqrt{kRT^*}} = Ma \sqrt{\frac{T}{T^*}}$$

where Ma is the local Mach number, T is the local temperature, and T^* is the critical temperature. Solving for T from Eq. 17-18 and for T^* from Eq. 17-21 and substituting, we get

$$Ma^* = Ma \sqrt{\frac{k+1}{2+(k-1)Ma^2}} \quad (17-28)$$

Values of Ma^* are also listed in Table A-32 versus the Mach number for $k = 1.4$ (Fig. 17-23). Note that the parameter Ma^* differs from the Mach number Ma in that Ma^* is the local velocity nondimensionalized with respect to the sonic velocity at the *throat*, whereas Ma is the local velocity nondimensionalized with respect to the *local* sonic velocity. (Recall that the sonic velocity in a nozzle varies with temperature and thus with location.)

Ma	Ma*	$\frac{A}{A^*}$	$\frac{P}{P_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$
⋮	⋮	⋮	⋮	⋮	⋮
0.90	0.9146	1.0089	0.5913	⋮	⋮
1.00	1.0000	1.0000	0.5283	⋮	⋮
1.10	1.0812	1.0079	0.4684	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮

FIGURE 17-23

Various property ratios for isentropic flow through nozzles and diffusers are listed in Table A-32 for $k = 1.4$ for convenience.

EXAMPLE 17-5 Effect of Back Pressure on Mass Flow Rate

Air at 1 MPa and 600°C enters a converging nozzle, shown in Fig. 17-24, with a velocity of 150 m/s. Determine the mass flow rate through the nozzle for a nozzle throat area of 50 cm² when the back pressure is (a) 0.7 MPa and (b) 0.4 MPa.

Solution Air enters a converging nozzle. The mass flow rate of air through the nozzle is to be determined for different back pressures.

Assumptions 1 Air is an ideal gas with constant specific heats at room temperature. 2 Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The constant-pressure specific heat and the specific heat ratio of air are $c_p = 1.005$ kJ/kg · K and $k = 1.4$, respectively (Table A-2a).

Analysis We use the subscripts i and t to represent the properties at the nozzle inlet and the throat, respectively. The stagnation temperature and pressure at the nozzle inlet are determined from Eqs. 17-4 and 17-5:

$$T_{0i} = T_i + \frac{V_i^2}{2c_p} = 873 \text{ K} + \frac{(150 \text{ m/s})^2}{2(1.005 \text{ kJ/kg} \cdot \text{K})} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 884 \text{ K}$$

$$P_{0i} = P_i \left(\frac{T_{0i}}{T_i} \right)^{k/(k-1)} = (1 \text{ MPa}) \left(\frac{884 \text{ K}}{873 \text{ K}} \right)^{1.4/(1.4-1)} = 1.045 \text{ MPa}$$

These stagnation temperature and pressure values remain constant throughout the nozzle since the flow is assumed to be isentropic. That is,

$$T_0 = T_{0i} = 884 \text{ K} \quad \text{and} \quad P_0 = P_{0i} = 1.045 \text{ MPa}$$

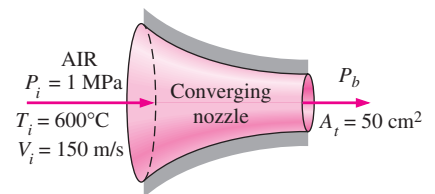


FIGURE 17-24

Schematic for Example 17-5.

The critical-pressure ratio is determined from Table 17–2 (or Eq. 17–22) to be $P^*/P_0 = 0.5283$.

(a) The back pressure ratio for this case is

$$\frac{P_b}{P_0} = \frac{0.7 \text{ MPa}}{1.045 \text{ MPa}} = 0.670$$

which is greater than the critical-pressure ratio, 0.5283. Thus the exit plane pressure (or throat pressure P_t) is equal to the back pressure in this case. That is, $P_t = P_b = 0.7 \text{ MPa}$, and $P_t/P_0 = 0.670$. Therefore, the flow is not choked. From Table A–32 at $P_t/P_0 = 0.670$, we read $\text{Ma}_t = 0.778$ and $T_t/T_0 = 0.892$.

The mass flow rate through the nozzle can be calculated from Eq. 17–24. But it can also be determined in a step-by-step manner as follows:

$$T_t = 0.892T_0 = 0.892(884 \text{ K}) = 788.5 \text{ K}$$

$$\rho_t = \frac{P_t}{RT_t} = \frac{700 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(788.5 \text{ K})} = 3.093 \text{ kg/m}^3$$

$$\begin{aligned} V_t &= \text{Ma}_t c_t = \text{Ma}_t \sqrt{kRT_t} \\ &= (0.778) \sqrt{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(788.5 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} \\ &= 437.9 \text{ m/s} \end{aligned}$$

Thus,

$$\dot{m} = \rho_t A_t V_t = (3.093 \text{ kg/m}^3)(50 \times 10^{-4} \text{ m}^2)(437.9 \text{ m/s}) = \mathbf{6.77 \text{ kg/s}}$$

(b) The back pressure ratio for this case is

$$\frac{P_b}{P_0} = \frac{0.4 \text{ MPa}}{1.045 \text{ MPa}} = 0.383$$

which is less than the critical-pressure ratio, 0.5283. Therefore, sonic conditions exist at the exit plane (throat) of the nozzle, and $\text{Ma} = 1$. The flow is choked in this case, and the mass flow rate through the nozzle can be calculated from Eq. 17–25:

$$\begin{aligned} \dot{m} &= A^* P_0 \sqrt{\frac{k}{RT_0} \left(\frac{2}{k+1} \right)^{(k+1)/(2(k-1))}} \\ &= (50 \times 10^{-4} \text{ m}^2)(1045 \text{ kPa}) \times \sqrt{\frac{1.4}{(0.287 \text{ kJ/kg} \cdot \text{K})(884 \text{ K})} \left(\frac{2}{1.4+1} \right)^{2.4/0.8}} \\ &= \mathbf{7.10 \text{ kg/s}} \end{aligned}$$

since $\text{kPa} \cdot \text{m}^2 / \sqrt{\text{kJ/kg}} = \sqrt{1000} \text{ kg/s}$.

Discussion This is the maximum mass flow rate through the nozzle for the specified inlet conditions and nozzle throat area.

EXAMPLE 17–6 Gas Flow through a Converging Nozzle

Nitrogen enters a duct with varying flow area at $T_1 = 400 \text{ K}$, $P_1 = 100 \text{ kPa}$, and $\text{Ma}_1 = 0.3$. Assuming steady isentropic flow, determine T_2 , P_2 , and Ma_2 at a location where the flow area has been reduced by 20 percent.

Solution Nitrogen gas enters a converging nozzle. The properties at the nozzle exit are to be determined.

Assumptions 1 Nitrogen is an ideal gas with $k = 1.4$. 2 Flow through the nozzle is steady, one-dimensional, and isentropic.

Analysis The schematic of the duct is shown in Fig. 17–25. For isentropic flow through a duct, the area ratio A/A^* (the flow area over the area of the throat where $\text{Ma} = 1$) is also listed in Table A–32. At the initial Mach number of $\text{Ma}_1 = 0.3$, we read

$$\frac{A_1}{A^*} = 2.0351 \quad \frac{T_1}{T_0} = 0.9823 \quad \frac{P_1}{P_0} = 0.9395$$

With a 20 percent reduction in flow area, $A_2 = 0.8A_1$, and

$$\frac{A_2}{A^*} = \frac{A_2}{A_1} \frac{A_1}{A^*} = (0.8)(2.0351) = 1.6281$$

For this value of A_2/A^* from Table A–32, we read

$$\frac{T_2}{T_0} = 0.9701 \quad \frac{P_2}{P_0} = 0.8993 \quad \text{Ma}_2 = \mathbf{0.391}$$

Here we chose the subsonic Mach number for the calculated A_2/A^* instead of the supersonic one because the duct is converging in the flow direction and the initial flow is subsonic. Since the stagnation properties are constant for isentropic flow, we can write

$$\frac{T_2}{T_1} = \frac{T_2/T_0}{T_1/T_0} \rightarrow T_2 = T_1 \left(\frac{T_2/T_0}{T_1/T_0} \right) = (400 \text{ K}) \left(\frac{0.9701}{0.9823} \right) = \mathbf{395 \text{ K}}$$

$$\frac{P_2}{P_1} = \frac{P_2/P_0}{P_1/P_0} \rightarrow P_2 = P_1 \left(\frac{P_2/P_0}{P_1/P_0} \right) = (100 \text{ kPa}) \left(\frac{0.8993}{0.9395} \right) = \mathbf{95.7 \text{ kPa}}$$

which are the temperature and pressure at the desired location.

Discussion Note that the temperature and pressure drop as the fluid accelerates in a converging nozzle.

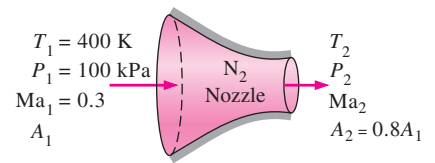


FIGURE 17–25

Schematic for Example 17–6 (not to scale).

Converging–Diverging Nozzles

When we think of nozzles, we ordinarily think of flow passages whose cross-sectional area decreases in the flow direction. However, the highest velocity to which a fluid can be accelerated in a converging nozzle is limited to the sonic velocity ($\text{Ma} = 1$), which occurs at the exit plane (throat) of the nozzle. Accelerating a fluid to supersonic velocities ($\text{Ma} > 1$) can be accomplished only by attaching a diverging flow section to the subsonic nozzle at the throat. The resulting combined flow section is a converging–diverging nozzle, which is standard equipment in supersonic aircraft and rocket propulsion (Fig. 17–26).

Forcing a fluid through a converging–diverging nozzle is no guarantee that the fluid will be accelerated to a supersonic velocity. In fact, the fluid may find itself decelerating in the diverging section instead of accelerating if the back pressure is not in the right range. The state of the nozzle flow is determined by the overall pressure ratio P_b/P_0 . Therefore, for given inlet conditions, the flow through a converging–diverging nozzle is governed by the back pressure P_b , as will be explained.

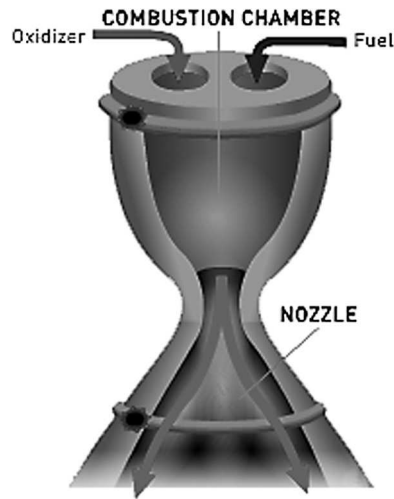


FIGURE 17–26

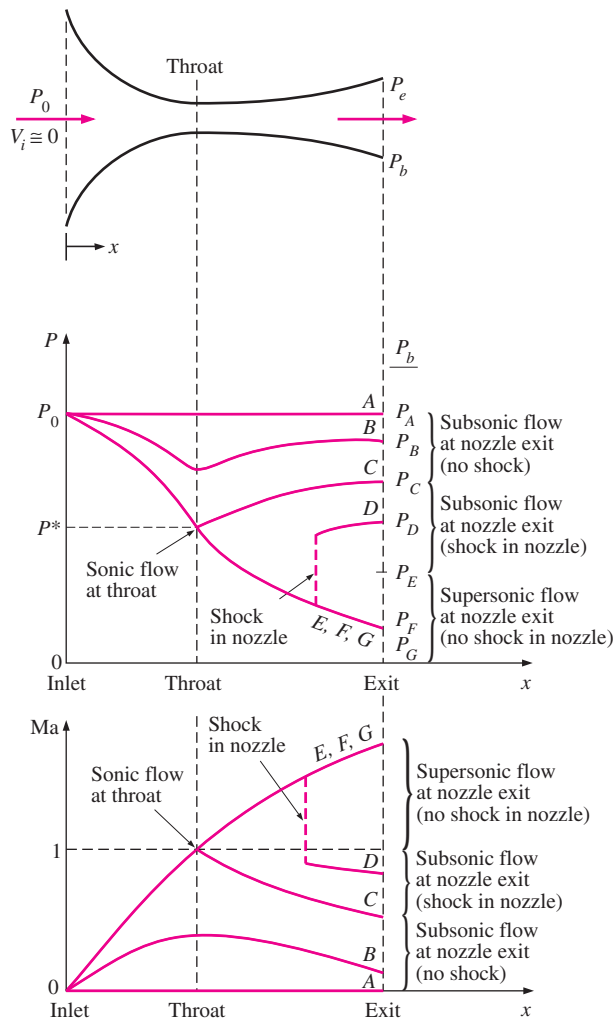
Converging–diverging nozzles are commonly used in rocket engines to provide high thrust.

Courtesy of Pratt and Whitney, www.pratt-whitney.com/how.htm. Used by permission.

Consider the converging–diverging nozzle shown in Fig. 17–27. A fluid enters the nozzle with a low velocity at stagnation pressure P_0 . When $P_b = P_0$ (case A), there will be no flow through the nozzle. This is expected since the flow in a nozzle is driven by the pressure difference between the nozzle inlet and the exit. Now let us examine what happens as the back pressure is lowered.

1. When $P_0 > P_b > P_C$, the flow remains subsonic throughout the nozzle, and the mass flow is less than that for choked flow. The fluid velocity increases in the first (converging) section and reaches a maximum at the throat (but $Ma < 1$). However, most of the gain in velocity is lost in the second (diverging) section of the nozzle, which acts as a diffuser. The pressure decreases in the converging section, reaches a minimum at the throat, and increases at the expense of velocity in the diverging section.
2. When $P_b = P_C$, the throat pressure becomes P^* and the fluid achieves sonic velocity at the throat. But the diverging section of the nozzle still acts as a diffuser, slowing the fluid to subsonic velocities. The mass flow rate that was increasing with decreasing P_b also reaches its maximum value.

Recall that P^* is the lowest pressure that can be obtained at the throat, and the sonic velocity is the highest velocity that can be achieved with a converging nozzle. Thus, lowering P_b further has no influence on the fluid flow in the converging part of the nozzle or the


FIGURE 17-27

The effects of back pressure on the flow through a converging–diverging nozzle.

mass flow rate through the nozzle. However, it does influence the character of the flow in the diverging section.

- When $P_C > P_b > P_E$, the fluid that achieved a sonic velocity at the throat continues accelerating to supersonic velocities in the diverging section as the pressure decreases. This acceleration comes to a sudden stop, however, as a **normal shock** develops at a section between the throat and the exit plane, which causes a sudden drop in velocity to subsonic levels and a sudden increase in pressure. The fluid then continues to decelerate further in the remaining part of the converging–diverging nozzle. Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic. The normal shock moves downstream away from the throat as P_b is decreased, and it approaches the nozzle exit plane as P_b approaches P_E .

When $P_b = P_E$, the normal shock forms at the exit plane of the nozzle. The flow is supersonic through the entire diverging section in this case, and it can be approximated as isentropic. However, the fluid velocity drops to subsonic levels just before leaving the nozzle as it

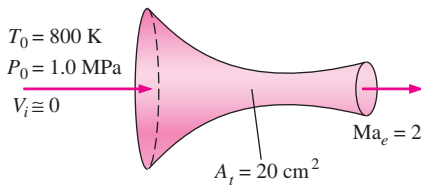


FIGURE 17-28
Schematic for Example 17-7.

crosses the normal shock. Normal shock waves are discussed in Section 17-5.

4. When $P_E > P_b > 0$, the flow in the diverging section is supersonic, and the fluid expands to P_F at the nozzle exit with no normal shock forming within the nozzle. Thus, the flow through the nozzle can be approximated as isentropic. When $P_b = P_F$, no shocks occur within or outside the nozzle. When $P_b < P_F$, irreversible mixing and expansion waves occur downstream of the exit plane of the nozzle. When $P_b > P_F$, however, the pressure of the fluid increases from P_F to P_b irreversibly in the wake of the nozzle exit, creating what are called *oblique shocks*.

EXAMPLE 17-7 Airflow through a Converging–Diverging Nozzle

Air enters a converging–diverging nozzle, shown in Fig. 17-28, at 1.0 MPa and 800 K with a negligible velocity. The flow is steady, one-dimensional, and isentropic with $k = 1.4$. For an exit Mach number of $Ma = 2$ and a throat area of 20 cm^2 , determine (a) the throat conditions, (b) the exit plane conditions, including the exit area, and (c) the mass flow rate through the nozzle.

Solution Air flows through a converging–diverging nozzle. The throat and the exit conditions and the mass flow rate are to be determined.

Assumptions 1 Air is an ideal gas with constant specific heats at room temperature. 2 Flow through the nozzle is steady, one-dimensional, and isentropic.

Properties The specific heat ratio of air is given to be $k = 1.4$. The gas constant of air is $0.287 \text{ kJ/kg} \cdot \text{K}$.

Analysis The exit Mach number is given to be 2. Therefore, the flow must be sonic at the throat and supersonic in the diverging section of the nozzle. Since the inlet velocity is negligible, the stagnation pressure and stagnation temperature are the same as the inlet temperature and pressure, $P_0 = 1.0 \text{ MPa}$ and $T_0 = 800 \text{ K}$. The stagnation density is

$$\rho_0 = \frac{P_0}{RT_0} = \frac{1000 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(800 \text{ K})} = 4.355 \text{ kg/m}^3$$

(a) At the throat of the nozzle $Ma = 1$, and from Table A-32 we read

$$\frac{P^*}{P_0} = 0.5283 \quad \frac{T^*}{T_0} = 0.8333 \quad \frac{\rho^*}{\rho_0} = 0.6339$$

Thus,

$$P^* = 0.5283P_0 = (0.5283)(1.0 \text{ MPa}) = \mathbf{0.5283 \text{ MPa}}$$

$$T^* = 0.8333T_0 = (0.8333)(800 \text{ K}) = \mathbf{666.6 \text{ K}}$$

$$\rho^* = 0.6339\rho_0 = (0.6339)(4.355 \text{ kg/m}^3) = \mathbf{2.761 \text{ kg/m}^3}$$

Also,

$$\begin{aligned} V^* = c^* &= \sqrt{kRT^*} = \sqrt{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(666.6 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} \\ &= \mathbf{517.5 \text{ m/s}} \end{aligned}$$

(b) Since the flow is isentropic, the properties at the exit plane can also be calculated by using data from Table A–32. For $Ma = 2$ we read

$$\frac{P_e}{P_0} = 0.1278 \quad \frac{T_e}{T_0} = 0.5556 \quad \frac{\rho_e}{\rho_0} = 0.2300 \quad Ma_t^* = 1.6330 \quad \frac{A_e}{A^*} = 1.6875$$

Thus,

$$P_e = 0.1278P_0 = (0.1278)(10 \text{ MPa}) = \mathbf{0.1278 \text{ MPa}}$$

$$T_e = 0.5556T_0 = (0.5556)(800 \text{ K}) = \mathbf{444.5 \text{ K}}$$

$$\rho_e = 0.2300\rho_0 = (0.2300)(4.355 \text{ kg/m}^3) = \mathbf{1.002 \text{ kg/m}^3}$$

$$A_e = 1.6875A^* = (1.6875)(20 \text{ cm}^2) = \mathbf{33.75 \text{ cm}^2}$$

and

$$V_e = Ma_e^*c_e^* = (1.6330)(517.5 \text{ m/s}) = \mathbf{845.1 \text{ m/s}}$$

The nozzle exit velocity could also be determined from $V_e = Ma_e c_e$, where c_e is the speed of sound at the exit conditions:

$$\begin{aligned} V_e = Ma_e c_e &= Ma_e \sqrt{kRT_e} = 2\sqrt{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(444.5 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} \\ &= \mathbf{845.2 \text{ m/s}} \end{aligned}$$

(c) Since the flow is steady, the mass flow rate of the fluid is the same at all sections of the nozzle. Thus it may be calculated by using properties at any cross section of the nozzle. Using the properties at the throat, we find that the mass flow rate is

$$\dot{m} = \rho^* A^* V^* = (2.761 \text{ kg/m}^3)(20 \times 10^{-4} \text{ m}^2)(517.5 \text{ m/s}) = \mathbf{2.86 \text{ kg/s}}$$

Discussion Note that this is the highest possible mass flow rate that can flow through this nozzle for the specified inlet conditions.

