n Chap. 6, we introduced the second law of thermodynamics and applied it to cycles and cyclic devices. In this chapter, we apply the second law to processes. The first law of thermodynamics deals with the property energy and the conservation of it. The second law leads to the definition of a new property called entropy. Entropy is a somewhat abstract property, and it is difficult to give a physical description of it without considering the microscopic state of the system. Entropy is best understood and appreciated by studying its uses in commonly encountered engineering processes, and this is what we intend to do.

This chapter starts with a discussion of the Clausius inequality, which forms the basis for the definition of entropy, and continues with the increase of entropy principle. Unlike energy, entropy is a nonconserved property, and there is no such thing as conservation of entropy. Next, the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases are discussed, and a special class of idealized processes, called isentropic processes, is examined. Then, the reversible steady-flow work and the isentropic efficiencies of various engineering devices such as turbines and compressors are considered. Finally, entropy balance is introduced and applied to various systems.

## Objectives

The objectives of Chapter 7 are to:

- Apply the second law of thermodynamics to processes.
- Define a new property called entropy to quantify the second-law effects.
- Establish the increase of entropy principle.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called isentropic processes, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

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FIGURE 7-1
The system considered in the development of the Clausius inequality.

## 7-1 • ENTROPY

The second law of thermodynamics often leads to expressions that involve inequalities. An irreversible (i.e., actual) heat engine, for example, is less efficient than a reversible one operating between the same two thermal energy reservoirs. Likewise, an irreversible refrigerator or a heat pump has a lower coefficient of performance (COP) than a reversible one operating between the same temperature limits. Another important inequality that has major consequences in thermodynamics is the Clausius inequality. It was first stated by the German physicist R. J. E. Clausius (1822-1888), one of the founders of thermodynamics, and is expressed as

$$
\oint \frac{\delta Q}{T} \leq 0
$$

That is, the cyclic integral of $\delta Q / T$ is always less than or equal to zero. This inequality is valid for all cycles, reversible or irreversible. The symbol $\oint$ (integral symbol with a circle in the middle) is used to indicate that the integration is to be performed over the entire cycle. Any heat transfer to or from a system can be considered to consist of differential amounts of heat transfer. Then the cyclic integral of $\delta Q / T$ can be viewed as the sum of all these differential amounts of heat transfer divided by the temperature at the boundary.

To demonstrate the validity of the Clausius inequality, consider a system connected to a thermal energy reservoir at a constant thermodynamic (i.e., absolute) temperature of $T_{R}$ through a reversible cyclic device (Fig. 7-1). The cyclic device receives heat $\delta Q_{R}$ from the reservoir and supplies heat $\delta Q$ to the system whose temperature at that part of the boundary is $T$ (a variable) while producing work $\delta W_{\text {rev }}$. The system produces work $\delta W_{\text {sys }}$ as a result of this heat transfer. Applying the energy balance to the combined system identified by dashed lines yields

$$
\delta W_{C}=\delta Q_{R}-d E_{C}
$$

where $\delta W_{C}$ is the total work of the combined system ( $\delta W_{\text {rev }}+\delta W_{\text {sys }}$ ) and $d E_{C}$ is the change in the total energy of the combined system. Considering that the cyclic device is a reversible one, we have

$$
\frac{\delta Q_{R}}{T_{R}}=\frac{\delta Q}{T}
$$

where the sign of $\delta Q$ is determined with respect to the system (positive if to the system and negative if from the system) and the sign of $\delta Q_{R}$ is determined with respect to the reversible cyclic device. Eliminating $\delta Q_{R}$ from the two relations above yields

$$
\delta W_{C}=T_{R} \frac{\delta Q}{T}-d E_{C}
$$

We now let the system undergo a cycle while the cyclic device undergoes an integral number of cycles. Then the preceding relation becomes

$$
W_{C}=T_{R} \oint \frac{\delta Q}{T}
$$

since the cyclic integral of energy (the net change in the energy, which is a property, during a cycle) is zero. Here $W_{C}$ is the cyclic integral of $\delta W_{C}$, and it represents the net work for the combined cycle.

It appears that the combined system is exchanging heat with a single thermal energy reservoir while involving (producing or consuming) work $W_{C}$ during a cycle. On the basis of the Kelvin-Planck statement of the second law, which states that no system can produce a net amount of work while operating in a cycle and exchanging heat with a single thermal energy reservoir, we reason that $W_{C}$ cannot be a work output, and thus it cannot be a positive quantity. Considering that $T_{R}$ is the thermodynamic temperature and thus a positive quantity, we must have

$$
\begin{equation*}
\oint \frac{\delta Q}{T} \leq 0 \tag{7-1}
\end{equation*}
$$

which is the Clausius inequality. This inequality is valid for all thermodynamic cycles, reversible or irreversible, including the refrigeration cycles.

If no irreversibilities occur within the system as well as the reversible cyclic device, then the cycle undergone by the combined system is internally reversible. As such, it can be reversed. In the reversed cycle case, all the quantities have the same magnitude but the opposite sign. Therefore, the work $W_{C}$, which could not be a positive quantity in the regular case, cannot be a negative quantity in the reversed case. Then it follows that $W_{C, \text { int rev }}=0$ since it cannot be a positive or negative quantity, and therefore

$$
\begin{equation*}
\oint\left(\frac{\delta Q}{T}\right)_{\mathrm{int} \mathrm{rev}}=0 \tag{7-2}
\end{equation*}
$$

for internally reversible cycles. Thus, we conclude that the equality in the Clausius inequality holds for totally or just internally reversible cycles and the inequality for the irreversible ones.
To develop a relation for the definition of entropy, let us examine Eq. 7-2 more closely. Here we have a quantity whose cyclic integral is zero. Let us think for a moment what kind of quantities can have this characteristic. We know that the cyclic integral of work is not zero. (It is a good thing that it is not. Otherwise, heat engines that work on a cycle such as steam power plants would produce zero net work.) Neither is the cyclic integral of heat.

Now consider the volume occupied by a gas in a piston-cylinder device undergoing a cycle, as shown in Fig. 7-2. When the piston returns to its initial position at the end of a cycle, the volume of the gas also returns to its initial value. Thus the net change in volume during a cycle is zero. This is also expressed as

$$
\begin{equation*}
\oint d V=0 \tag{7-3}
\end{equation*}
$$

That is, the cyclic integral of volume (or any other property) is zero. Conversely, a quantity whose cyclic integral is zero depends on the state only and not the process path, and thus it is a property. Therefore, the quantity $(\delta Q / T)_{\text {int rev }}$ must represent a property in the differential form.

Clausius realized in 1865 that he had discovered a new thermodynamic property, and he chose to name this property entropy. It is designated $S$ and is defined as

$$
\begin{equation*}
d S=\left(\frac{\delta Q}{T}\right)_{\mathrm{int} \mathrm{rev}} \quad(\mathrm{~kJ} / \mathrm{K}) \tag{7-4}
\end{equation*}
$$



FIGURE 7-2
The net change in volume (a property) during a cycle is always zero.


FIGURE 7-3
The entropy change between two specified states is the same whether the process is reversible or irreversible.

Entropy is an extensive property of a system and sometimes is referred to as total entropy. Entropy per unit mass, designated $s$, is an intensive property and has the unit $\mathrm{kJ} / \mathrm{kg} \cdot \mathrm{K}$. The term entropy is generally used to refer to both total entropy and entropy per unit mass since the context usually clarifies which one is meant.