17–5 ■ SHOCK WAVES AND EXPANSION WAVES

We have seen that sound waves are caused by infinitesimally small pressure disturbances, and they travel through a medium at the speed of sound. We have also seen that for some back pressure values, abrupt changes in fluid properties occur in a very thin section of a converging–diverging nozzle under supersonic flow conditions, creating a **shock wave**. It is of interest to study the conditions under which shock waves develop and how they affect the flow.

Normal Shocks

First we consider shock waves that occur in a plane normal to the direction of flow, called **normal shock waves**. The flow process through the shock wave is highly irreversible and *cannot* be approximated as being isentropic.

Next we follow the footsteps of Pierre Laplace (1749–1827), G. F. Bernhard Riemann (1826–1866), William Rankine (1820–1872), Pierre Henry Hugoniot (1851–1887), Lord Rayleigh (1842–1919), and G. I. Taylor

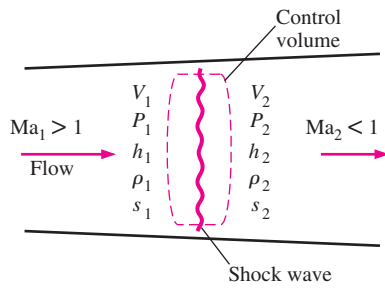


FIGURE 17-29

Control volume for flow across a normal shock wave.

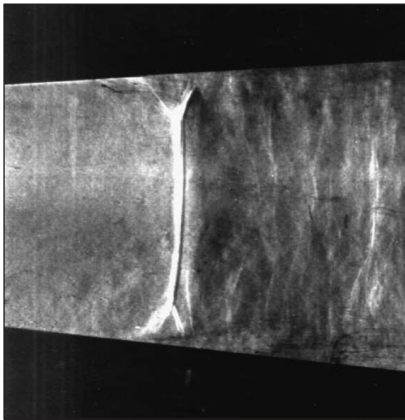


FIGURE 17-30

Schlieren image of a normal shock in a Laval nozzle. The Mach number in the nozzle just upstream (to the left) of the shock wave is about 1.3. Boundary layers distort the shape of the normal shock near the walls and lead to flow separation beneath the shock.

Photo by G. S. Settles, Penn State University. Used by permission.

(1886–1975) and develop relationships for the flow properties before and after the shock. We do this by applying the conservation of mass, momentum, and energy relations as well as some property relations to a stationary control volume that contains the shock, as shown in Fig. 17–29. The normal shock waves are extremely thin, so the entrance and exit flow areas for the control volume are approximately equal (Fig. 17–30).

We assume steady flow with no heat and work interactions and no potential energy changes. Denoting the properties upstream of the shock by the subscript 1 and those downstream of the shock by 2, we have the following:

$$\text{Conservation of mass:} \quad \rho_1 A V_1 = \rho_2 A V_2 \quad (17-29)$$

or

$$\rho_1 V_1 = \rho_2 V_2$$

$$\text{Conservation of energy:} \quad h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad (17-30)$$

or

$$h_{01} = h_{02} \quad (17-31)$$

Conservation of momentum: Rearranging Eq. 17–14 and integrating yield

$$A(P_1 - P_2) = \dot{m}(V_2 - V_1) \quad (17-32)$$

$$\text{Increase of entropy:} \quad s_2 - s_1 \geq 0 \quad (17-33)$$

We can combine the conservation of mass and energy relations into a single equation and plot it on an h - s diagram, using property relations. The resultant curve is called the **Fanno line**, and it is the locus of states that have the same value of stagnation enthalpy and mass flux (mass flow per unit flow area). Likewise, combining the conservation of mass and momentum equations into a single equation and plotting it on the h - s diagram yield a curve called the **Rayleigh line**. Both these lines are shown on the h - s diagram in Fig. 17–31. As proved later in Example 17–8, the points of maximum entropy on these lines (points a and b) correspond to $Ma = 1$. The state on the upper part of each curve is subsonic and on the lower part supersonic.

The Fanno and Rayleigh lines intersect at two points (points 1 and 2), which represent the two states at which all three conservation equations are satisfied. One of these (state 1) corresponds to the state before the shock, and the other (state 2) corresponds to the state after the shock. Note that the flow is supersonic before the shock and subsonic afterward. Therefore the flow must change from supersonic to subsonic if a shock is to occur. The larger the Mach number before the shock, the stronger the shock will be. In the limiting case of $Ma = 1$, the shock wave simply becomes a sound wave. Notice from Fig. 17–31 that $s_2 > s_1$. This is expected since the flow through the shock is adiabatic but irreversible.

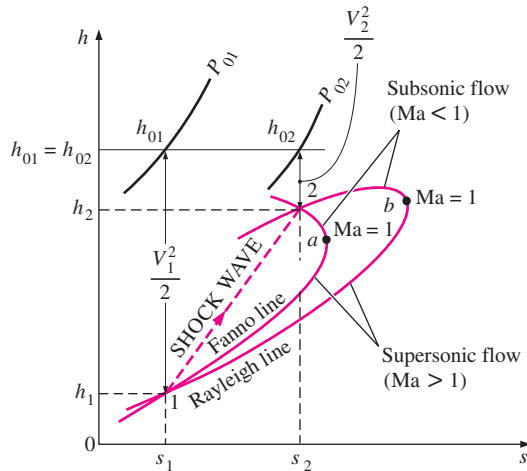


FIGURE 17-31

The h - s diagram for flow across a normal shock.

The conservation of energy principle (Eq. 17–31) requires that the stagnation enthalpy remain constant across the shock; $h_{01} = h_{02}$. For ideal gases $h = h(T)$, and thus

$$T_{01} = T_{02} \tag{17-34}$$

That is, the stagnation temperature of an ideal gas also remains constant across the shock. Note, however, that the stagnation pressure decreases across the shock because of the irreversibilities, while the thermodynamic temperature rises drastically because of the conversion of kinetic energy into enthalpy due to a large drop in fluid velocity (see Fig. 17–32).

We now develop relations between various properties before and after the shock for an ideal gas with constant specific heats. A relation for the ratio of the thermodynamic temperatures T_2/T_1 is obtained by applying Eq. 17–18 twice:

$$\frac{T_{01}}{T_1} = 1 + \left(\frac{k-1}{2}\right)Ma_1^2 \quad \text{and} \quad \frac{T_{02}}{T_2} = 1 + \left(\frac{k-1}{2}\right)Ma_2^2$$

Dividing the first equation by the second one and noting that $T_{01} = T_{02}$, we have

$$\frac{T_2}{T_1} = \frac{1 + Ma_1^2(k-1)/2}{1 + Ma_2^2(k-1)/2} \tag{17-35}$$

From the ideal-gas equation of state,

$$\rho_1 = \frac{P_1}{RT_1} \quad \text{and} \quad \rho_2 = \frac{P_2}{RT_2}$$

Substituting these into the conservation of mass relation $\rho_1 V_1 = \rho_2 V_2$ and noting that $Ma = V/c$ and $c = \sqrt{kRT}$, we have

$$\frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1} = \frac{P_2 Ma_2 c_2}{P_1 Ma_1 c_1} = \frac{P_2 Ma_2 \sqrt{T_2}}{P_1 Ma_1 \sqrt{T_1}} = \left(\frac{P_2}{P_1}\right)^2 \left(\frac{Ma_2}{Ma_1}\right)^2 \tag{17-36}$$

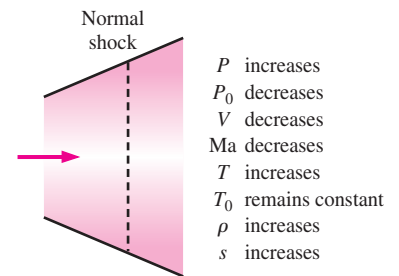


FIGURE 17-32

Variation of flow properties across a normal shock.

Combining Eqs. 17–35 and 17–36 gives the pressure ratio across the shock:

$$\frac{P_2}{P_1} = \frac{\text{Ma}_1 \sqrt{1 + \text{Ma}_1^2(k-1)/2}}{\text{Ma}_2 \sqrt{1 + \text{Ma}_2^2(k-1)/2}} \quad (17-37)$$

Equation 17–37 is a combination of the conservation of mass and energy equations; thus, it is also the equation of the Fanno line for an ideal gas with constant specific heats. A similar relation for the Rayleigh line can be obtained by combining the conservation of mass and momentum equations. From Eq. 17–32,

$$P_1 - P_2 = \frac{\dot{m}}{A} (V_2 - V_1) = \rho_2 V_2^2 - \rho_1 V_1^2$$

However,

$$\rho V^2 = \left(\frac{P}{RT} \right) (\text{Ma}c)^2 = \left(\frac{P}{RT} \right) (\text{Ma} \sqrt{kRT})^2 = Pk\text{Ma}^2$$

Thus,

$$P_1(1 + k\text{Ma}_1^2) = P_2(1 + k\text{Ma}_2^2)$$

or

$$\frac{P_2}{P_1} = \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} \quad (17-38)$$

Combining Eqs. 17–37 and 17–38 yields

$$\text{Ma}_2^2 = \frac{\text{Ma}_1^2 + 2/(k-1)}{2\text{Ma}_1^2 k/(k-1) - 1} \quad (17-39)$$

This represents the intersections of the Fanno and Rayleigh lines and relates the Mach number upstream of the shock to that downstream of the shock.

The occurrence of shock waves is not limited to supersonic nozzles only. This phenomenon is also observed at the engine inlet of a supersonic aircraft, where the air passes through a shock and decelerates to subsonic velocities before entering the diffuser of the engine. Explosions also produce powerful expanding spherical normal shocks, which can be very destructive (Fig. 17–33).

Various flow property ratios across the shock are listed in Table A–33 for an ideal gas with $k = 1.4$. Inspection of this table reveals that Ma_2 (the Mach number after the shock) is always less than 1 and that the larger the supersonic Mach number before the shock, the smaller the subsonic Mach number after the shock. Also, we see that the static pressure, temperature, and density all increase after the shock while the stagnation pressure decreases.

The entropy change across the shock is obtained by applying the entropy-change equation for an ideal gas across the shock:

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad (17-40)$$

which can be expressed in terms of k , R , and Ma_1 by using the relations developed earlier in this section. A plot of nondimensional entropy change



FIGURE 17–33

Schlieren image of the blast wave (expanding spherical normal shock) produced by the explosion of a firecracker detonated inside a metal can that sat on a stool. The shock expanded radially outward in all directions at a supersonic speed that decreased with radius from the center of the explosion. The microphone at the lower right sensed the sudden change in pressure of the passing shock wave and triggered the microsecond flashlamp that exposed the photograph.

Photo by G. S. Settles, Penn State University. Used by permission.

across the normal shock $(s_2 - s_1)/R$ versus Ma_1 is shown in Fig. 17–34. Since the flow across the shock is adiabatic and irreversible, the second law requires that the entropy increase across the shock wave. Thus, a shock wave cannot exist for values of Ma_1 less than unity where the entropy change would be negative. For adiabatic flows, shock waves can exist only for supersonic flows, $Ma_1 > 1$.

EXAMPLE 17–8 The Point of Maximum Entropy on the Fanno Line

Show that the point of maximum entropy on the Fanno line (point *b* of Fig. 17–31) for the adiabatic steady flow of a fluid in a duct corresponds to the sonic velocity, $Ma = 1$.

Solution It is to be shown that the point of maximum entropy on the Fanno line for steady adiabatic flow corresponds to sonic velocity.

Assumptions The flow is steady, adiabatic, and one-dimensional.

Analysis In the absence of any heat and work interactions and potential energy changes, the steady-flow energy equation reduces to

$$h + \frac{V^2}{2} = \text{constant}$$

Differentiating yields

$$dh + V dV = 0$$

For a very thin shock with negligible change of duct area across the shock, the steady-flow continuity (conservation of mass) equation can be expressed as

$$\rho V = \text{constant}$$

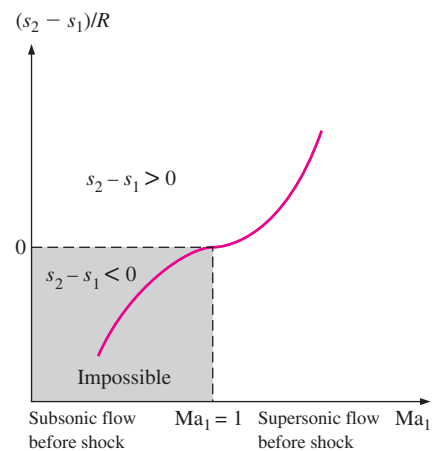


FIGURE 17–34

Entropy change across the normal shock.

Differentiating, we have

$$\rho dV + V d\rho = 0$$

Solving for dV gives

$$dV = -V \frac{d\rho}{\rho}$$

Combining this with the energy equation, we have

$$dh - V^2 \frac{d\rho}{\rho} = 0$$

which is the equation for the Fanno line in differential form. At point a (the point of maximum entropy) $ds = 0$. Then from the second $T ds$ relation ($T ds = dh - v dP$) we have $dh = v dP = dP/\rho$. Substituting yields

$$\frac{dP}{\rho} - V^2 \frac{d\rho}{\rho} = 0 \quad \text{at } s = \text{constant}$$

Solving for V , we have

$$V = \left(\frac{\partial P}{\partial \rho} \right)_s^{1/2}$$

which is the relation for the speed of sound, Eq. 17-9. Thus the proof is complete.

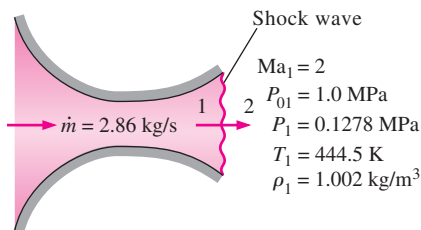


FIGURE 17-35

Schematic for Example 17-9.

EXAMPLE 17-9 Shock Wave in a Converging–Diverging Nozzle

If the air flowing through the converging–diverging nozzle of Example 17-7 experiences a normal shock wave at the nozzle exit plane (Fig. 17-35), determine the following after the shock: (a) the stagnation pressure, static pressure, static temperature, and static density; (b) the entropy change across the shock; (c) the exit velocity; and (d) the mass flow rate through the nozzle. Assume steady, one-dimensional, and isentropic flow with $k = 1.4$ from the nozzle inlet to the shock location.

Solution Air flowing through a converging–diverging nozzle experiences a normal shock at the exit. The effect of the shock wave on various properties is to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats at room temperature. **2** Flow through the nozzle is steady, one-dimensional, and isentropic before the shock occurs. **3** The shock wave occurs at the exit plane.

Properties The constant-pressure specific heat and the specific heat ratio of air are $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$ and $k = 1.4$. The gas constant of air is $0.287 \text{ kJ/kg} \cdot \text{K}$ (Table A-2a).

Analysis (a) The fluid properties at the exit of the nozzle just before the shock (denoted by subscript 1) are those evaluated in Example 17-7 at the nozzle exit to be

$$P_{01} = 1.0 \text{ MPa} \quad P_1 = 0.1278 \text{ MPa} \quad T_1 = 444.5 \text{ K} \quad \rho_1 = 1.002 \text{ kg/m}^3$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A–33. For $Ma_1 = 2.0$, we read

$$Ma_2 = 0.5774 \quad \frac{P_{02}}{P_{01}} = 0.7209 \quad \frac{P_2}{P_1} = 4.5000 \quad \frac{T_2}{T_1} = 1.6875 \quad \frac{\rho_2}{\rho_1} = 2.6667$$

Then the stagnation pressure P_{02} , static pressure P_2 , static temperature T_2 , and static density ρ_2 after the shock are

$$P_{02} = 0.7209P_{01} = (0.7209)(1.0 \text{ MPa}) = \mathbf{0.721 \text{ MPa}}$$

$$P_2 = 4.5000P_1 = (4.5000)(0.1278 \text{ MPa}) = \mathbf{0.575 \text{ MPa}}$$

$$T_2 = 1.6875T_1 = (1.6875)(444.5 \text{ K}) = \mathbf{750 \text{ K}}$$

$$\rho_2 = 2.6667\rho_1 = (2.6667)(1.002 \text{ kg/m}^3) = \mathbf{2.67 \text{ kg/m}^3}$$

(b) The entropy change across the shock is

$$\begin{aligned} s_2 - s_1 &= c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \\ &= (1.005 \text{ kJ/kg} \cdot \text{K}) \ln(1.6875) - (0.287 \text{ kJ/kg} \cdot \text{K}) \ln(4.5000) \\ &= \mathbf{0.0942 \text{ kJ/kg} \cdot \text{K}} \end{aligned}$$

Thus, the entropy of the air increases as it experiences a normal shock, which is highly irreversible.

(c) The air velocity after the shock can be determined from $V_2 = Ma_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock:

$$\begin{aligned} V_2 &= Ma_2 c_2 = Ma_2 \sqrt{kRT_2} \\ &= (0.5774) \sqrt{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(750 \text{ K}) \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} \\ &= \mathbf{317 \text{ m/s}} \end{aligned}$$

(d) The mass flow rate through a converging–diverging nozzle with sonic conditions at the throat is not affected by the presence of shock waves in the nozzle. Therefore, the mass flow rate in this case is the same as that determined in Example 17–7:

$$\dot{m} = \mathbf{2.86 \text{ kg/s}}$$

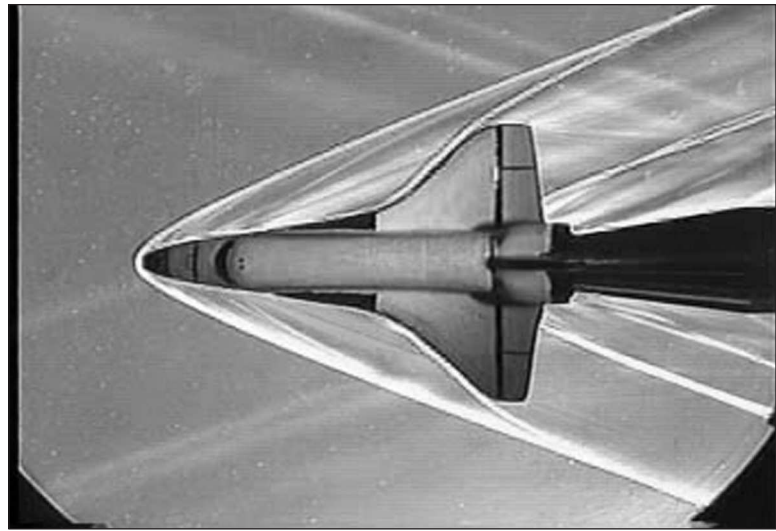
Discussion This result can easily be verified by using property values at the nozzle exit after the shock at all Mach numbers significantly greater than unity.

Example 17–9 illustrates that the stagnation pressure and velocity decrease while the static pressure, temperature, density, and entropy increase across the shock. The rise in the temperature of the fluid downstream of a shock wave is of major concern to the aerospace engineer because it creates heat transfer problems on the leading edges of wings and nose cones of space reentry vehicles and the recently proposed hypersonic space planes. Overheating, in fact, led to the tragic loss of the space shuttle *Columbia* in February of 2003 as it was reentering earth’s atmosphere.

FIGURE 17-36

Schlieren image of a small model of the space shuttle *Orbiter* being tested at Mach 3 in the supersonic wind tunnel of the Penn State Gas Dynamics Lab. Several *oblique shocks* are seen in the air surrounding the spacecraft.

Photo by G. S. Settles, Penn State University. Used by permission.