The entropy change of a system during a process can be determined by integrating Eq. $7-4$ between the initial and the final states:

$$
\begin{equation*}
\Delta S=S_{2}-S_{1}=\int_{1}^{2}\left(\frac{\delta Q}{T}\right)_{\text {int rev }} \quad(\mathrm{kJ} / \mathrm{K}) \tag{7-5}
\end{equation*}
$$

Notice that we have actually defined the change in entropy instead of entropy itself, just as we defined the change in energy instead of the energy itself when we developed the first-law relation. Absolute values of entropy are determined on the basis of the third law of thermodynamics, which is discussed later in this chapter. Engineers are usually concerned with the changes in entropy. Therefore, the entropy of a substance can be assigned a zero value at some arbitrarily selected reference state, and the entropy values at other states can be determined from Eq. 7-5 by choosing state 1 to be the reference state $(S=0)$ and state 2 to be the state at which entropy is to be determined.
To perform the integration in Eq. 7-5, one needs to know the relation between $Q$ and $T$ during a process. This relation is often not available, and the integral in Eq. $7-5$ can be performed for a few cases only. For the majority of cases we have to rely on tabulated data for entropy.

Note that entropy is a property, and like all other properties, it has fixed values at fixed states. Therefore, the entropy change $\Delta S$ between two specified states is the same no matter what path, reversible or irreversible, is followed during a process (Fig. 7-3).

Also note that the integral of $\delta Q / T$ gives us the value of entropy change only if the integration is carried out along an internally reversible path between the two states. The integral of $\delta Q / T$ along an irreversible path is not a property, and in general, different values will be obtained when the integration is carried out along different irreversible paths. Therefore, even for irreversible processes, the entropy change should be determined by carrying out this integration along some convenient imaginary internally reversible path between the specified states.

## A Special Case: Internally Reversible Isothermal Heat Transfer Processes

Recall that isothermal heat transfer processes are internally reversible. Therefore, the entropy change of a system during an internally reversible isothermal heat transfer process can be determined by performing the integration in Eq. 7-5:

$$
\Delta S=\int_{1}^{2}\left(\frac{\delta Q}{T}\right)_{\text {int rev }}=\int_{1}^{2}\left(\frac{\delta Q}{T_{0}}\right)_{\text {int rev }}=\frac{1}{T_{0}} \int_{1}^{2}(\delta Q)_{\text {intrev }}
$$

which reduces to

$$
\begin{equation*}
\Delta S=\frac{Q}{T_{0}} \quad(\mathrm{~kJ} / \mathrm{K}) \tag{7-6}
\end{equation*}
$$

where $T_{0}$ is the constant temperature of the system and $Q$ is the heat transfer for the internally reversible process. Equation 7-6 is particularly useful for determining the entropy changes of thermal energy reservoirs that can absorb or supply heat indefinitely at a constant temperature.

Notice that the entropy change of a system during an internally reversible isothermal process can be positive or negative, depending on the direction of heat transfer. Heat transfer to a system increases the entropy of a system, whereas heat transfer from a system decreases it. In fact, losing heat is the only way the entropy of a system can be decreased.

## EXAMPLE 7-1 Entropy Change during an Isothermal Process

A piston-cylinder device contains a liquid-vapor mixture of water at 300 K . During a constant-pressure process, 750 kJ of heat is transferred to the water. As a result, part of the liquid in the cylinder vaporizes. Determine the entropy change of the water during this process.

Solution Heat is transferred to a liquid-vapor mixture of water in a pistoncylinder device at constant pressure. The entropy change of water is to be determined.
Assumptions No irreversibilities occur within the system boundaries during the process.
Analysis We take the entire water (liquid + vapor) in the cylinder as the system (Fig. 7-4). This is a closed system since no mass crosses the system boundary during the process. We note that the temperature of the system remains constant at 300 K during this process since the temperature of a pure substance remains constant at the saturation value during a phasechange process at constant pressure.

The system undergoes an internally reversible, isothermal process, and thus its entropy change can be determined directly from Eq. 7-6 to be

$$
\Delta S_{\mathrm{sys}, \text { isothermal }}=\frac{Q}{T_{\mathrm{sys}}}=\frac{750 \mathrm{~kJ}}{300 \mathrm{~K}}=2.5 \mathrm{~kJ} / \mathrm{K}
$$

Discussion Note that the entropy change of the system is positive, as expected, since heat transfer is to the system.

## 7-2 • THE INCREASE OF ENTROPY PRINCIPLE

Consider a cycle that is made up of two processes: process 1-2, which is arbitrary (reversible or irreversible), and process $2-1$, which is internally reversible, as shown in Figure 7-5. From the Clausius inequality,

$$
\oint \frac{\delta Q}{T} \leq 0
$$

or

$$
\int_{1}^{2} \frac{\delta Q}{T}+\int_{2}^{1}\left(\frac{\delta Q}{T}\right)_{\text {int rev }} \leq 0
$$



FIGURE 7-4
Schematic for Example 7-1.

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FIGURE 7-5
A cycle composed of a reversible and an irreversible process.

The second integral in the previous relation is recognized as the entropy change $S_{1}-S_{2}$. Therefore,

$$
\int_{1}^{2} \frac{\delta Q}{T}+S_{1}-S_{2} \leq 0
$$

which can be rearranged as

$$
\begin{equation*}
S_{2}-S_{1} \geq \int_{1}^{2} \frac{\delta Q}{T} \tag{7-7}
\end{equation*}
$$

It can also be expressed in differential form as

$$
\begin{equation*}
d S \geq \frac{\delta Q}{T} \tag{7-8}
\end{equation*}
$$

where the equality holds for an internally reversible process and the inequality for an irreversible process. We may conclude from these equations that the entropy change of a closed system during an irreversible process is greater than the integral of $\delta Q / T$ evaluated for that process. In the limiting case of a reversible process, these two quantities become equal. We again emphasize that $T$ in these relations is the thermodynamic temperature at the boundary where the differential heat $\delta Q$ is transferred between the system and the surroundings.

The quantity $\Delta S=S_{2}-S_{1}$ represents the entropy change of the system. For a reversible process, it becomes equal to $\int_{1}^{2} \delta Q / T$, which represents the entropy transfer with heat.

The inequality sign in the preceding relations is a constant reminder that the entropy change of a closed system during an irreversible process is always greater than the entropy transfer. That is, some entropy is generated or created during an irreversible process, and this generation is due entirely to the presence of irreversibilities. The entropy generated during a process is called entropy generation and is denoted by $S_{\text {gen }}$. Noting that the difference between the entropy change of a closed system and the entropy transfer is equal to entropy generation, Eq. 7-7 can be rewritten as an equality as

$$
\begin{equation*}
\Delta S_{\mathrm{sys}}=S_{2}-S_{1}=\int_{1}^{2} \frac{\delta Q}{T}+S_{\mathrm{gen}} \tag{7-9}
\end{equation*}
$$

Note that the entropy generation $S_{\text {gen }}$ is always a positive quantity or zero. Its value depends on the process, and thus it is not a property of the system. Also, in the absence of any entropy transfer, the entropy change of a system is equal to the entropy generation.

Equation 7-7 has far-reaching implications in thermodynamics. For an isolated system (or simply an adiabatic closed system), the heat transfer is zero, and Eq. 7-7 reduces to

$$
\begin{equation*}
\Delta S_{\text {isolated }} \geq 0 \tag{7-10}
\end{equation*}
$$

This equation can be expressed as the entropy of an isolated system during a process always increases or, in the limiting case of a reversible process, remains constant. In other words, it never decreases. This is known as the increase of entropy principle. Note that in the absence of any heat transfer, entropy change is due to irreversibilities only, and their effect is always to increase entropy.

Entropy is an extensive property, and thus the total entropy of a system is equal to the sum of the entropies of the parts of the system. An isolated system may consist of any number of subsystems (Fig. 7-6). A system and its surroundings, for example, constitute an isolated system since both can be enclosed by a sufficiently large arbitrary boundary across which there is no heat, work, or mass transfer (Fig. 7-7). Therefore, a system and its surroundings can be viewed as the two subsystems of an isolated system, and the entropy change of this isolated system during a process is the sum of the entropy changes of the system and its surroundings, which is equal to the entropy generation since an isolated system involves no entropy transfer. That is,

$$
\begin{equation*}
S_{\text {gen }}=\Delta S_{\text {total }}=\Delta S_{\text {sys }}+\Delta S_{\text {surr }} \geq 0 \tag{7-11}
\end{equation*}
$$

where the equality holds for reversible processes and the inequality for irreversible ones. Note that $\Delta S_{\text {surr }}$ refers to the change in the entropy of the surroundings as a result of the occurrence of the process under consideration.