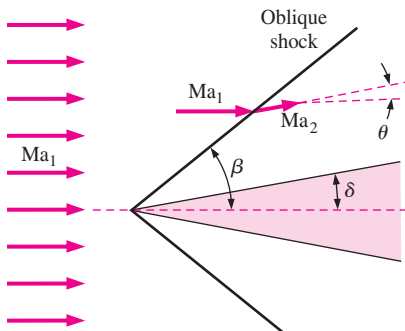


Oblique Shocks

Not all shock waves are normal shocks (perpendicular to the flow direction). For example, when the space shuttle travels at supersonic speeds through the atmosphere, it produces a complicated shock pattern consisting of inclined shock waves called **oblique shocks** (Fig. 17-36). As you can see, some portions of an oblique shock are curved, while other portions are straight.

First, we consider straight oblique shocks, like that produced when a uniform supersonic flow ($Ma_1 > 1$) impinges on a slender, two-dimensional wedge of half-angle δ (Fig. 17-37). Since information about the wedge cannot travel upstream in a supersonic flow, the fluid “knows” nothing about the wedge until it hits the nose. At that point, since the fluid cannot flow *through* the wedge, it turns suddenly through an angle called the **turning angle** or **deflection angle** θ . The result is a straight oblique shock wave, aligned at **shock angle** or **wave angle** β , measured relative to the oncoming flow (Fig. 17-38). To conserve mass, β must obviously be greater than δ . Since the Reynolds number of supersonic flows is typically large, the boundary layer growing along the wedge is very thin, and we ignore its effects. The flow therefore turns by the same angle as the wedge; namely, deflection angle θ is equal to wedge half-angle δ . If we take into account the displacement thickness effect of the boundary layer, the deflection angle θ of the oblique shock turns out to be slightly greater than wedge half-angle δ .

Like normal shocks, the Mach number decreases across an oblique shock, and oblique shocks are possible only if the upstream flow is supersonic. However, unlike normal shocks, in which the downstream Mach number is always subsonic, Ma_2 downstream of an oblique shock can be subsonic, sonic, or supersonic, depending on the upstream Mach number Ma_1 and the turning angle.

**FIGURE 17-37**

An oblique shock of *shock angle* β formed by a slender, two-dimensional wedge of half-angle δ . The flow is turned by *deflection angle* θ downstream of the shock, and the Mach number decreases.

We analyze a straight oblique shock in Fig. 17–38 by decomposing the velocity vectors upstream and downstream of the shock into normal and tangential components, and considering a small control volume around the shock. Upstream of the shock, all fluid properties (velocity, density, pressure, etc.) along the lower left face of the control volume are identical to those along the upper right face. The same is true downstream of the shock. Therefore, the mass flow rates entering and leaving those two faces cancel each other out, and conservation of mass reduces to

$$\rho_1 V_{1,n} A = \rho_2 V_{2,n} A \rightarrow \rho_1 V_{1,n} = \rho_2 V_{2,n} \quad (17-41)$$

where A is the area of the control surface that is parallel to the shock. Since A is the same on either side of the shock, it has dropped out of Eq. 17–41.

As you might expect, the tangential component of velocity (parallel to the oblique shock) does not change across the shock (i.e., $V_{1,t} = V_{2,t}$). This is easily proven by applying the tangential momentum equation to the control volume.

When we apply conservation of momentum in the direction *normal* to the oblique shock, the only forces are pressure forces, and we get

$$P_1 A - P_2 A = \rho V_{2,n} A V_{2,n} - \rho V_{1,n} A V_{1,n} \rightarrow P_1 - P_2 = \rho_2 V_{2,n}^2 - \rho_1 V_{1,n}^2 \quad (17-42)$$

Finally, since there is no work done by the control volume and no heat transfer into or out of the control volume, stagnation enthalpy does *not* change across an oblique shock, and conservation of energy yields

$$h_{01} = h_{02} = h_0 \rightarrow h_1 + \frac{1}{2} V_{1,n}^2 + \frac{1}{2} V_{1,t}^2 = h_2 + \frac{1}{2} V_{2,n}^2 + \frac{1}{2} V_{2,t}^2$$

But since $V_{1,t} = V_{2,t}$, this equation reduces to

$$h_1 + \frac{1}{2} V_{1,n}^2 = h_2 + \frac{1}{2} V_{2,n}^2 \quad (17-43)$$

Careful comparison reveals that the equations for conservation of mass, momentum, and energy (Eqs. 17–41 through 17–43) across an oblique shock are identical to those across a normal shock, except that they are written in terms of the *normal* velocity component only. Therefore, the normal shock relations derived previously apply to oblique shocks as well, but must be written in terms of Mach numbers $Ma_{1,n}$ and $Ma_{2,n}$ normal to the oblique shock. This is most easily visualized by rotating the velocity vectors in Fig. 17–38 by angle $\pi/2 - \beta$, so that the oblique shock appears to be vertical (Fig. 17–39). Trigonometry yields

$$Ma_{1,n} = Ma_1 \sin \beta \quad \text{and} \quad Ma_{2,n} = Ma_2 \sin(\beta - \theta) \quad (17-44)$$

where $Ma_{1,n} = V_{1,n}/c_1$ and $Ma_{2,n} = V_{2,n}/c_2$. From the point of view shown in Fig. 17–40, we see what looks like a normal shock, but with some superposed tangential flow “coming along for the ride.” Thus,

All the equations, shock tables, etc., for normal shocks apply to oblique shocks as well, provided that we use only the **normal** components of the Mach number.

In fact, you may think of normal shocks as special oblique shocks in which shock angle $\beta = \pi/2$, or 90° . We recognize immediately that an oblique shock can exist only if $Ma_{1,n} > 1$, and $Ma_{2,n} < 1$. The normal shock

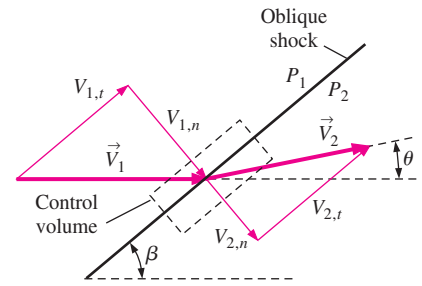


FIGURE 17–38

Velocity vectors through an oblique shock of shock angle β and deflection angle θ .

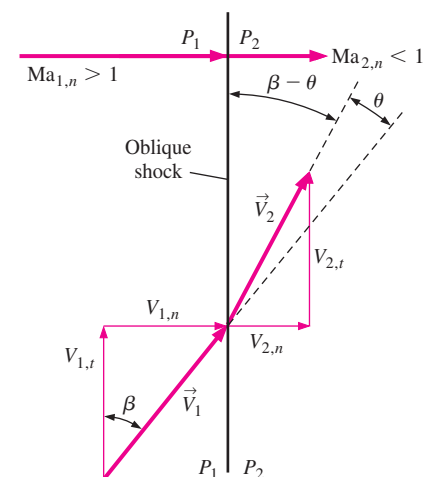


FIGURE 17–39

The same velocity vectors of Fig. 17–38, but rotated by angle $\pi/2 - \beta$, so that the oblique shock is vertical. Normal Mach numbers $Ma_{1,n}$ and $Ma_{2,n}$ are also defined.

FIGURE 17-40

Relationships across an oblique shock for an ideal gas in terms of the normal component of upstream Mach number $Ma_{1,n}$.

equations appropriate for oblique shocks in an ideal gas are summarized in Fig. 17-40 in terms of $Ma_{1,n}$.

For known shock angle β and known upstream Mach number Ma_1 , we use the first part of Eq. 17-44 to calculate $Ma_{1,n}$, and then use the normal shock tables (or their corresponding equations) to obtain $Ma_{2,n}$. If we also knew the deflection angle θ , we could calculate Ma_2 from the second part of Eq. 17-44. But, in a typical application, we know either β or θ , but not both. Fortunately, a bit more algebra provides us with a relationship between θ , β , and Ma_1 . We begin by noting that $\tan \beta = V_{1,n}/V_{1,t}$ and $\tan(\beta - \theta) = V_{2,n}/V_{2,t}$ (Fig. 17-39). But since $V_{1,t} = V_{2,t}$, we combine these two expressions to yield

$$\frac{V_{2,n}}{V_{1,n}} = \frac{\tan(\beta - \theta)}{\tan \beta} = \frac{2 + (k-1)Ma_{1,n}^2}{(k+1)Ma_{1,n}^2} = \frac{2 + (k-1)Ma_1^2 \sin^2 \beta}{(k+1)Ma_1^2 \sin^2 \beta} \quad (17-45)$$

where we have also used Eq. 17-44 and the fourth equation of Fig. 17-40. We apply trigonometric identities for $\cos 2\beta$ and $\tan(\beta - \theta)$, namely,

$$\cos 2\beta = \cos^2 \beta - \sin^2 \beta \quad \text{and} \quad \tan(\beta - \theta) = \frac{\tan \beta - \tan \theta}{1 + \tan \beta \tan \theta}$$

After some algebra, Eq. 17-45 reduces to

$$\text{The } \theta\text{-}\beta\text{-}Ma \text{ relationship:} \quad \tan \theta = \frac{2 \cot \beta (Ma_1^2 \sin^2 \beta - 1)}{Ma_1^2 (k + \cos 2\beta) + 2} \quad (17-46)$$

Equation 17-46 provides deflection angle θ as a unique function of shock angle β , specific heat ratio k , and upstream Mach number Ma_1 . For air ($k = 1.4$), we plot θ versus β for several values of Ma_1 in Fig. 17-41. We note that this plot is often presented with the axes reversed (β versus θ) in compressible flow textbooks, since, physically, shock angle β is determined by deflection angle θ .

Much can be learned by studying Fig. 17-41, and we list some observations here:

- Figure 17-41 displays the full range of possible shock waves at a given free-stream Mach number, from the weakest to the strongest. For any value of Mach number Ma_1 greater than 1, the possible values of θ range from $\theta = 0^\circ$ at some value of β between 0 and 90° , to a maximum value $\theta = \theta_{\max}$ at an intermediate value of β , and then back to $\theta = 0^\circ$ at $\beta = 90^\circ$. Straight oblique shocks for θ or β outside of this range *cannot* and *do not* exist. At $Ma_1 = 1.5$, for example, straight oblique shocks cannot exist in air with shock angle β less than about 42° , nor with deflection angle θ greater than about 12° . If the wedge half-angle is greater than θ_{\max} , the shock becomes curved and detaches from the nose of the wedge, forming what is called a **detached oblique shock** or a **bow wave** (Fig. 17-42). The shock angle β of the detached shock is 90° at the nose, but β decreases as the shock curves downstream. Detached shocks are much more complicated than simple straight oblique shocks to analyze. In fact, no simple solutions exist, and prediction of detached shocks requires computational methods.
- Similar oblique shock behavior is observed in *axisymmetric flow* over cones, as in Fig. 17-43, although the θ - β - Ma relationship for axisymmetric flows differs from that of Eq. 17-46.

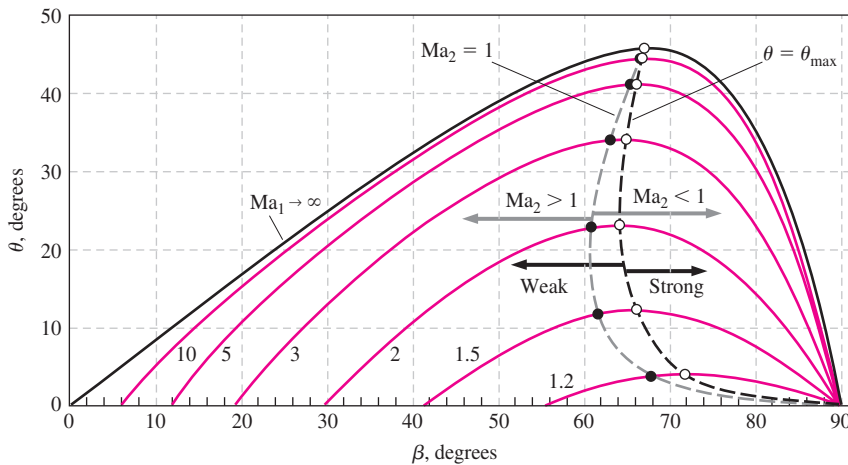


FIGURE 17-41

The dependence of straight oblique shock deflection angle θ on shock angle β for several values of upstream Mach number Ma_1 . Calculations are for an ideal gas with $k = 1.4$. The dashed black line connects points of maximum deflection angle ($\theta = \theta_{\max}$). *Weak oblique shocks* are to the left of this line, while *strong oblique shocks* are to the right of this line. The dashed gray line connects points where the downstream Mach number is sonic ($Ma_2 = 1$). *Supersonic downstream flow* ($Ma_2 > 1$) is to the left of this line, while *subsonic downstream flow* ($Ma_2 < 1$) is to the right of this line.

- When supersonic flow impinges on a blunt body—a body *without* a sharply pointed nose, the wedge half-angle δ at the nose is 90° , and an attached oblique shock cannot exist, regardless of Mach number. In fact, a detached oblique shock occurs in front of *all* such blunt-nosed bodies, whether two-dimensional, axisymmetric, or fully three-dimensional. For example, a detached oblique shock is seen in front of the space shuttle model in Fig. 17–36 and in front of a sphere in Fig. 17–44.
- While θ is a unique function of Ma_1 and β for a given value of k , there are *two* possible values of β for $\theta < \theta_{\max}$. The dashed black line in Fig. 17–41 passes through the locus of θ_{\max} values, dividing the shocks into **weak oblique shocks** (the smaller value of β) and **strong oblique shocks** (the larger value of β). At a given value of θ , the weak shock is more common and is “preferred” by the flow unless the downstream pressure conditions are high enough for the formation of a strong shock.
- For a given upstream Mach number Ma_1 , there is a unique value of θ for which the downstream Mach number Ma_2 is exactly 1. The dashed gray line in Fig. 17–41 passes through the locus of values where $Ma_2 = 1$. To the left of this line, $Ma_2 > 1$, and to the right of this line, $Ma_2 < 1$. Downstream sonic conditions occur on the weak shock side of the plot, with θ very close to θ_{\max} . Thus, the flow downstream of a strong oblique shock is *always subsonic* ($Ma_2 < 1$). The flow downstream of a weak oblique shock remains *supersonic*, except for a narrow range of θ just below θ_{\max} , where it is subsonic, although it is still called a weak oblique shock.
- As the upstream Mach number approaches infinity, straight oblique shocks become possible for any β between 0 and 90° , but the maximum possible turning angle for $k = 1.4$ (air) is $\theta_{\max} \cong 45.6^\circ$, which occurs at $\beta = 67.8^\circ$. Straight oblique shocks with turning angles above this value of θ_{\max} are not possible, regardless of the Mach number.
- For a given value of upstream Mach number, there are two shock angles where there is *no turning of the flow* ($\theta = 0^\circ$): the strong case, $\beta = 90^\circ$,

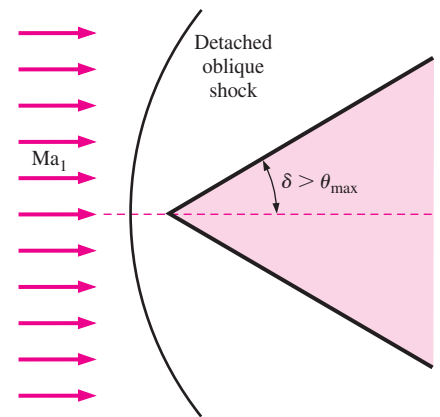


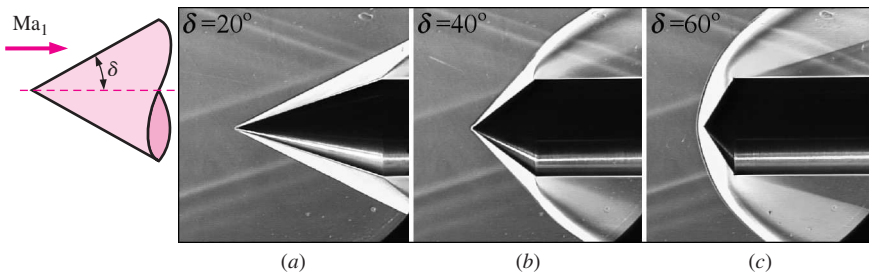
FIGURE 17-42

A *detached oblique shock* occurs upstream of a two-dimensional wedge of half-angle δ when δ is greater than the maximum possible deflection angle θ . A shock of this kind is called a *bow wave* because of its resemblance to the water wave that forms at the bow of a ship.

FIGURE 17-43

Still frames from schlieren videography illustrating the detachment of an oblique shock from a cone with increasing cone half-angle δ in air at Mach 3. At (a) $\delta = 20^\circ$ and (b) $\delta = 40^\circ$, the oblique shock remains attached, but by (c) $\delta = 60^\circ$, the oblique shock has detached, forming a bow wave.

Photos by G. S. Settles, Penn State University. Used by permission.



corresponds to a *normal shock*, and the weak case, $\beta = \beta_{\min}$, represents the weakest possible oblique shock at that Mach number, which is called a **Mach wave**. Mach waves are caused, for example, by very small nonuniformities on the walls of a supersonic wind tunnel (several can be seen in Figs. 17-36 and 17-43). Mach waves have no effect on the flow, since the shock is vanishingly weak. In fact, in the limit, Mach waves are *isentropic*. The shock angle for Mach waves is a unique function of the Mach number and is given the symbol μ , not to be confused with the coefficient of viscosity. Angle μ is called the **Mach angle** and is found by setting θ equal to zero in Eq. 17-46, solving for $\beta = \mu$, and taking the smaller root. We get

$$\text{Mach angle:} \quad \mu = \sin^{-1}(1/Ma_1) \quad (17-47)$$

Since the specific heat ratio appears only in the denominator of Eq. 17-46, μ is independent of k . Thus, we can estimate the Mach number of any supersonic flow simply by measuring the Mach angle and applying Eq. 17-47.

Prandtl–Meyer Expansion Waves

We now address situations where supersonic flow is turned in the *opposite* direction, such as in the upper portion of a two-dimensional wedge at an angle of attack greater than its half-angle δ (Fig. 17-45). We refer to this type of flow as an **expanding flow**, whereas a flow that produces an oblique shock may be called a **compressing flow**. As previously, the flow changes direction to conserve mass. However, unlike a compressing flow, an expanding flow does *not* result in a shock wave. Rather, a continuous expanding region called an **expansion fan** appears, composed of an infinite number of Mach waves called **Prandtl–Meyer expansion waves**. In other words, the flow does not turn suddenly, as through a shock, but *gradually*—each successive Mach wave turns the flow by an infinitesimal amount. Since each individual expansion wave is isentropic, the flow across the entire expansion fan is also isentropic. The Mach number downstream of the expansion *increases* ($Ma_2 > Ma_1$), while pressure, density, and temperature *decrease*, just as they do in the supersonic (expanding) portion of a converging–diverging nozzle.

Prandtl–Meyer expansion waves are inclined at the local Mach angle μ , as sketched in Fig. 17-45. The Mach angle of the first expansion wave is easily determined as $\mu_1 = \sin^{-1}(1/Ma_1)$. Similarly, $\mu_2 = \sin^{-1}(1/Ma_2)$,

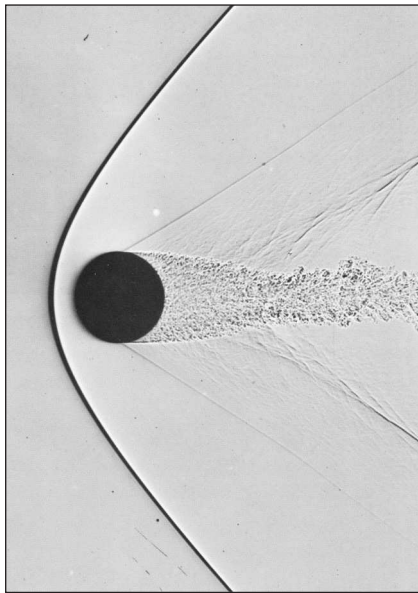


FIGURE 17-44

Shadowgram of a one-half-in diameter sphere in free flight through air at $Ma = 1.53$. The flow is subsonic behind the part of the bow wave that is ahead of the sphere and over its surface back to about 45° . At about 90° the laminar boundary layer separates through an oblique shock wave and quickly becomes turbulent. The fluctuating wake generates a system of weak disturbances that merge into the second “recompression” shock wave.

Photo by A. C. Charters, Army Ballistic Research Laboratory.

where we must be careful to measure the angle relative to the *new* direction of flow downstream of the expansion, namely, parallel to the upper wall of the wedge in Fig. 17–45 if we neglect the influence of the boundary layer along the wall. But how do we determine Ma_2 ? It turns out that the turning angle θ across the expansion fan can be calculated by integration, making use of the isentropic flow relationships. For an ideal gas, the result is (Anderson, 2003),

Turning angle across an expansion fan: $\theta = \nu(Ma_2) - \nu(Ma_1)$ (17–48)

where $\nu(Ma)$ is an angle called the **Prandtl–Meyer function** (not to be confused with the kinematic viscosity),

$$\nu(Ma) = \sqrt{\frac{k+1}{k-1}} \tan^{-1} \left[\sqrt{\frac{k-1}{k+1}} (Ma^2 - 1) \right] - \tan^{-1} \left(\sqrt{Ma^2 - 1} \right) \quad (17-49)$$

Note that $\nu(Ma)$ is an angle, and can be calculated in either degrees or radians. Physically, $\nu(Ma)$ is the angle through which the flow must expand, starting with $\nu = 0$ at $Ma = 1$, in order to reach a supersonic Mach number, $Ma > 1$.

To find Ma_2 for known values of Ma_1 , k , and θ , we calculate $\nu(Ma_1)$ from Eq. 17–49, $\nu(Ma_2)$ from Eq. 17–48, and then Ma_2 from Eq. 17–49, noting that the last step involves solving an implicit equation for Ma_2 . Since there is no heat transfer or work, and the flow is isentropic through the expansion, T_0 and P_0 remain constant, and we use the isentropic flow relations derived previously to calculate other flow properties downstream of the expansion, such as T_2 , ρ_2 , and P_2 .

Prandtl–Meyer expansion fans also occur in axisymmetric supersonic flows, as in the corners and trailing edges of a cone-cylinder (Fig. 17–46). Some very complex and, to some of us, beautiful interactions involving both shock waves and expansion waves occur in the supersonic jet produced by an “overexpanded” nozzle, as in Fig. 17–47. Analysis of such flows is beyond the scope of the present text; interested readers are referred to compressible flow textbooks such as Thompson (1972) and Anderson (2003).

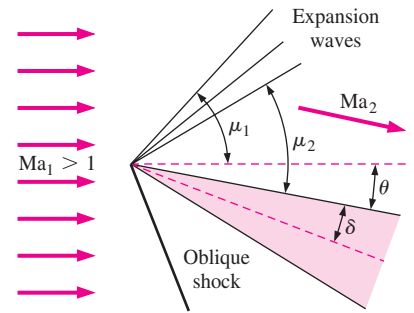


FIGURE 17–45

An expansion fan in the upper portion of the flow formed by a two-dimensional wedge at the angle of attack in a supersonic flow. The flow is turned by angle θ , and the Mach number increases across the expansion fan. Mach angles upstream and downstream of the expansion fan are indicated. Only three expansion waves are shown for simplicity, but in fact, there are an infinite number of them. (An oblique shock is present in the bottom portion of this flow.)

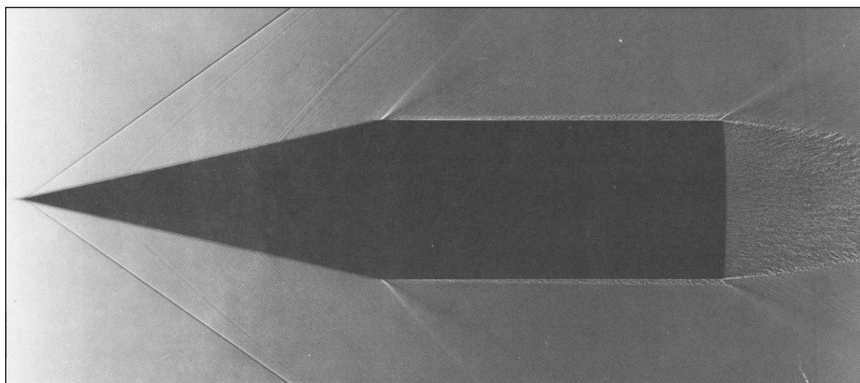


FIGURE 17–46

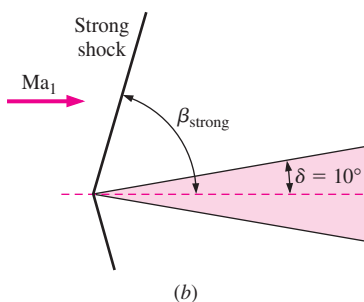
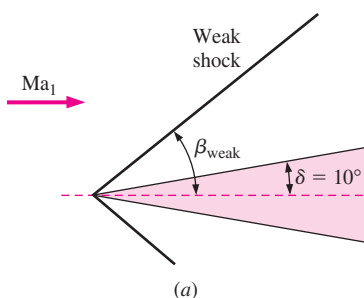
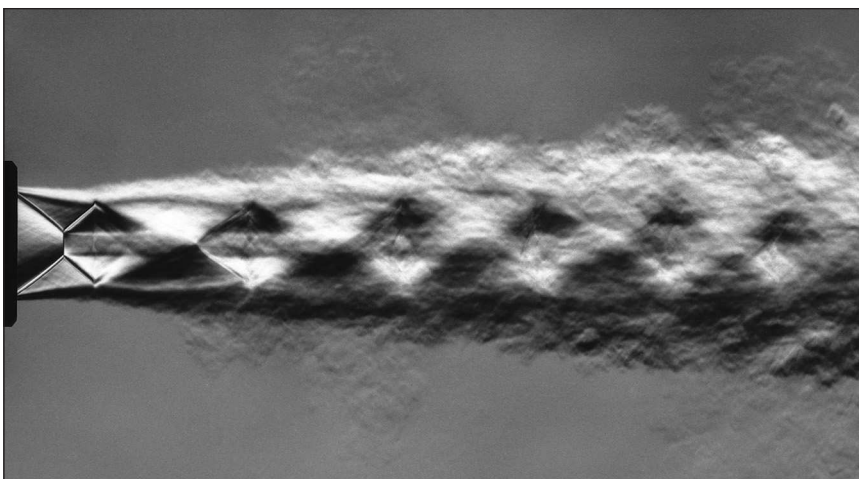
A cone-cylinder of 12.5° half-angle in a Mach number 1.84 flow. The boundary layer becomes turbulent shortly downstream of the nose, generating Mach waves that are visible in this shadowgraph. Expansion waves are seen at the corners and at the trailing edge of the cone.

Photo by A. C. Charters, Army Ballistic Research Laboratory.

FIGURE 17-47

The complex interactions between shock waves and expansion waves in an “overexpanded” supersonic jet. The flow is visualized by a schlieren-like differential interferogram.

Photo by H. Oertel sen. Reproduced by courtesy of the French-German Research Institute of Saint-Louis, ISL. Used with permission.

**FIGURE 17-48**

Two possible oblique shock angles, (a) β_{weak} and (b) β_{strong} , formed by a two-dimensional wedge of half-angle $\delta = 10^\circ$.

EXAMPLE 17-10 Estimation of the Mach Number from Mach Lines

Estimate the Mach number of the free-stream flow upstream of the space shuttle in Fig. 17-36 from the figure alone. Compare with the known value of Mach number provided in the figure caption.

Solution We are to estimate the Mach number from a figure and compare it to the known value.

Analysis Using a protractor, we measure the angle of the Mach lines in the free-stream flow: $\mu \cong 19^\circ$. The Mach number is obtained from Eq. 17-47,

$$\mu = \sin^{-1}\left(\frac{1}{\text{Ma}_1}\right) \rightarrow \text{Ma}_1 = \frac{1}{\sin 19^\circ} \rightarrow \text{Ma}_1 = 3.07$$

Our estimated Mach number agrees with the experimental value of 3.0 ± 0.1 .

Discussion The result is independent of the fluid properties.

EXAMPLE 17-11 Oblique Shock Calculations

Supersonic air at $\text{Ma}_1 = 2.0$ and 75.0 kPa impinges on a two-dimensional wedge of half-angle $\delta = 10^\circ$ (Fig. 17-48). Calculate the two possible oblique shock angles, β_{weak} and β_{strong} , that could be formed by this wedge. For each case, calculate the pressure and Mach number downstream of the oblique shock, compare, and discuss.

Solution We are to calculate the shock angle, Mach number, and pressure downstream of the weak and strong oblique shocks formed by a two-dimensional wedge.

Assumptions 1 The flow is steady. 2 The boundary layer on the wedge is very thin.

Properties The fluid is air with $k = 1.4$.

Analysis Because of assumption 2, we approximate the oblique shock deflection angle as equal to the wedge half-angle, i.e., $\theta \cong \delta = 10^\circ$. With $Ma_1 = 2.0$ and $\theta = 10^\circ$, we solve Eq. 17-46 for the two possible values of oblique shock angle β : $\beta_{\text{weak}} = 39.3^\circ$ and $\beta_{\text{strong}} = 83.7^\circ$. From these values, we use the first part of Eq. 17-44 to calculate the upstream normal Mach number $Ma_{1,n}$,

$$\text{Weak shock: } Ma_{1,n} = Ma_1 \sin \beta \rightarrow Ma_{1,n} = 2.0 \sin 39.3^\circ = 1.267$$

$$\text{Strong shock: } Ma_{1,n} = Ma_1 \sin \beta \rightarrow Ma_{1,n} = 2.0 \sin 83.7^\circ = 1.988$$

We substitute these values of $Ma_{1,n}$ into the second equation of Fig. 17-40 to calculate the downstream normal Mach number $Ma_{2,n}$. For the weak shock, $Ma_{2,n} = 0.8032$, and for the strong shock, $Ma_{2,n} = 0.5794$. We also calculate the downstream pressure for each case, using the third equation of Fig. 17-40, which gives

Weak shock:

$$\frac{P_2}{P_1} = \frac{2kMa_{1,n}^2 - k + 1}{k + 1} \rightarrow P_2 = (75.0 \text{ kPa}) \frac{2(1.4)(1.267)^2 - 1.4 + 1}{1.4 + 1} = 128 \text{ kPa}$$

Strong shock:

$$\frac{P_2}{P_1} = \frac{2kMa_{1,n}^2 - k + 1}{k + 1} \rightarrow P_2 = (75.0 \text{ kPa}) \frac{2(1.4)(1.988)^2 - 1.4 + 1}{1.4 + 1} = 333 \text{ kPa}$$

Finally, we use the second part of Eq. 17-44 to calculate the downstream Mach number,

$$\text{Weak shock: } Ma_2 = \frac{Ma_{2,n}}{\sin(\beta - \theta)} = \frac{0.8032}{\sin(39.3^\circ - 10^\circ)} = 1.64$$

$$\text{Strong shock: } Ma_2 = \frac{Ma_{2,n}}{\sin(\beta - \theta)} = \frac{0.5794}{\sin(83.7^\circ - 10^\circ)} = 0.604$$

The changes in Mach number and pressure across the strong shock are much greater than the changes across the weak shock, as expected.

Discussion Since Eq. 17-46 is implicit in β , we solve it by an iterative approach or with an equation solver such as EES. For both the weak and strong oblique shock cases, $Ma_{1,n}$ is supersonic and $Ma_{2,n}$ is subsonic. However, Ma_2 is *supersonic* across the weak oblique shock, but *subsonic* across the strong oblique shock. We could also use the normal shock tables in place of the equations, but with loss of precision.

EXAMPLE 17-12 Prandtl–Meyer Expansion Wave Calculations

Supersonic air at $Ma_1 = 2.0$ and 230 kPa flows parallel to a flat wall that suddenly expands by $\delta = 10^\circ$ (Fig. 17-49). Ignoring any effects caused by the boundary layer along the wall, calculate downstream Mach number Ma_2 and pressure P_2 .

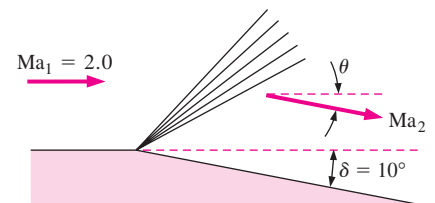


FIGURE 17-49

An expansion fan caused by the sudden expansion of a wall with $\delta = 10^\circ$.

Solution We are to calculate the Mach number and pressure downstream of a sudden expansion along a wall.

Assumptions **1** The flow is steady. **2** The boundary layer on the wall is very thin.

Properties The fluid is air with $k = 1.4$.

Analysis Because of assumption 2, we approximate the total deflection angle as equal to the wall expansion angle (i.e., $\theta \cong \delta = 10^\circ$). With $Ma_1 = 2.0$, we solve Eq. 17–49 for the upstream Prandtl–Meyer function,

$$\begin{aligned} \nu(Ma) &= \sqrt{\frac{k+1}{k-1}} \tan^{-1} \left[\sqrt{\frac{k-1}{k+1}} (Ma^2 - 1) \right] - \tan^{-1} \left(\sqrt{Ma^2 - 1} \right) \\ &= \sqrt{\frac{1.4+1}{1.4-1}} \tan^{-1} \left[\sqrt{\frac{1.4-1}{1.4+1}} (2.0^2 - 1) \right] - \tan^{-1} \left(\sqrt{2.0^2 - 1} \right) = 26.38^\circ \end{aligned}$$

Next, we use Eq. 17–48 to calculate the downstream Prandtl–Meyer function,

$$\theta = \nu(Ma_2) - \nu(Ma_1) \rightarrow \nu(Ma_2) = \theta + \nu(Ma_1) = 10^\circ + 26.38^\circ = 36.38^\circ$$

Ma_2 is found by solving Eq. 17–49, which is implicit—an equation solver is helpful. We get $Ma_2 = 2.385$. There are also compressible flow calculators on the Internet that solve these implicit equations, along with both normal and oblique shock equations; e.g., see www.aoe.vt.edu/~devenpor/aoe3114/calc.html.

We use the isentropic relations to calculate the downstream pressure,

$$P_2 = \frac{P_2/P_0}{P_1/P_0} P_1 = \frac{\left[1 + \left(\frac{k-1}{2} \right) Ma_2^2 \right]^{-k/(k-1)}}{\left[1 + \left(\frac{k-1}{2} \right) Ma_1^2 \right]^{-k/(k-1)}} (230 \text{ kPa}) = \mathbf{126 \text{ kPa}}$$

Since this is an expansion, Mach number increases and pressure decreases, as expected.

Discussion We could also solve for downstream temperature, density, etc., using the appropriate isentropic relations.

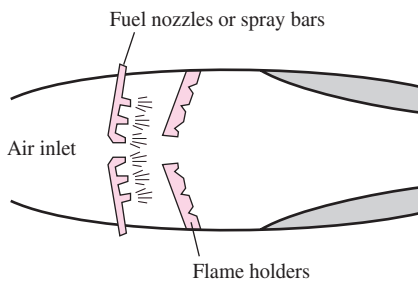


FIGURE 17–50

Many practical compressible flow problems involve combustion, which may be modeled as heat gain through the duct wall.

17–6 ■ DUCT FLOW WITH HEAT TRANSFER AND NEGLIGIBLE FRICTION (RAYLEIGH FLOW)

So far we have limited our consideration mostly to *isentropic flow*, also called *reversible adiabatic flow* since it involves no heat transfer and no irreversibilities such as friction. Many compressible flow problems encountered in practice involve chemical reactions such as combustion, nuclear reactions, evaporation, and condensation as well as heat gain or heat loss through the duct wall. Such problems are difficult to analyze exactly since they may involve significant changes in chemical composition during flow, and the conversion of latent, chemical, and nuclear energies to thermal energy (Fig. 17–50).

The essential features of such complex flows can still be captured by a simple analysis by modeling the generation or absorption of thermal energy

as heat transfer through the duct wall at the same rate and disregarding any changes in chemical composition. This simplified problem is still too complicated for an elementary treatment of the topic since the flow may involve friction, variations in duct area, and multidimensional effects. In this section, we limit our consideration to one-dimensional flow in a duct of constant cross-sectional area with negligible frictional effects.

Consider steady one-dimensional flow of an ideal gas with constant specific heats through a constant-area duct with heat transfer, but with negligible friction. Such flows are referred to as **Rayleigh flows** after Lord Rayleigh (1842–1919). The conservation of mass, momentum, and energy equations for the control volume shown in Fig. 17–51 can be written as follows:

Mass equation Noting that the duct cross-sectional area A is constant, the relation $\dot{m}_1 = \dot{m}_2$ or $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ reduces to

$$\rho_1 V_1 = \rho_2 V_2 \quad (17-50)$$

x-Momentum equation Noting that the frictional effects are negligible and thus there are no shear forces, and assuming there are no external

and body forces, the momentum equation $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$

in the flow (or x -) direction becomes a balance between static pressure forces and momentum transfer. Noting that the flows are high speed and turbulent, the momentum flux correction factor is approximately 1 ($\beta \cong 1$) and thus can be neglected. Then,

$$P_1 A_1 - P_2 A_2 = \dot{m} V_2 - \dot{m} V_1 \rightarrow P_1 - P_2 = (\rho_2 V_2) V_2 - (\rho_1 V_1) V_1$$

or

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 \quad (17-51)$$

Energy equation The control volume involves no shear, shaft, or other forms of work, and the potential energy change is negligible. If the rate of heat transfer is \dot{Q} and the heat transfer per unit mass of fluid is $q = \dot{Q}/\dot{m}$, the steady-flow energy balance $\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$ becomes

$$\dot{Q} + \dot{m} \left(h_1 + \frac{V_1^2}{2} \right) = \dot{m} \left(h_2 + \frac{V_2^2}{2} \right) \rightarrow q + h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad (17-52)$$

For an ideal gas with constant specific heats, $\Delta h = c_p \Delta T$, and thus

$$q = c_p (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} \quad (17-53)$$

or

$$q = h_{02} - h_{01} = c_p (T_{02} - T_{01}) \quad (17-54)$$

Therefore, the stagnation enthalpy h_0 and stagnation temperature T_0 change during Rayleigh flow (both increase when heat is transferred to the fluid and thus q is positive, and both decrease when heat is transferred from the fluid and thus q is negative).

Entropy change In the absence of any irreversibilities such as friction, the entropy of a system changes by heat transfer only: it increases with heat gain, and decreases with heat loss. Entropy is a property and thus

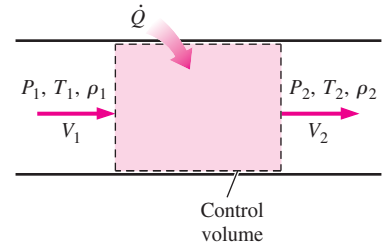


FIGURE 17-51

Control volume for flow in a constant-area duct with heat transfer and negligible friction.

a state function, and the entropy change of an ideal gas with constant specific heats during a change of state from 1 to 2 is given by

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad (17-55)$$

The entropy of a fluid may increase or decrease during Rayleigh flow, depending on the direction of heat transfer.

Equation of state Noting that $P = \rho RT$, the properties P , ρ , and T of an ideal gas at states 1 and 2 are related to each other by

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2} \quad (17-56)$$

Consider a gas with known properties R , k , and c_p . For a specified inlet state 1, the inlet properties P_1 , T_1 , ρ_1 , V_1 , and s_1 are known. The five exit properties P_2 , T_2 , ρ_2 , V_2 , and s_2 can be determined from the five equations 17-50, 17-51, 17-53, 17-55, and 17-56 for any specified value of heat transfer q . When the velocity and temperature are known, the Mach number can be determined from $Ma = V/c = V/\sqrt{kRT}$.

Obviously there is an infinite number of possible downstream states 2 corresponding to a given upstream state 1. A practical way of determining these downstream states is to assume various values of T_2 , and calculate all other properties as well as the heat transfer q for each assumed T_2 from the Eqs. 17-50 through 17-56. Plotting the results on a T - s diagram gives a curve passing through the specified inlet state, as shown in Fig. 17-52. The plot of Rayleigh flow on a T - s diagram is called the **Rayleigh line**, and several important observations can be made from this plot and the results of the calculations:

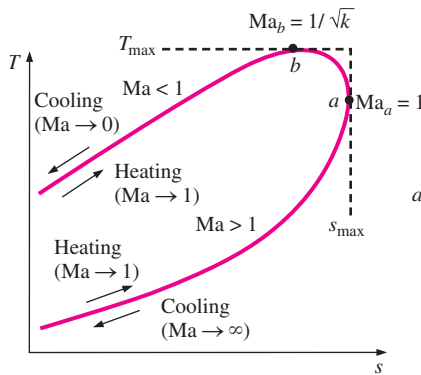


FIGURE 17-52

T - s diagram for flow in a constant-area duct with heat transfer and negligible friction (Rayleigh flow).

1. All the states that satisfy the conservation of mass, momentum, and energy equations as well as the property relations are on the Rayleigh line. Therefore, for a given initial state, the fluid cannot exist at any downstream state outside the Rayleigh line on a T - s diagram. In fact, the Rayleigh line is the locus of all physically attainable downstream states corresponding to an initial state.
2. Entropy increases with heat gain, and thus we proceed to the right on the Rayleigh line as heat is transferred to the fluid. The Mach number is $Ma = 1$ at point a , which is the point of maximum entropy (see Example 17-13 for proof). The states on the upper arm of the Rayleigh line above point a are subsonic, and the states on the lower arm below point a are supersonic. Therefore, a process proceeds to the right on the Rayleigh line with heat addition and to the left with heat rejection regardless of the initial value of the Mach number.
3. Heating increases the Mach number for subsonic flow, but decreases it for supersonic flow. The flow Mach number approaches $Ma = 1$ in both cases (from 0 in subsonic flow and from ∞ in supersonic flow) during heating.
4. It is clear from the energy balance $q = c_p(T_{02} - T_{01})$ that heating increases the stagnation temperature T_0 for both subsonic and supersonic flows, and cooling decreases it. (The maximum value of T_0 occurs at $Ma = 1$.) This is also the case for the thermodynamic temperature T

except for the narrow Mach number range of $1/\sqrt{k} < Ma < 1$ in subsonic flow (see Example 17–13). Both temperature and the Mach number increase with heating in subsonic flow, but T reaches a maximum T_{max} at $Ma = 1/\sqrt{k}$ (which is 0.845 for air), and then decreases. It may seem peculiar that the temperature of a fluid drops as heat is transferred to it. But this is no more peculiar than the fluid velocity increasing in the diverging section of a converging–diverging nozzle. The cooling effect in this region is due to the large increase in the fluid velocity and the accompanying drop in temperature in accordance with the relation $T_0 = T + V^2/2c_p$. Note also that heat rejection in the region $1/\sqrt{k} < Ma < 1$ causes the fluid temperature to increase (Fig. 17–53).

5. The momentum equation $P + KV = \text{constant}$, where $K = \rho V = \text{constant}$ (from the conservation of mass equation), reveals that velocity and static pressure have opposite trends. Therefore, static pressure decreases with heat gain in subsonic flow (since velocity and the Mach number increase), but increases with heat gain in supersonic flow (since velocity and the Mach number decrease).
6. The continuity equation $\rho V = \text{constant}$ indicates that density and velocity are inversely proportional. Therefore, density decreases with heat transfer to the fluid in subsonic flow (since velocity and the Mach number increase), but increases with heat gain in supersonic flow (since velocity and the Mach number decrease).
7. On the left half of Fig. 17–52, the lower arm of the Rayleigh line is steeper (in terms of s as a function of T), which indicates that the entropy change corresponding to a specified temperature change (and thus a given amount of heat transfer) is larger in supersonic flow.

The effects of heating and cooling on the properties of Rayleigh flow are listed in Table 17–3. Note that heating or cooling has opposite effects on most properties. Also, the stagnation pressure decreases during heating and increases during cooling regardless of whether the flow is subsonic or supersonic.

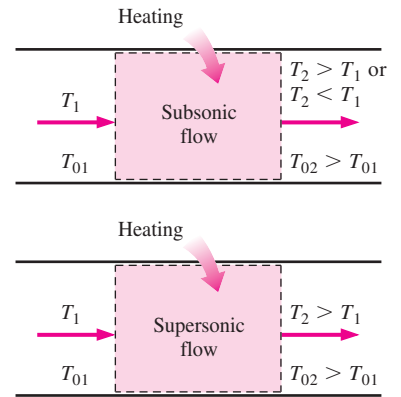


FIGURE 17–53 During heating, fluid temperature always increases if the Rayleigh flow is supersonic, but the temperature may actually drop if the flow is subsonic.

TABLE 17–3

The effects of heating and cooling on the properties of Rayleigh flow

Property	Heating		Cooling	
	Subsonic	Supersonic	Subsonic	Supersonic
Velocity, V	Increase	Decrease	Decrease	Increase
Mach number, Ma	Increase	Decrease	Decrease	Increase
Stagnation temperature, T_0	Increase	Increase	Decrease	Decrease
Temperature, T	Increase for $Ma < 1/k^{1/2}$ Decrease for $Ma > 1/k^{1/2}$	Increase	Decrease for $Ma < 1/k^{1/2}$ Increase for $Ma > 1/k^{1/2}$	Decrease
Density, ρ	Decrease	Increase	Increase	Decrease
Stagnation pressure, P_0	Decrease	Decrease	Increase	Increase
Pressure, P	Decrease	Increase	Increase	Decrease
Entropy, s	Increase	Increase	Decrease	Decrease

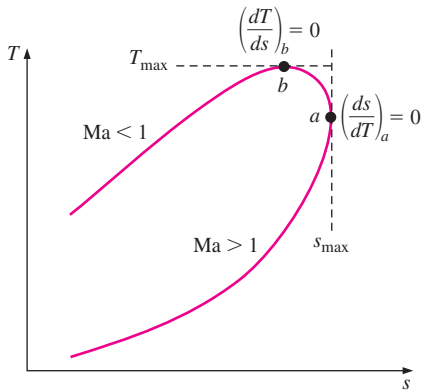


FIGURE 17-54

The T - s diagram of Rayleigh flow considered in Example 17-13.

EXAMPLE 17-13 Extrema of Rayleigh Line

Consider the T - s diagram of Rayleigh flow, as shown in Fig. 17-54. Using the differential forms of the conservation equations and property relations, show that the Mach number is $Ma_a = 1$ at the point of maximum entropy (point a), and $Ma_b = 1/\sqrt{k}$ at the point of maximum temperature (point b).

Solution It is to be shown that $Ma_a = 1$ at the point of maximum entropy and $Ma_b = 1/\sqrt{k}$ at the point of maximum temperature on the Rayleigh line.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional-area duct with negligible frictional effects) are valid.

Analysis The differential forms of the mass ($\rho V = \text{constant}$), momentum [rearranged as $P + (\rho V)V = \text{constant}$], ideal gas ($P = \rho RT$), and enthalpy change ($\Delta h = c_p \Delta T$) equations can be expressed as

$$\rho V = \text{constant} \rightarrow \rho dV + V d\rho = 0 \rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V} \quad (1)$$

$$P + (\rho V)V = \text{constant} \rightarrow dP + (\rho V) dV = 0 \rightarrow \frac{dP}{dV} = -\rho V \quad (2)$$

$$P = \rho RT \rightarrow dP = \rho R dT + RT d\rho \rightarrow \frac{dP}{P} = \frac{dT}{T} + \frac{d\rho}{\rho} \quad (3)$$

The differential form of the entropy change relation (Eq. 17-40) of an ideal gas with constant specific heats is

$$ds = c_p \frac{dT}{T} - R \frac{dP}{P} \quad (4)$$

Substituting Eq. 3 into Eq. 4 gives

$$ds = c_p \frac{dT}{T} - R \left(\frac{dT}{T} + \frac{d\rho}{\rho} \right) = (c_p - R) \frac{dT}{T} - R \frac{d\rho}{\rho} = \frac{R}{k-1} \frac{dT}{T} - R \frac{d\rho}{\rho} \quad (5)$$

since

$$c_p - R = c_v \rightarrow kc_v - R = c_v \rightarrow c_v = R/(k-1)$$

Dividing both sides of Eq. 5 by dT and combining with Eq. 1,

$$\frac{ds}{dT} = \frac{R}{T(k-1)} + \frac{R}{V} \frac{dV}{dT} \quad (6)$$

Dividing Eq. 3 by dV and combining it with Eqs. 1 and 2 give, after rearranging,

$$\frac{dT}{dV} = \frac{T}{V} - \frac{V}{R} \quad (7)$$

Substituting Eq. 7 into Eq. 6 and rearranging,

$$\frac{ds}{dT} = \frac{R}{T(k-1)} + \frac{R}{T - V^2/R} = \frac{R^2(kRT - V^2)}{T(k-1)(RT - V^2)} \quad (8)$$

Setting $ds/dT = 0$ and solving the resulting equation $R^2(kRT - V^2) = 0$ for V give the velocity at point a to be

$$V_a = \sqrt{kRT_a} \quad \text{and} \quad Ma_a = \frac{V_a}{c_a} = \frac{\sqrt{kRT_a}}{\sqrt{kRT_a}} = 1 \quad (9)$$

Therefore, sonic conditions exist at point a , and thus the Mach number is 1.

Setting $dT/ds = (ds/dT)^{-1} = 0$ and solving the resulting equation $T(k-1)(RT - V^2) = 0$ for velocity at point b give

$$V_b = \sqrt{RT_b} \quad \text{and} \quad \text{Ma}_b = \frac{V_b}{c_b} = \frac{\sqrt{RT_b}}{\sqrt{kRT_b}} = \frac{1}{\sqrt{k}} \quad (10)$$

Therefore, the Mach number at point b is $\text{Ma}_b = 1/\sqrt{k}$. For air, $k = 1.4$ and thus $\text{Ma}_b = 0.845$.

Discussion Note that in Rayleigh flow, sonic conditions are reached as the entropy reaches its maximum value, and maximum temperature occurs during subsonic flow.

EXAMPLE 17-14 Effect of Heat Transfer on Flow Velocity

Starting with the differential form of the energy equation, show that the flow velocity increases with heat addition in subsonic Rayleigh flow, but decreases in supersonic Rayleigh flow.

Solution It is to be shown that flow velocity increases with heat addition in subsonic Rayleigh flow and that the opposite occurs in supersonic flow.

Assumptions 1 The assumptions associated with Rayleigh flow are valid.

2 There are no work interactions and potential energy changes are negligible.

Analysis Consider heat transfer to the fluid in the differential amount of δq . The differential form of the energy equations can be expressed as

$$\delta q = dh_0 = d\left(h + \frac{V^2}{2}\right) = c_p dT + V dV \quad (1)$$

Dividing by $c_p T$ and factoring out dV/V give

$$\frac{\delta q}{c_p T} = \frac{dT}{T} + \frac{V dV}{c_p T} = \frac{dV}{V} \left[\frac{V}{dV} \frac{dT}{T} + \frac{(k-1)V^2}{kRT} \right] \quad (2)$$

where we also used $c_p = kR/(k-1)$. Noting that $\text{Ma}^2 = V^2/c^2 = V^2/kRT$ and using Eq. 7 for dT/dV from Example 17-13 give

$$\frac{\delta q}{c_p T} = \frac{dV}{V} \left[\frac{V}{T} \left(\frac{T}{V} - \frac{V}{R} \right) + (k-1)\text{Ma}^2 \right] = \frac{dV}{V} \left(1 - \frac{V^2}{TR} + k\text{Ma}^2 - \text{Ma}^2 \right) \quad (3)$$

Canceling the two middle terms in Eq. 3 since $V^2/TR = k\text{Ma}^2$ and rearranging give the desired relation,

$$\frac{dV}{V} = \frac{\delta q}{c_p T} \frac{1}{(1 - \text{Ma}^2)} \quad (4)$$

In subsonic flow, $1 - \text{Ma}^2 > 0$ and thus heat transfer and velocity change have the same sign. As a result, heating the fluid ($\delta q > 0$) increases the flow velocity while cooling decreases it. In supersonic flow, however, $1 - \text{Ma}^2 < 0$ and heat transfer and velocity change have opposite signs. As a result, heating the fluid ($\delta q > 0$) decreases the flow velocity while cooling increases it (Fig. 17-55).

Discussion Note that heating the fluid has the opposite effect on flow velocity in subsonic and supersonic Rayleigh flows.

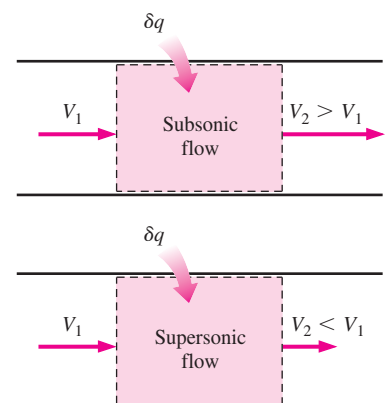


FIGURE 17-55

Heating increases the flow velocity in subsonic flow, but decreases it in supersonic flow.

Property Relations for Rayleigh Flow

It is often desirable to express the variations in properties in terms of the Mach number Ma . Noting that $Ma = V/c = V/\sqrt{kRT}$ and thus $V = Ma\sqrt{kRT}$,

$$\rho V^2 = \rho kRT Ma^2 = kP Ma^2 \quad (17-57)$$

since $P = \rho RT$. Substituting into the momentum equation (Eq. 17-51) gives $P_1 + kP_1 Ma_1^2 = P_2 + kP_2 Ma_2^2$, which can be rearranged as

$$\frac{P_2}{P_1} = \frac{1 + kMa_1^2}{1 + kMa_2^2} \quad (17-58)$$

Again utilizing $V = Ma\sqrt{kRT}$, the continuity equation $\rho_1 V_1 = \rho_2 V_2$ can be expressed as

$$\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} = \frac{Ma_2 \sqrt{kRT_2}}{Ma_1 \sqrt{kRT_1}} = \frac{Ma_2 \sqrt{T_2}}{Ma_1 \sqrt{T_1}} \quad (17-59)$$

Then the ideal-gas relation (Eq. 17-56) becomes

$$\frac{T_2}{T_1} = \frac{P_2 \rho_1}{P_1 \rho_2} = \left(\frac{1 + kMa_1^2}{1 + kMa_2^2} \right) \left(\frac{Ma_2 \sqrt{T_2}}{Ma_1 \sqrt{T_1}} \right) \quad (17-60)$$

Solving Eq. 17-60 for the temperature ratio T_2/T_1 gives

$$\frac{T_2}{T_1} = \left[\frac{Ma_2(1 + kMa_1^2)}{Ma_1(1 + kMa_2^2)} \right]^2 \quad (17-61)$$

Substituting this relation into Eq. 17-59 gives the density or velocity ratio as

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{Ma_1^2(1 + kMa_2^2)}{Ma_2^2(1 + kMa_1^2)} \quad (17-62)$$

Flow properties at sonic conditions are usually easy to determine, and thus the critical state corresponding to $Ma = 1$ serves as a convenient reference point in compressible flow. Taking state 2 to be the sonic state ($Ma_2 = 1$, and superscript * is used) and state 1 to be any state (no subscript), the property relations in Eqs. 17-58, 17-61, and 17-62 reduce to (Fig. 17-56)

$$\frac{P}{P^*} = \frac{1 + k}{1 + kMa^2} \quad \frac{T}{T^*} = \left[\frac{Ma(1 + k)}{1 + kMa^2} \right]^2 \quad \text{and} \quad \frac{V}{V^*} = \frac{\rho^*}{\rho} = \frac{(1 + k)Ma^2}{1 + kMa^2} \quad (17-63)$$

Similar relations can be obtained for dimensionless stagnation temperature and stagnation pressure as follows:

$$\frac{T_0}{T_0^*} = \frac{T_0}{T} \frac{T}{T^*} = \left(1 + \frac{k-1}{2} Ma^2 \right) \left[\frac{Ma(1 + k)}{1 + kMa^2} \right]^2 \left(1 + \frac{k-1}{2} \right)^{-1} \quad (17-64)$$

which simplifies to

$$\frac{T_0}{T_0^*} = \frac{(k + 1)Ma^2[2 + (k - 1)Ma^2]}{(1 + kMa^2)^2} \quad (17-65)$$

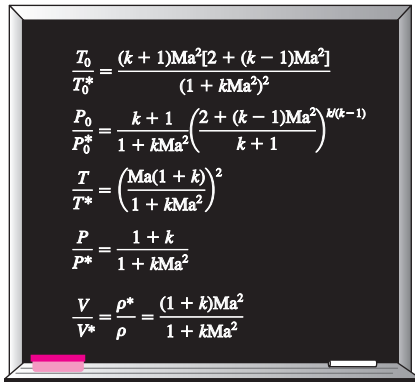


FIGURE 17-56

Summary of relations for Rayleigh flow.

Also,

$$\frac{P_0}{P_0^*} = \frac{P_0}{P} \frac{P}{P^*} \frac{P^*}{P_0^*} = \left(1 + \frac{k-1}{2} \text{Ma}^2\right)^{k/(k-1)} \left(\frac{1+k}{1+k\text{Ma}^2}\right) \left(1 + \frac{k-1}{2}\right)^{-k/(k-1)} \quad (17-66)$$

which simplifies to

$$\frac{P_0}{P_0^*} = \frac{k+1}{1+k\text{Ma}^2} \left[\frac{2+(k-1)\text{Ma}^2}{k+1} \right]^{k/(k-1)} \quad (17-67)$$

The five relations in Eqs. 17-63, 17-65, and 17-67 enable us to calculate the dimensionless pressure, temperature, density, velocity, stagnation temperature, and stagnation pressure for Rayleigh flow of an ideal gas with a specified k for any given Mach number. Representative results are given in tabular form in Table A-34 for $k = 1.4$.

Choked Rayleigh Flow

It is clear from the earlier discussions that subsonic Rayleigh flow in a duct may accelerate to sonic velocity ($\text{Ma} = 1$) with heating. What happens if we continue to heat the fluid? Does the fluid continue to accelerate to supersonic velocities? An examination of the Rayleigh line indicates that the fluid at the critical state of $\text{Ma} = 1$ cannot be accelerated to supersonic velocities by heating. Therefore, the flow is *choked*. This is analogous to not being able to accelerate a fluid to supersonic velocities in a converging nozzle by simply extending the converging flow section. If we keep heating the fluid, we will simply move the critical state further downstream and reduce the flow rate since fluid density at the critical state will now be lower. Therefore, for a given inlet state, the corresponding critical state fixes the maximum possible heat transfer for steady flow (Fig. 17-57). That is,

$$q_{\max} = h_0^* - h_{01} = c_p(T_0^* - T_{01}) \quad (17-68)$$

Further heat transfer causes choking and thus the inlet state to change (e.g., inlet velocity will decrease), and the flow no longer follows the same Rayleigh line. Cooling the subsonic Rayleigh flow reduces the velocity, and the Mach number approaches zero as the temperature approaches absolute zero. Note that the stagnation temperature T_0 is maximum at the critical state of $\text{Ma} = 1$.

In supersonic Rayleigh flow, heating decreases the flow velocity. Further heating simply increases the temperature and moves the critical state further downstream, resulting in a reduction in the mass flow rate of the fluid. It may seem like supersonic Rayleigh flow can be cooled indefinitely, but it turns out that there is a limit. Taking the limit of Eq. 17-65 as the Mach number approaches infinity gives

$$\lim_{\text{Ma} \rightarrow \infty} \frac{T_0}{T_0^*} = 1 - \frac{1}{k^2} \quad (17-69)$$

which yields $T_0/T_0^* = 0.49$ for $k = 1.4$. Therefore, if the critical stagnation temperature is 1000 K, air cannot be cooled below 490 K in Rayleigh flow. Physically this means that the flow velocity reaches infinity by the time the temperature reaches 490 K—a physical impossibility. When supersonic flow cannot be sustained, the flow undergoes a normal shock wave and becomes subsonic.

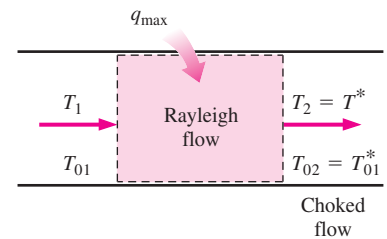


FIGURE 17-57

For a given inlet state, the maximum possible heat transfer occurs when sonic conditions are reached at the exit state.

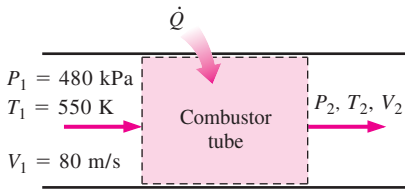


FIGURE 17-58

Schematic of the combustor tube analyzed in Example 17-15.

EXAMPLE 17-15 Rayleigh Flow in a Tubular Combustor

A combustion chamber consists of tubular combustors of 15-cm diameter. Compressed air enters the tubes at 550 K, 480 kPa, and 80 m/s (Fig. 17-58). Fuel with a heating value of 42,000 kJ/kg is injected into the air and is burned with an air–fuel mass ratio of 40. Approximating combustion as a heat transfer process to air, determine the temperature, pressure, velocity, and Mach number at the exit of the combustion chamber.

Solution Fuel is burned in a tubular combustion chamber with compressed air. The exit temperature, pressure, velocity, and Mach number are to be determined.

Assumptions 1 The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional-area duct with negligible frictional effects) are valid. 2 Combustion is complete, and it is treated as a heat transfer process, with no change in the chemical composition of the flow. 3 The increase in mass flow rate due to fuel injection is disregarded.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$, and $R = 0.287 \text{ kJ/kg} \cdot \text{K}$ (Table A-2a).

Analysis The inlet density and mass flow rate of air are

$$\rho_1 = \frac{P_1}{RT_1} = \frac{480 \text{ kPa}}{(0.287 \text{ kJ/kg} \cdot \text{K})(550 \text{ K})} = 3.041 \text{ kg/m}^3$$

$$\dot{m}_{\text{air}} = \rho_1 A_1 V_1 = (3.041 \text{ kg/m}^3)[\pi(0.15 \text{ m})^2/4](80 \text{ m/s}) = 4.299 \text{ kg/s}$$

The mass flow rate of fuel and the rate of heat transfer are

$$\dot{m}_{\text{fuel}} = \frac{\dot{m}_{\text{air}}}{\text{AF}} = \frac{4.299 \text{ kg/s}}{40} = 0.1075 \text{ kg/s}$$

$$\dot{Q} = \dot{m}_{\text{fuel}} \text{HV} = (0.1075 \text{ kg/s})(42,000 \text{ kJ/kg}) = 4515 \text{ kW}$$

$$q = \frac{\dot{Q}}{\dot{m}_{\text{air}}} = \frac{4515 \text{ kJ/s}}{4.299 \text{ kg/s}} = 1050 \text{ kJ/kg}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 550 \text{ K} + \frac{(80 \text{ m/s})^2}{2(1.005 \text{ kJ/kg} \cdot \text{K})} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 553.2 \text{ K}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(550 \text{ K})} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right) = 470.1 \text{ m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{80 \text{ m/s}}{470.1 \text{ m/s}} = 0.1702$$

The exit stagnation temperature is, from the energy equation $q = c_p(T_{02} - T_{01})$,

$$T_{02} = T_{01} + \frac{q}{c_p} = 553.2 \text{ K} + \frac{1050 \text{ kJ/kg}}{1.005 \text{ kJ/kg} \cdot \text{K}} = 1598 \text{ K}$$

The maximum value of stagnation temperature T_0^* occurs at $Ma = 1$, and its value can be determined from Table A-34 or from Eq. 17-65. At $Ma_1 = 0.1702$ we read $T_0/T_0^* = 0.1291$. Therefore,

$$T_0^* = \frac{T_{01}}{0.1291} = \frac{553.2 \text{ K}}{0.1291} = 4285 \text{ K}$$

The stagnation temperature ratio at the exit state and the Mach number corresponding to it are, from Table A-34,

$$\frac{T_{02}}{T_0^*} = \frac{1598 \text{ K}}{4285 \text{ K}} = 0.3729 \rightarrow Ma_2 = \mathbf{0.3142}$$

The Rayleigh flow relations corresponding to the inlet and exit Mach numbers are (Table A-34):

$$Ma_1 = 0.1702: \quad \frac{T_1}{T^*} = 0.1541 \quad \frac{P_1}{P^*} = 2.3065 \quad \frac{V_1}{V^*} = 0.0668$$

$$Ma_2 = 0.3142: \quad \frac{T_2}{T^*} = 0.4389 \quad \frac{P_2}{P^*} = 2.1086 \quad \frac{V_2}{V^*} = 0.2082$$

Then the exit temperature, pressure, and velocity are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.4389}{0.1541} = 2.848 \rightarrow T_2 = 2.848T_1 = 2.848(550 \text{ K}) = \mathbf{1566 \text{ K}}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{2.1086}{2.3065} = 0.9142 \rightarrow P_2 = 0.9142P_1 = 0.9142(480 \text{ kPa}) = \mathbf{439 \text{ kPa}}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{0.2082}{0.0668} = 3.117 \rightarrow V_2 = 3.117V_1 = 3.117(80 \text{ m/s}) = \mathbf{249 \text{ m/s}}$$

Discussion Note that the temperature and velocity increase and pressure decreases during this subsonic Rayleigh flow with heating, as expected. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

17-7 ■ STEAM NOZZLES

We have seen in Chapter 3 that water vapor at moderate or high pressures deviates considerably from ideal-gas behavior, and thus most of the relations developed in this chapter are not applicable to the flow of steam through the nozzles or blade passages encountered in steam turbines. Given that the steam properties such as enthalpy are functions of pressure as well as temperature and that no simple property relations exist, an accurate analysis of steam flow through the nozzles is no easy matter. Often it becomes necessary to use steam tables, an h - s diagram, or a computer program for the properties of steam.

A further complication in the expansion of steam through nozzles occurs as the steam expands into the saturation region, as shown in Fig. 17-59. As the steam expands in the nozzle, its pressure and temperature drop, and

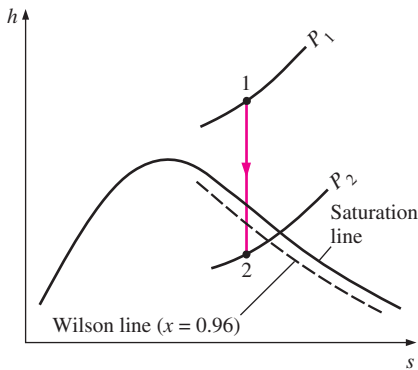


FIGURE 17-59

The h - s diagram for the isentropic expansion of steam in a nozzle.

ordinarily one would expect the steam to start condensing when it strikes the saturation line. However, this is not always the case. Owing to the high speeds, the residence time of the steam in the nozzle is small, and there may not be sufficient time for the necessary heat transfer and the formation of liquid droplets. Consequently, the condensation of the steam may be delayed for a little while. This phenomenon is known as **supersaturation**, and the steam that exists in the wet region without containing any liquid is called **supersaturated steam**. Supersaturation states are nonequilibrium (or metastable) states.

During the expansion process, the steam reaches a temperature lower than that normally required for the condensation process to begin. Once the temperature drops a sufficient amount below the saturation temperature corresponding to the local pressure, groups of steam moisture droplets of sufficient size are formed, and condensation occurs rapidly. The locus of points where condensation takes place regardless of the initial temperature and pressure at the nozzle entrance is called the **Wilson line**. The Wilson line lies between the 4 and 5 percent moisture curves in the saturation region on the h - s diagram for steam, and it is often approximated by the 4 percent moisture line. Therefore, steam flowing through a high-velocity nozzle is assumed to begin condensation when the 4 percent moisture line is crossed.

The critical-pressure ratio P^*/P_0 for steam depends on the nozzle inlet state as well as on whether the steam is superheated or saturated at the nozzle inlet. However, the ideal-gas relation for the critical-pressure ratio, Eq. 17-22, gives reasonably good results over a wide range of inlet states. As indicated in Table 17-2, the specific heat ratio of superheated steam is approximated as $k = 1.3$. Then the critical-pressure ratio becomes

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1} \right)^{k/(k-1)} = 0.546$$

When steam enters the nozzle as a saturated vapor instead of superheated vapor (a common occurrence in the lower stages of a steam turbine), the critical-pressure ratio is taken to be 0.576, which corresponds to a specific heat ratio of $k = 1.14$.

EXAMPLE 17-16 Steam Flow through a Converging–Diverging Nozzle

Steam enters a converging–diverging nozzle at 2 MPa and 400°C with a negligible velocity and a mass flow rate of 2.5 kg/s, and it exits at a pressure of 300 kPa. The flow is isentropic between the nozzle entrance and throat, and the overall nozzle efficiency is 93 percent. Determine (a) the throat and exit areas and (b) the Mach number at the throat and the nozzle exit.

Solution Steam enters a converging–diverging nozzle with a low velocity. The throat and exit areas and the Mach number are to be determined.

Assumptions 1 Flow through the nozzle is one-dimensional. 2 The flow is isentropic between the inlet and the throat, and is adiabatic and irreversible between the throat and the exit. 3 The inlet velocity is negligible.

Analysis We denote the entrance, throat, and exit states by 1, t , and 2, respectively, as shown in Fig. 17–60.

(a) Since the inlet velocity is negligible, the inlet stagnation and static states are identical. The ratio of the exit-to-inlet stagnation pressure is

$$\frac{P_2}{P_{01}} = \frac{300 \text{ kPa}}{2000 \text{ kPa}} = 0.15$$

It is much smaller than the critical-pressure ratio, which is taken to be $P^*/P_{01} = 0.546$ since the steam is superheated at the nozzle inlet. Therefore, the flow surely is supersonic at the exit. Then the velocity at the throat is the sonic velocity, and the throat pressure is

$$P_t = 0.546P_{01} = (0.546)(2 \text{ MPa}) = 1.09 \text{ MPa}$$

At the inlet,

$$\left. \begin{array}{l} P_1 = P_{01} = 2 \text{ MPa} \\ T_1 = T_{01} = 400^\circ\text{C} \end{array} \right\} \begin{array}{l} h_1 = h_{01} = 3248.4 \text{ kJ/kg} \\ s_1 = s_t = s_{2s} = 7.1292 \text{ kJ/kg} \cdot \text{K} \end{array}$$

Also, at the throat,

$$\left. \begin{array}{l} P_t = 1.09 \text{ MPa} \\ s_t = 7.1292 \text{ kJ/kg} \cdot \text{K} \end{array} \right\} \begin{array}{l} h_t = 3076.8 \text{ kJ/kg} \\ v_t = 0.24196 \text{ m}^3/\text{kg} \end{array}$$

Then the throat velocity is determined from Eq. 17–3 to be

$$V_t = \sqrt{2(h_{01} - h_t)} = \sqrt{[2(3248.4 - 3076.8) \text{ kJ/kg}] \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 585.8 \text{ m/s}$$

The flow area at the throat is determined from the mass flow rate relation:

$$A_t = \frac{\dot{m}v_t}{V_t} = \frac{(2.5 \text{ kg/s})(0.2420 \text{ m}^3/\text{kg})}{585.8 \text{ m/s}} = 10.33 \times 10^{-4} \text{ m}^2 = \mathbf{10.33 \text{ cm}^2}$$

At state $2s$,

$$\left. \begin{array}{l} P_{2s} = P_2 = 300 \text{ kPa} \\ s_{2s} = s_1 = 7.1292 \text{ kJ/kg} \cdot \text{K} \end{array} \right\} h_{2s} = 2783.6 \text{ kJ/kg}$$

The enthalpy of the steam at the actual exit state is (see Chap. 7)

$$\eta_N = \frac{h_{01} - h_2}{h_{01} - h_{2s}}$$

$$0.93 = \frac{3248.4 - h_2}{3248.4 - 2783.6} \longrightarrow h_2 = 2816.1 \text{ kJ/kg}$$

Therefore,

$$\left. \begin{array}{l} P_2 = 300 \text{ kPa} \\ h_2 = 2816.1 \text{ kJ/kg} \end{array} \right\} \begin{array}{l} v_2 = 0.67723 \text{ m}^3/\text{kg} \\ s_2 = 7.2019 \text{ kJ/kg} \cdot \text{K} \end{array}$$

Then the exit velocity and the exit area become

$$V_2 = \sqrt{2(h_{01} - h_2)} = \sqrt{[2(3248.4 - 2816.1) \text{ kJ/kg}] \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 929.8 \text{ m/s}$$

$$A_2 = \frac{\dot{m}v_2}{V_2} = \frac{(2.5 \text{ kg/s})(0.67723 \text{ m}^3/\text{kg})}{929.8 \text{ m/s}} = 18.21 \times 10^{-4} \text{ m}^2 = \mathbf{18.21 \text{ cm}^2}$$

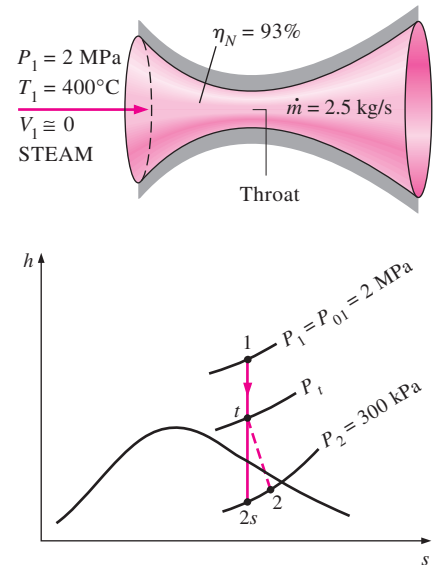


FIGURE 17–60

Schematic and h - s diagram for Example 17–16.

(b) The velocity of sound and the Mach numbers at the throat and the exit of the nozzle are determined by replacing differential quantities with differences,

$$c = \left(\frac{\partial P}{\partial \rho} \right)_s \cong \left[\frac{\Delta P}{\Delta(1/v)} \right]_s^{1/2}$$

The velocity of sound at the throat is determined by evaluating the specific volume at $s_t = 7.1292 \text{ kJ/kg} \cdot \text{K}$ and at pressures of 1.115 and 1.065 MPa ($P_t \pm 25 \text{ kPa}$):

$$c = \sqrt{\frac{(1115 - 1065) \text{ kPa}}{(1/0.23776 - 1/0.24633) \text{ kg/m}^3} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kPa} \cdot \text{m}^3/\text{kg}} \right)} = 584.6 \text{ m/s}$$

The Mach number at the throat is determined from Eq. 17–12 to be

$$\text{Ma} = \frac{V}{c} = \frac{585.8 \text{ m/s}}{584.6 \text{ m/s}} = \mathbf{1.002}$$

Thus, the flow at the throat is sonic, as expected. The slight deviation of the Mach number from unity is due to replacing the derivatives by differences.

The velocity of sound and the Mach number at the nozzle exit are determined by evaluating the specific volume at $s_2 = 7.2019 \text{ kJ/kg} \cdot \text{K}$ and at pressures of 325 and 275 kPa ($P_2 \pm 25 \text{ kPa}$):

$$c = \sqrt{\frac{(325 - 275) \text{ kPa}}{(1/0.63596 - 1/0.72245) \text{ kg/m}^3} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kPa} \cdot \text{m}^3/\text{kg}} \right)} = 515.4 \text{ m/s}$$

and

$$\text{Ma} = \frac{V}{c} = \frac{929.8 \text{ m/s}}{515.4 \text{ m/s}} = \mathbf{1.804}$$

Thus the flow of steam at the nozzle exit is supersonic.

SUMMARY

In this chapter the effects of compressibility on gas flow are examined. When dealing with compressible flow, it is convenient to combine the enthalpy and the kinetic energy of the fluid into a single term called *stagnation* (or *total*) *enthalpy* h_0 , defined as

$$h_0 = h + \frac{V^2}{2}$$

The properties of a fluid at the stagnation state are called *stagnation properties* and are indicated by the subscript zero. The *stagnation temperature* of an ideal gas with constant specific heats is

$$T_0 = T + \frac{V^2}{2c_p}$$

which represents the temperature an ideal gas would attain if it is brought to rest adiabatically. The stagnation properties of an ideal gas are related to the static properties of the fluid by

$$\frac{P_0}{P} = \left(\frac{T_0}{T} \right)^{k/(k-1)} \quad \text{and} \quad \frac{\rho_0}{\rho} = \left(\frac{T_0}{T} \right)^{1/(k-1)}$$

The speed at which an infinitesimally small pressure wave travels through a medium is the *speed of sound*. For an ideal gas it is expressed as

$$c = \sqrt{\left(\frac{\partial P}{\partial \rho} \right)_s} = \sqrt{kRT}$$

The *Mach number* is the ratio of the actual velocity of the fluid to the speed of sound at the same state:

$$\text{Ma} = \frac{V}{c}$$

The flow is called *sonic* when $\text{Ma} = 1$, *subsonic* when $\text{Ma} < 1$, *supersonic* when $\text{Ma} > 1$, *hypersonic* when $\text{Ma} \gg 1$, and *transonic* when $\text{Ma} \cong 1$.

Nozzles whose flow area decreases in the flow direction are called *converging nozzles*. Nozzles whose flow area first decreases and then increases are called *converging–diverging nozzles*. The location of the smallest flow area of a nozzle is called the *throat*. The highest velocity to which a fluid can be accelerated in a converging nozzle is the sonic velocity. Accelerating a fluid to supersonic velocities is possible only in converging–diverging nozzles. In all supersonic converging–diverging nozzles, the flow velocity at the throat is the speed of sound.

The ratios of the stagnation to static properties for ideal gases with constant specific heats can be expressed in terms of the Mach number as

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right) \text{Ma}^2$$

$$\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2}\right) \text{Ma}^2\right]^{k/(k-1)}$$

and
$$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{k-1}{2}\right) \text{Ma}^2\right]^{1/(k-1)}$$

When $\text{Ma} = 1$, the resulting static-to-stagnation property ratios for the temperature, pressure, and density are called *critical ratios* and are denoted by the superscript asterisk:

$$\frac{T^*}{T_0} = \frac{2}{k+1} \quad \frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

and
$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{1/(k-1)}$$

The pressure outside the exit plane of a nozzle is called the *back pressure*. For all back pressures lower than P^* , the pres-

sure at the exit plane of the converging nozzle is equal to P^* , the Mach number at the exit plane is unity, and the mass flow rate is the maximum (or choked) flow rate.

In some range of back pressure, the fluid that achieved a sonic velocity at the throat of a converging–diverging nozzle and is accelerating to supersonic velocities in the diverging section experiences a *normal shock*, which causes a sudden rise in pressure and temperature and a sudden drop in velocity to subsonic levels. Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic. The properties of an ideal gas with constant specific heats before (subscript 1) and after (subscript 2) a shock are related by

$$T_{01} = T_{02} \quad \text{Ma}_2 = \sqrt{\frac{(k-1)\text{Ma}_1^2 + 2}{2k\text{Ma}_1^2 - k + 1}}$$

$$\frac{T_2}{T_1} = \frac{2 + \text{Ma}_1^2(k-1)}{2 + \text{Ma}_2^2(k-1)}$$

and
$$\frac{P_2}{P_1} = \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{2k\text{Ma}_1^2 - k + 1}{k + 1}$$

These equations also hold across an oblique shock, provided that the component of the Mach number *normal* to the oblique shock is used in place of the Mach number.

Steady one-dimensional flow of an ideal gas with constant specific heats through a constant-area duct with heat transfer and negligible friction is referred to as *Rayleigh flow*. The property relations and curves for Rayleigh flow are given in Table A–34. Heat transfer during Rayleigh flow can be determined from

$$q = c_p(T_{02} - T_{01}) = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

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PROBLEMS*

Stagnation Properties

17-1C A high-speed aircraft is cruising in still air. How will the temperature of air at the nose of the aircraft differ from the temperature of air at some distance from the aircraft?

17-2C How and why is the stagnation enthalpy h_0 defined? How does it differ from ordinary (static) enthalpy?

17-3C What is dynamic temperature?

17-4C In air-conditioning applications, the temperature of air is measured by inserting a probe into the flow stream. Thus, the probe actually measures the stagnation temperature. Does this cause any significant error?

17-5 Determine the stagnation temperature and stagnation pressure of air that is flowing at 44 kPa, 245.9 K, and 470 m/s. *Answers: 355.8 K, 160.3 kPa*

17-6 Air at 300 K is flowing in a duct at a velocity of (a) 1, (b) 10, (c) 100, and (d) 1000 m/s. Determine the temperature that a stationary probe inserted into the duct will read for each case.

17-7 Calculate the stagnation temperature and pressure for the following substances flowing through a duct: (a) helium at 0.25 MPa, 50°C, and 240 m/s; (b) nitrogen at 0.15 MPa, 50°C, and 300 m/s; and (c) steam at 0.1 MPa, 350°C, and 480 m/s.

17-8 Air enters a compressor with a stagnation pressure of 100 kPa and a stagnation temperature of 27°C, and it is compressed to a stagnation pressure of 900 kPa. Assuming the compression process to be isentropic, determine the power input to the compressor for a mass flow rate of 0.02 kg/s. *Answer: 5.27 kW*

17-9E Steam flows through a device with a stagnation pressure of 120 psia, a stagnation temperature of 700°F, and a velocity of 900 ft/s. Assuming ideal-gas behavior, determine the static pressure and temperature of the steam at this state.

17-10 Products of combustion enter a gas turbine with a stagnation pressure of 1.0 MPa and a stagnation temperature of 750°C, and they expand to a stagnation pressure of 100 kPa. Taking $k = 1.33$ and $R = 0.287$ kJ/kg · K for the products of combustion, and assuming the expansion process to

be isentropic, determine the power output of the turbine per unit mass flow.

17-11 Air flows through a device such that the stagnation pressure is 0.6 MPa, the stagnation temperature is 400°C, and the velocity is 570 m/s. Determine the static pressure and temperature of the air at this state. *Answers: 518.6 K, 0.23 MPa*

Speed of Sound and Mach Number

17-12C What is sound? How is it generated? How does it travel? Can sound waves travel in a vacuum?

17-13C Is it realistic to assume that the propagation of sound waves is an isentropic process? Explain.

17-14C Is the sonic velocity in a specified medium a fixed quantity, or does it change as the properties of the medium change? Explain.

17-15C In which medium does a sound wave travel faster: in cool air or in warm air?

17-16C In which medium will sound travel fastest for a given temperature: air, helium, or argon?

17-17C In which medium does a sound wave travel faster: in air at 20°C and 1 atm or in air at 20°C and 5 atm?

17-18C Does the Mach number of a gas flowing at a constant velocity remain constant? Explain.



17-19 Determine the speed of sound in air at (a) 300 K and (b) 1000 K. Also determine the Mach number of an aircraft moving in air at a velocity of 280 m/s for both cases.

17-20 Carbon dioxide enters an adiabatic nozzle at 1200 K with a velocity of 50 m/s and leaves at 400 K. Assuming constant specific heats at room temperature, determine the Mach number (a) at the inlet and (b) at the exit of the nozzle. Assess the accuracy of the constant specific heat assumption. *Answers: (a) 0.0925, (b) 3.73*


17-21 Nitrogen enters a steady-flow heat exchanger at 150 kPa, 10°C, and 100 m/s, and it receives heat in the amount of 120 kJ/kg as it flows through it. Nitrogen leaves the heat exchanger at 100 kPa with a velocity of 200 m/s. Determine the Mach number of the nitrogen at the inlet and the exit of the heat exchanger.

17-22 Assuming ideal-gas behavior, determine the speed of sound in refrigerant-134a at 0.1 MPa and 60°C.

17-23 The Airbus A-340 passenger plane has a maximum takeoff weight of about 260,000 kg, a length of 64 m, a wing span of 60 m, a maximum cruising speed of 945 km/h, a seating capacity of 271 passengers, maximum cruising altitude of 14,000 m, and a maximum range of 12,000 km. The air temperature at the cruising altitude is about -60°C. Determine the Mach number of this plane for the stated limiting conditions.

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with a CD-EES icon  are solved using EES, and complete solutions together with parametric studies are included on the enclosed DVD. Problems with a computer-EES icon  are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

17-24E Steam flows through a device with a pressure of 120 psia, a temperature of 700°F, and a velocity of 900 ft/s. Determine the Mach number of the steam at this state by assuming ideal-gas behavior with $k = 1.3$. *Answer: 0.441*

17-25E  Reconsider Prob. 17-24E. Using EES (or other) software, compare the Mach number of steam flow over the temperature range 350 to 700°F. Plot the Mach number as a function of temperature.

17-26 The isentropic process for an ideal gas is expressed as $PV^k = \text{constant}$. Using this process equation and the definition of the speed of sound (Eq. 17-9), obtain the expression for the speed of sound for an ideal gas (Eq. 17-11).

17-27 Air expands isentropically from 1.5 MPa and 60°C to 0.4 MPa. Calculate the ratio of the initial to final speed of sound. *Answer: 1.21*

17-28 Repeat Prob. 17-27 for helium gas.

17-29E Air expands isentropically from 170 psia and 200°F to 60 psia. Calculate the ratio of the initial to final speed of sound. *Answer: 1.16*

One-Dimensional Isentropic Flow

17-30C Consider a converging nozzle with sonic velocity at the exit plane. Now the nozzle exit area is reduced while the nozzle inlet conditions are maintained constant. What will happen to (a) the exit velocity and (b) the mass flow rate through the nozzle?

17-31C A gas initially at a supersonic velocity enters an adiabatic converging duct. Discuss how this affects (a) the velocity, (b) the temperature, (c) the pressure, and (d) the density of the fluid.

17-32C A gas initially at a supersonic velocity enters an adiabatic diverging duct. Discuss how this affects (a) the velocity, (b) the temperature, (c) the pressure, and (d) the density of the fluid.

17-33C A gas initially at a supersonic velocity enters an adiabatic converging duct. Discuss how this affects (a) the velocity, (b) the temperature, (c) the pressure, and (d) the density of the fluid.

17-34C A gas initially at a subsonic velocity enters an adiabatic diverging duct. Discuss how this affects (a) the velocity, (b) the temperature, (c) the pressure, and (d) the density of the fluid.

17-35C A gas at a specified stagnation temperature and pressure is accelerated to $Ma = 2$ in a converging–diverging nozzle and to $Ma = 3$ in another nozzle. What can you say about the pressures at the throats of these two nozzles?

17-36C Is it possible to accelerate a gas to a supersonic velocity in a converging nozzle?

17-37 Air enters a converging–diverging nozzle at a pressure of 1.2 MPa with negligible velocity. What is the lowest


pressure that can be obtained at the throat of the nozzle? *Answer: 634 kPa*

17-38 Helium enters a converging–diverging nozzle at 0.7 MPa, 800 K, and 100 m/s. What are the lowest temperature and pressure that can be obtained at the throat of the nozzle?

17-39 Calculate the critical temperature, pressure, and density of (a) air at 200 kPa, 100°C, and 250 m/s, and (b) helium at 200 kPa, 40°C, and 300 m/s.

17-40 Quiescent carbon dioxide at 600 kPa and 400 K is accelerated isentropically to a Mach number of 0.5. Determine the temperature and pressure of the carbon dioxide after acceleration. *Answers: 388 K, 514 kPa*

17-41 Air at 200 kPa, 100°C, and Mach number $Ma = 0.8$ flows through a duct. Find the velocity and the stagnation pressure, temperature, and density of the air.

17-42  Reconsider Prob. 17-41. Using EES (or other) software, study the effect of Mach numbers in the range 0.1 to 2 on the velocity, stagnation pressure, temperature, and density of air. Plot each parameter as a function of the Mach number.

17-43E Air at 30 psia, 212°F, and Mach number $Ma = 0.8$ flows through a duct. Calculate the velocity and the stagnation pressure, temperature, and density of air.

Answers: 1016 ft/s, 45.7 psia, 758 R, 0.163 lbm/ft³

17-44 An aircraft is designed to cruise at Mach number $Ma = 1.2$ at 8000 m where the atmospheric temperature is 236.15 K. Determine the stagnation temperature on the leading edge of the wing.

Isentropic Flow through Nozzles

17-45C Consider subsonic flow in a converging nozzle with fixed inlet conditions. What is the effect of dropping the back pressure to the critical pressure on (a) the exit velocity, (b) the exit pressure, and (c) the mass flow rate through the nozzle?

17-46C Consider subsonic flow in a converging nozzle with specified conditions at the nozzle inlet and critical pressure at the nozzle exit. What is the effect of dropping the back pressure well below the critical pressure on (a) the exit velocity, (b) the exit pressure, and (c) the mass flow rate through the nozzle?

17-47C Consider a converging nozzle and a converging–diverging nozzle having the same throat areas. For the same inlet conditions, how would you compare the mass flow rates through these two nozzles?

17-48C Consider gas flow through a converging nozzle with specified inlet conditions. We know that the highest velocity the fluid can have at the nozzle exit is the sonic velocity, at which point the mass flow rate through the nozzle is a maximum. If it were possible to achieve hypersonic

velocities at the nozzle exit, how would it affect the mass flow rate through the nozzle?

17-49C How does the parameter Ma^* differ from the Mach number Ma ?

17-50C What would happen if we attempted to decelerate a supersonic fluid with a diverging diffuser?

17-51C What would happen if we tried to further accelerate a supersonic fluid with a diverging diffuser?

17-52C Consider the isentropic flow of a fluid through a converging–diverging nozzle with a subsonic velocity at the throat. How does the diverging section affect (a) the velocity, (b) the pressure, and (c) the mass flow rate of the fluid?

17-53C Is it possible to accelerate a fluid to supersonic velocities with a velocity other than the sonic velocity at the throat? Explain.

17-54 Explain why the maximum flow rate per unit area for a given gas depends only on $P_0/\sqrt{T_0}$. For an ideal gas with $k = 1.4$ and $R = 0.287$ kJ/kg · K, find the constant a such that $\dot{m}/A^* = aP_0/\sqrt{T_0}$.

17-55 For an ideal gas obtain an expression for the ratio of the velocity of sound where $Ma = 1$ to the speed of sound based on the stagnation temperature, c^*/c_0 .

17-56 An ideal gas flows through a passage that first converges and then diverges during an adiabatic, reversible, steady-flow process. For subsonic flow at the inlet, sketch the variation of pressure, velocity, and Mach number along the length of the nozzle when the Mach number at the minimum flow area is equal to unity.

17-57 Repeat Prob. 17-56 for supersonic flow at the inlet.

17-58 Air enters a nozzle at 0.2 MPa, 350 K, and a velocity of 150 m/s. Assuming isentropic flow, determine the pressure and temperature of air at a location where the air velocity equals the speed of sound. What is the ratio of the area at this location to the entrance area?

Answers: 0.118 MPa, 301 K, 0.629

17-59 Repeat Prob. 17-58 assuming the entrance velocity is negligible.

17-60E Air enters a nozzle at 30 psia, 630 R, and a velocity of 450 ft/s. Assuming isentropic flow, determine the pressure and temperature of air at a location where the air velocity equals the speed of sound. What is the ratio of the area at this location to the entrance area?


Answers: 17.4 psia, 539 R, 0.574


17-61 Air enters a converging–diverging nozzle at 0.5 MPa with a negligible velocity. Assuming the flow to be isentropic, determine the back pressure that will result in an exit Mach number of 1.8. *Answer: 0.087 MPa*

17-62 Nitrogen enters a converging–diverging nozzle at 700 kPa and 450 K with a negligible velocity. Determine the critical velocity, pressure, temperature, and density in the nozzle.

17-63 An ideal gas with $k = 1.4$ is flowing through a nozzle such that the Mach number is 2.4 where the flow area is 25 cm². Assuming the flow to be isentropic, determine the flow area at the location where the Mach number is 1.2.

17-64 Repeat Prob. 17-63 for an ideal gas with $k = 1.33$.

17-65  Air at 900 kPa and 400 K enters a converging nozzle with a negligible velocity. The throat area of the nozzle is 10 cm². Assuming isentropic flow, calculate and plot the exit pressure, the exit velocity, and the mass flow rate versus the back pressure P_b for $0.9 \geq P_b \geq 0.1$ MPa.

17-66  Reconsider Prob. 17-65. Using EES (or other software, solve the problem for the inlet conditions of 1 MPa and 1000 K.

17-67E Air enters a converging–diverging nozzle of a supersonic wind tunnel at 150 psia and 100°F with a low velocity. The flow area of the test section is equal to the exit area of the nozzle, which is 5 ft². Calculate the pressure, temperature, velocity, and mass flow rate in the test section for a Mach number $Ma = 2$. Explain why the air must be very dry for this application. *Answers: 19.1 psia, 311 R, 1729 ft/s, 1435 lbm/s*

Shock Waves and Expansion Waves

17-68C Can a shock wave develop in the converging section of a converging–diverging nozzle? Explain.

17-69C What do the states on the Fanno line and the Rayleigh line represent? What do the intersection points of these two curves represent?

17-70C Can the Mach number of a fluid be greater than 1 after a shock wave? Explain.

17-71C How does the normal shock affect (a) the fluid velocity, (b) the static temperature, (c) the stagnation temperature, (d) the static pressure, and (e) the stagnation pressure?

17-72C How do oblique shocks occur? How do oblique shocks differ from normal shocks?

17-73C For an oblique shock to occur, does the upstream flow have to be supersonic? Does the flow downstream of an oblique shock have to be subsonic?

17-74C It is claimed that an oblique shock can be analyzed like a normal shock provided that the normal component of velocity (normal to the shock surface) is used in the analysis. Do you agree with this claim?

17-75C Consider supersonic airflow approaching the nose of a two-dimensional wedge and experiencing an oblique shock. Under what conditions does an oblique shock detach from the nose of the wedge and form a bow wave? What is the numerical value of the shock angle of the detached shock at the nose?

17-76C Consider supersonic flow impinging on the rounded nose of an aircraft. Will the oblique shock that forms in front of the nose be an attached or detached shock? Explain.

17-77C Are the isentropic relations of ideal gases applicable for flows across (a) normal shock waves, (b) oblique shock waves, and (c) Prandtl–Meyer expansion waves?

17-78 For an ideal gas flowing through a normal shock, develop a relation for V_2/V_1 in terms of k , Ma_1 , and Ma_2 .


17-79 Air enters a converging–diverging nozzle of a supersonic wind tunnel at 1.5 MPa and 350 K with a low velocity. If a normal shock wave occurs at the exit plane of the nozzle at $Ma = 2$, determine the pressure, temperature, Mach number, velocity, and stagnation pressure after the shock wave.
Answers: 0.863 MPa, 328 K, 0.577, 210 m/s, 1.081 MPa


17-80 Air enters a converging–diverging nozzle with low velocity at 2.0 MPa and 100°C. If the exit area of the nozzle is 3.5 times the throat area, what must the back pressure be to produce a normal shock at the exit plane of the nozzle?
Answer: 0.661 MPa

17-81 What must the back pressure be in Prob. 17-80 for a normal shock to occur at a location where the cross-sectional area is twice the throat area?

17-82 Air flowing steadily in a nozzle experiences a normal shock at a Mach number of $Ma = 2.5$. If the pressure and temperature of air are 61.64 kPa and 262.15 K, respectively, upstream of the shock, calculate the pressure, temperature, velocity, Mach number, and stagnation pressure downstream of the shock. Compare these results to those for helium undergoing a normal shock under the same conditions.


17-83 Calculate the entropy change of air across the normal shock wave in Prob. 17-82.

17-84E  Air flowing steadily in a nozzle experiences a normal shock at a Mach number of $Ma = 2.5$. If the pressure and temperature of air are 10.0 psia and 440.5 R, respectively, upstream of the shock, calculate the pressure, temperature, velocity, Mach number, and stagnation pressure downstream of the shock. Compare these results to those for helium undergoing a normal shock under the same conditions.

17-85E  Reconsider Prob. 17-84E. Using EES (or other) software, study the effects of both air and helium flowing steadily in a nozzle when there is a normal shock at a Mach number in the range $2 < Ma_1 < 3.5$. In addition to the required information, calculate the entropy change of the air and helium across the normal shock. Tabulate the results in a parametric table.

17-86 Air enters a normal shock at 22.6 kPa, 217 K, and 680 m/s. Calculate the stagnation pressure and Mach number upstream of the shock, as well as pressure, temperature, velocity, Mach number, and stagnation pressure downstream of the shock.

17-87 Calculate the entropy change of air across the normal shock wave in Prob. 17-86. *Answer: 0.155 kJ/kg · K*

17-88  Using EES (or other) software, calculate and plot the entropy change of air across the nor-

mal shock for upstream Mach numbers between 0.5 and 1.5 in increments of 0.1. Explain why normal shock waves can occur only for upstream Mach numbers greater than $Ma = 1$.

17-89 Consider supersonic airflow approaching the nose of a two-dimensional wedge at a Mach number of 5. Using Fig. 17-41, determine the minimum shock angle and the maximum deflection angle a straight oblique shock can have.

17-90 Air flowing at 60 kPa, 240 K, and a Mach number of 3.4 impinges on a two-dimensional wedge of half-angle 12° . Determine the two possible oblique shock angles, β_{weak} and β_{strong} , that could be formed by this wedge. For each case, calculate the pressure, temperature, and Mach number downstream of the oblique shock.

17-91 Consider the supersonic flow of air at upstream conditions of 70 kPa and 260 K and a Mach number of 2.4 over a two-dimensional wedge of half-angle 10° . If the axis of the wedge is tilted 25° with respect to the upstream airflow, determine the downstream Mach number, pressure, and temperature above the wedge. *Answers: 3.105, 23.8 kPa, 191 K*

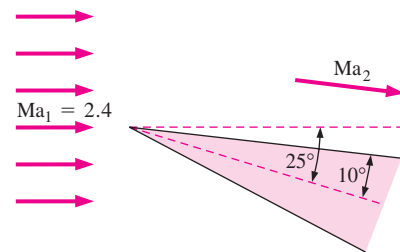


FIGURE P17-91

17-92 Reconsider Prob. 17-91. Determine the downstream Mach number, pressure, and temperature below the wedge for a strong oblique shock for an upstream Mach number of 5.

17-93E Air at 8 psia, 20°F, and a Mach number of 2.0 is forced to turn upward by a ramp that makes an 8° angle off the flow direction. As a result, a weak oblique shock forms. Determine the wave angle, Mach number, pressure, and temperature after the shock.

17-94 Air flowing at $P_1 = 40$ kPa, $T_1 = 280$ K, and $Ma_1 = 3.6$ is forced to undergo an expansion turn of 15° . Determine the Mach number, pressure, and temperature of air after the expansion. *Answers: 4.81, 8.31 kPa, 179 K*

17-95E Air flowing at $P_1 = 6$ psia, $T_1 = 480$ R, and $Ma_1 = 2.0$ is forced to undergo a compression turn of 15° . Determine the Mach number, pressure, and temperature of air after the compression.

Duct Flow with Heat Transfer and Negligible Friction (Rayleigh Flow)

17-96C What is the characteristic aspect of Rayleigh flow? What are the main assumptions associated with Rayleigh flow?

17-97C On a T - s diagram of Rayleigh flow, what do the points on the Rayleigh line represent?

17-98C What is the effect of heat gain and heat loss on the entropy of the fluid during Rayleigh flow?

17-99C Consider subsonic Rayleigh flow of air with a Mach number of 0.92. Heat is now transferred to the fluid and the Mach number increases to 0.95. Will the temperature T of the fluid increase, decrease, or remain constant during this process? How about the stagnation temperature T_0 ?

17-100C What is the effect of heating the fluid on the flow velocity in subsonic Rayleigh flow? Answer the same questions for supersonic Rayleigh flow.

17-101C Consider subsonic Rayleigh flow that is accelerated to sonic velocity ($Ma = 1$) at the duct exit by heating. If the fluid continues to be heated, will the flow at duct exit be supersonic, subsonic, or remain sonic?

17-102 Consider a 12-cm-diameter tubular combustion chamber. Air enters the tube at 500 K, 400 kPa, and 70 m/s. Fuel with a heating value of 39,000 kJ/kg is burned by spraying it into the air. If the exit Mach number is 0.8, determine the rate at which the fuel is burned and the exit temperature. Assume complete combustion and disregard the increase in the mass flow rate due to the fuel mass.

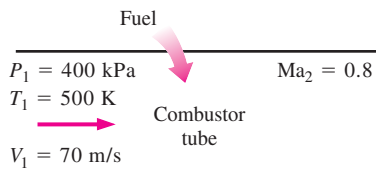


FIGURE P17-102

17-103 Air enters a rectangular duct at $T_1 = 300$ K, $P_1 = 420$ kPa, and $Ma_1 = 2$. Heat is transferred to the air in the amount of 55 kJ/kg as it flows through the duct. Disregarding frictional losses, determine the temperature and Mach number at the duct exit. *Answers: 386 K, 1.64*

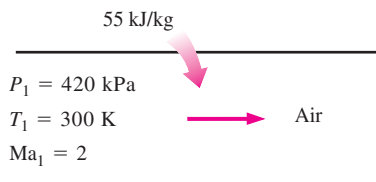



FIGURE P17-103

17-104 Repeat Prob. 17-103 assuming air is cooled in the amount of 55 kJ/kg.

17-105 Air is heated as it flows subsonically through a duct. When the amount of heat transfer reaches 60 kJ/kg, the

flow is observed to be choked, and the velocity and the static pressure are measured to be 620 m/s and 270 kPa. Disregarding frictional losses, determine the velocity, static temperature, and static pressure at the duct inlet.

17-106E Air flows with negligible friction through a 4-in-diameter duct at a rate of 5 lbm/s. The temperature and pressure at the inlet are $T_1 = 800$ R and $P_1 = 30$ psia, and the Mach number at the exit is $Ma_2 = 1$. Determine the rate of heat transfer and the pressure drop for this section of the duct.

17-107  Air enters a frictionless duct with $V_1 = 70$ m/s, $T_1 = 600$ K, and $P_1 = 350$ kPa. Letting the exit temperature T_2 vary from 600 to 5000 K, evaluate the entropy change at intervals of 200 K, and plot the Rayleigh line on a T - s diagram.

17-108E Air is heated as it flows through a 4 in \times 4 in square duct with negligible friction. At the inlet, air is at $T_1 = 700$ R, $P_1 = 80$ psia, and $V_1 = 260$ ft/s. Determine the rate at which heat must be transferred to the air to choke the flow at the duct exit, and the entropy change of air during this process.

17-109 Compressed air from the compressor of a gas turbine enters the combustion chamber at $T_1 = 550$ K, $P_1 = 600$ kPa, and $Ma_1 = 0.2$ at a rate of 0.3 kg/s. Via combustion, heat is transferred to the air at a rate of 200 kJ/s as it flows through the duct with negligible friction. Determine the Mach number at the duct exit and the drop in stagnation pressure $P_{01} - P_{02}$ during this process. *Answers: 0.319, 21.8 kPa*

17-110 Repeat Prob. 17-109 for a heat transfer rate of 300 kJ/s.

17-111 Argon gas enters a constant cross-sectional-area duct at $Ma_1 = 0.2$, $P_1 = 320$ kPa, and $T_1 = 400$ K at a rate of 0.8 kg/s. Disregarding frictional losses, determine the highest rate of heat transfer to the argon without reducing the mass flow rate.

17-112 Consider supersonic flow of air through a 6-cm-diameter duct with negligible friction. Air enters the duct at $Ma_1 = 1.8$, $P_{01} = 210$ kPa, and $T_{01} = 600$ K, and it is decelerated by heating. Determine the highest temperature that air can be heated by heat addition while the mass flow rate remains constant.

Steam Nozzles

17-113C What is supersaturation? Under what conditions does it occur?

17-114 Steam enters a converging nozzle at 3.0 MPa and 500°C with a negligible velocity, and it exits at 1.8 MPa. For a nozzle exit area of 32 cm², determine the exit velocity, mass flow rate, and exit Mach number if the nozzle (a) is isentropic and (b) has an efficiency of 90 percent. *Answers: (a) 580 m/s, 10.7 kg/s, 0.918, (b) 551 m/s, 10.1 kg/s, 0.865*

17-115E Steam enters a converging nozzle at 450 psia and 900°F with a negligible velocity, and it exits at 275 psia. For a nozzle exit area of 3.75 in², determine the exit velocity, mass flow rate, and exit Mach number if the nozzle (a) is isentropic and (b) has an efficiency of 90 percent. *Answers:* (a) 1847 ft/s, 18.7 lbm/s, 0.900, (b) 1752 ft/s, 17.5 lbm/s, 0.849

17-116 Steam enters a converging–diverging nozzle at 1 MPa and 500°C with a negligible velocity at a mass flow rate of 2.5 kg/s, and it exits at a pressure of 200 kPa. Assuming the flow through the nozzle to be isentropic, determine the exit area and the exit Mach number. *Answers:* 31.5 cm², 1.738

17-117 Repeat Prob. 17-116 for a nozzle efficiency of 95 percent.

Review Problems

17-118 Air in an automobile tire is maintained at a pressure of 220 kPa (gauge) in an environment where the atmospheric pressure is 94 kPa. The air in the tire is at the ambient temperature of 25°C. Now a 4-mm-diameter leak develops in the tire as a result of an accident. Assuming isentropic flow, determine the initial mass flow rate of air through the leak. *Answer:* 0.554 kg/min

17-119 The thrust developed by the engine of a Boeing 777 is about 380 kN. Assuming choked flow in the nozzles, determine the mass flow rate of air through the nozzle. Take the ambient conditions to be 265 K and 85 kPa.

17-120 A stationary temperature probe inserted into a duct where air is flowing at 250 m/s reads 85°C. What is the actual temperature of air? *Answer:* 53.9°C

17-121 Nitrogen enters a steady-flow heat exchanger at 150 kPa, 10°C, and 100 m/s, and it receives heat in the amount of 125 kJ/kg as it flows through it. The nitrogen leaves the heat exchanger at 100 kPa with a velocity of 180 m/s. Determine the stagnation pressure and temperature of the nitrogen at the inlet and exit states.

17-122 Derive an expression for the speed of sound based on van der Waals' equation of state $P = RT/(v - b) - a/v^2$. Using this relation, determine the speed of sound in carbon dioxide at 50°C and 200 kPa, and compare your result to that obtained by assuming ideal-gas behavior. The van der Waals constants for carbon dioxide are $a = 364.3 \text{ kPa} \cdot \text{m}^6/\text{kmol}^2$ and $b = 0.0427 \text{ m}^3/\text{kmol}$.

17-123 Obtain Eq. 17-10 by starting with Eq. 17-9 and using the cyclic rule and the thermodynamic property relations

$$\frac{c_p}{T} = \left(\frac{\partial s}{\partial T} \right)_p \quad \text{and} \quad \frac{c_v}{T} = \left(\frac{\partial s}{\partial T} \right)_v$$

17-124 For ideal gases undergoing isentropic flows, obtain expressions for P/P^* , T/T^* , and ρ/ρ^* as functions of k and Ma .

17-125 Using Eqs. 17-4, 17-13, and 17-14, verify that for the steady flow of ideal gases $dT_0/T = dA/A + (1 - \text{Ma}^2)$


dV/V . Explain the effect of heating and area changes on the velocity of an ideal gas in steady flow for (a) subsonic flow and (b) supersonic flow.


17-126 A subsonic airplane is flying at a 3000-m altitude where the atmospheric conditions are 70.109 kPa and 268.65 K. A Pitot static probe measures the difference between the static and stagnation pressures to be 35 kPa. Calculate the speed of the airplane and the flight Mach number. *Answers:* 257 m/s, 0.783

17-127 Plot the mass flow parameter $\dot{m}\sqrt{RT_0}/(AP_0)$ versus the Mach number for $k = 1.2, 1.4, \text{ and } 1.6$ in the range of $0 \leq \text{Ma} \leq 1$.

17-128 Helium enters a nozzle at 0.8 MPa, 500 K, and a velocity of 120 m/s. Assuming isentropic flow, determine the pressure and temperature of helium at a location where the velocity equals the speed of sound. What is the ratio of the area at this location to the entrance area?

17-129 Repeat Prob. 17-128 assuming the entrance velocity is negligible.

17-130  Air at 0.9 MPa and 400 K enters a converging nozzle with a velocity of 180 m/s. The throat area is 10 cm². Assuming isentropic flow, calculate and plot the mass flow rate through the nozzle, the exit velocity, the exit Mach number, and the exit pressure–stagnation pressure ratio versus the back pressure–stagnation pressure ratio for a back pressure range of $0.9 \geq P_b \geq 0.1 \text{ MPa}$.

17-131  Steam at 6.0 MPa and 700 K enters a converging nozzle with a negligible velocity. The nozzle throat area is 8 cm². Assuming isentropic flow, plot the exit pressure, the exit velocity, and the mass flow rate through the nozzle versus the back pressure P_b for $6.0 \geq P_b \geq 3.0 \text{ MPa}$. Treat the steam as an ideal gas with $k = 1.3$, $c_p = 1.872 \text{ kJ/kg} \cdot \text{K}$, and $R = 0.462 \text{ kJ/kg} \cdot \text{K}$.


17-132 Find the expression for the ratio of the stagnation pressure after a shock wave to the static pressure before the shock wave as a function of k and the Mach number upstream of the shock wave Ma_1 .


17-133 Nitrogen enters a converging–diverging nozzle at 700 kPa and 300 K with a negligible velocity, and it experiences a normal shock at a location where the Mach number is $\text{Ma} = 3.0$. Calculate the pressure, temperature, velocity, Mach number, and stagnation pressure downstream of the shock. Compare these results to those of air undergoing a normal shock at the same conditions.

17-134 An aircraft flies with a Mach number $\text{Ma}_1 = 0.8$ at an altitude of 7000 m where the pressure is 41.1 kPa and the temperature is 242.7 K. The diffuser at the engine inlet has an exit Mach number of $\text{Ma}_2 = 0.3$. For a mass flow rate of 65 kg/s, determine the static pressure rise across the diffuser and the exit area.


17-135 Helium expands in a nozzle from 1 MPa, 500 K, and negligible velocity to 0.1 MPa. Calculate the throat and exit areas for a mass flow rate of 0.25 kg/s, assuming the nozzle is isentropic. Why must this nozzle be converging–diverging? *Answers: 3.51 cm², 5.84 cm²*


17-136E Helium expands in a nozzle from 150 psia, 900 R, and negligible velocity to 15 psia. Calculate the throat and exit areas for a mass flow rate of 0.2 lbm/s, assuming the nozzle is isentropic. Why must this nozzle be converging–diverging?


17-137  Using the EES software and the relations in Table A–32, calculate the one-dimensional compressible flow functions for an ideal gas with $k = 1.667$, and present your results by duplicating Table A–32.


17-138  Using the EES software and the relations in Table A–33, calculate the one-dimensional normal shock functions for an ideal gas with $k = 1.667$, and present your results by duplicating Table A–33.


17-139 Consider an equimolar mixture of oxygen and nitrogen. Determine the critical temperature, pressure, and density for stagnation temperature and pressure of 800 K and 500 kPa.

17-140  Using EES (or other) software, determine the shape of a converging–diverging nozzle for air for a mass flow rate of 3 kg/s and inlet stagnation conditions of 1400 kPa and 200°C. Assume the flow is isentropic. Repeat the calculations for 50-kPa increments of pressure drops to an exit pressure of 100 kPa. Plot the nozzle to scale. Also, calculate and plot the Mach number along the nozzle.

17-141  Using EES (or other) software and the relations given in Table A–32, calculate the one-dimensional isentropic compressible-flow functions by varying the upstream Mach number from 1 to 10 in increments of 0.5 for air with $k = 1.4$.

17-142  Repeat Prob. 17–141 for methane with $k = 1.3$.

17-143  Using EES (or other) software and the relations given in Table A–33, generate the one-dimensional normal shock functions by varying the upstream Mach number from 1 to 10 in increments of 0.5 for air with $k = 1.4$.

17-144  Repeat Prob. 17–143 for methane with $k = 1.3$.

17-145 Air is cooled as it flows through a 20-cm-diameter duct. The inlet conditions are $Ma_1 = 1.2$, $T_{01} = 350$ K, and $P_{01} = 240$ kPa and the exit Mach number is $Ma_2 = 2.0$. Disregarding frictional effects, determine the rate of cooling of air.

17-146 Air is heated as it flows subsonically through a 10 cm × 10 cm square duct. The properties of air at the inlet are maintained at $Ma_1 = 0.4$, $P_1 = 400$ kPa, and $T_1 = 360$ K

at all times. Disregarding frictional losses, determine the highest rate of heat transfer to the air in the duct without affecting the inlet conditions. *Answer: 1958 kW*

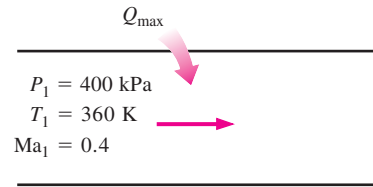


FIGURE P17-146

17-147 Repeat Prob. 17–146 for helium.

17-148 Air is accelerated as it is heated in a duct with negligible friction. Air enters at $V_1 = 100$ m/s, $T_1 = 400$ K, and $P_1 = 35$ kPa and then exits at a Mach number of $Ma_2 = 0.8$. Determine the heat transfer to the air, in kJ/kg. Also determine the maximum amount of heat transfer without reducing the mass flow rate of air.

17-149 Air at sonic conditions and static temperature and pressure of 500 K and 420 kPa, respectively, is to be accelerated to a Mach number of 1.6 by cooling it as it flows through a channel with constant cross-sectional area. Disregarding frictional effects, determine the required heat transfer from the air, in kJ/kg. *Answer: 69.8 kJ/kg*

17-150 Saturated steam enters a converging–diverging nozzle at 3.0 MPa, 5 percent moisture, and negligible velocity, and it exits at 1.2 MPa. For a nozzle exit area of 16 cm², determine the throat area, exit velocity, mass flow rate, and exit Mach number if the nozzle (a) is isentropic and (b) has an efficiency of 90 percent.

Fundamentals of Engineering (FE) Exam Problems

17-151 An aircraft is cruising in still air at 5°C at a velocity of 400 m/s. The air temperature at the nose of the aircraft where stagnation occurs is

- (a) 5°C (b) 25°C (c) 55°C (d) 80°C (e) 85°C

17-152 Air is flowing in a wind tunnel at 15°C, 80 kPa, and 200 m/s. The stagnation pressure at the probe inserted into the flow section is

- (a) 82 kPa (b) 91 kPa (c) 96 kPa
(d) 101 kPa (e) 114 kPa

17-153 An aircraft is reported to be cruising in still air at –20°C and 40 kPa at a Mach number of 0.86. The velocity of the aircraft is

- (a) 91 m/s (b) 220 m/s (c) 186 m/s
(d) 280 m/s (e) 378 m/s

17-154 Air is flowing in a wind tunnel at 12°C and 66 kPa at a velocity of 230 m/s. The Mach number of the flow is

- (a) 0.54 m/s (b) 0.87 m/s (c) 3.3 m/s
(d) 0.36 m/s (e) 0.68 m/s

17-155 Consider a converging nozzle with a low velocity at the inlet and sonic velocity at the exit plane. Now the nozzle exit diameter is reduced by half while the nozzle inlet temperature and pressure are maintained the same. The nozzle exit velocity will

- (a) remain the same (b) double (c) quadruple
(d) go down by half (e) go down to one-fourth

17-156 Air is approaching a converging–diverging nozzle with a low velocity at 20°C and 300 kPa, and it leaves the nozzle at a supersonic velocity. The velocity of air at the throat of the nozzle is

- (a) 290 m/s (b) 98 m/s (c) 313 m/s
(d) 343 m/s (e) 412 m/s

17-157 Argon gas is approaching a converging–diverging nozzle with a low velocity at 20°C and 120 kPa, and it leaves the nozzle at a supersonic velocity. If the cross-sectional area of the throat is 0.015 m^2 , the mass flow rate of argon through the nozzle is

- (a) 0.41 kg/s (b) 3.4 kg/s (c) 5.3 kg/s
(d) 17 kg/s (e) 22 kg/s

17-158 Carbon dioxide enters a converging–diverging nozzle at 60 m/s, 310°C , and 300 kPa, and it leaves the nozzle at a supersonic velocity. The velocity of carbon dioxide at the throat of the nozzle is

- (a) 125 m/s (b) 225 m/s (c) 312 m/s
(d) 353 m/s (e) 377 m/s

17-159 Consider gas flow through a converging–diverging nozzle. Of the five following statements, select the one that is incorrect:

- (a) The fluid velocity at the throat can never exceed the speed of sound.
(b) If the fluid velocity at the throat is below the speed of sound, the diversion section will act like a diffuser.
(c) If the fluid enters the diverging section with a Mach number greater than one, the flow at the nozzle exit will be supersonic.

(d) There will be no flow through the nozzle if the back pressure equals the stagnation pressure.

(e) The fluid velocity decreases, the entropy increases, and stagnation enthalpy remains constant during flow through a normal shock.

17-160 Combustion gases with $k = 1.33$ enter a converging nozzle at stagnation temperature and pressure of 400°C and 800 kPa, and are discharged into the atmospheric air at 20°C and 100 kPa. The lowest pressure that will occur within the nozzle is

- (a) 26 kPa (b) 100 kPa (c) 321 kPa
(d) 432 kPa (e) 272 kPa

Design and Essay Problems

17-161 Find out if there is a supersonic wind tunnel on your campus. If there is, obtain the dimensions of the wind tunnel and the temperatures and pressures as well as the Mach number at several locations during operation. For what typical experiments is the wind tunnel used?

17-162 Assuming you have a thermometer and a device to measure the speed of sound in a gas, explain how you can determine the mole fraction of helium in a mixture of helium gas and air.

17-163 Design a 1-m-long cylindrical wind tunnel whose diameter is 25 cm operating at a Mach number of 1.8. Atmospheric air enters the wind tunnel through a converging–diverging nozzle where it is accelerated to supersonic velocities. Air leaves the tunnel through a converging–diverging diffuser where it is decelerated to a very low velocity before entering the fan section. Disregard any irreversibilities. Specify the temperatures and pressures at several locations as well as the mass flow rate of air at steady-flow conditions. Why is it often necessary to dehumidify the air before it enters the wind tunnel?

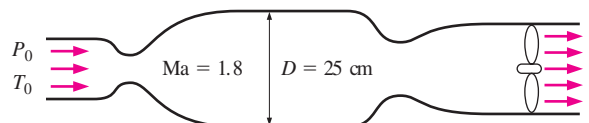


FIGURE P17-163