Since no actual process is truly reversible, we can conclude that some entropy is generated during a process, and therefore the entropy of the universe, which can be considered to be an isolated system, is continuously increasing. The more irreversible a process, the larger the entropy generated during that process. No entropy is generated during reversible processes ( $S_{\text {gen }}=0$ ).
Entropy increase of the universe is a major concern not only to engineers but also to philosophers, theologians, economists, and environmentalists since entropy is viewed as a measure of the disorder (or "mixed-up-ness") in the universe.

The increase of entropy principle does not imply that the entropy of a system cannot decrease. The entropy change of a system can be negative during a process (Fig. 7-8), but entropy generation cannot. The increase of entropy principle can be summarized as follows:

$$
S_{\text {gen }}\left\{\begin{array}{lll}
> & 0 \text { Irreversible process } \\
= & 0 \text { Reversible process } \\
< & 0 \text { Impossible process }
\end{array}\right.
$$

This relation serves as a criterion in determining whether a process is reversible, irreversible, or impossible.

Things in nature have a tendency to change until they attain a state of equilibrium. The increase of entropy principle dictates that the entropy of an isolated system increases until the entropy of the system reaches a maximum value. At that point, the system is said to have reached an equilibrium state since the increase of entropy principle prohibits the system from undergoing any change of state that results in a decrease in entropy.

## Some Remarks about Entropy

In light of the preceding discussions, we draw the following conclusions:

1. Processes can occur in a certain direction only, not in any direction. A process must proceed in the direction that complies with the increase of entropy principle, that is, $S_{\text {gen }} \geq 0$. A process that violates this principle is impossible. This principle often forces chemical reactions to come to a halt before reaching completion.
(Isolated)


FIGURE 7-6
The entropy change of an isolated system is the sum of the entropy changes of its components, and is never less than zero


FIGURE 7-7
A system and its surroundings form an isolated system.


$$
S_{\text {gen }}=\Delta S_{\text {total }}=\Delta S_{\text {sys }}+\Delta S_{\text {surr }}=1 \mathrm{~kJ} / \mathrm{K}
$$

## FIGURE 7-8

The entropy change of a system can be negative, but the entropy generation cannot.


FIGURE 7-9
Schematic for Example 7-2.
2. Entropy is a nonconserved property, and there is no such thing as the conservation of entropy principle. Entropy is conserved during the idealized reversible processes only and increases during all actual processes.
3. The performance of engineering systems is degraded by the presence of irreversibilities, and entropy generation is a measure of the magnitudes of the irreversibilities present during that process. The greater the extent of irreversibilities, the greater the entropy generation. Therefore, entropy generation can be used as a quantitative measure of irreversibilities associated with a process. It is also used to establish criteria for the performance of engineering devices. This point is illustrated further in Example 7-2.

## EXAMPLE 7-2 Entropy Generation during Heat Transfer Processes

A heat source at 800 K loses 2000 kJ of heat to a sink at (a) 500 K and (b) 750 K . Determine which heat transfer process is more irreversible.

Solution Heat is transferred from a heat source to two heat sinks at different temperatures. The heat transfer process that is more irreversible is to be determined.
Analysis A sketch of the reservoirs is shown in Fig. 7-9. Both cases involve heat transfer through a finite temperature difference, and therefore both are irreversible. The magnitude of the irreversibility associated with each process can be determined by calculating the total entropy change for each case. The total entropy change for a heat transfer process involving two reservoirs (a source and a sink) is the sum of the entropy changes of each reservoir since the two reservoirs form an adiabatic system.
Or do they? The problem statement gives the impression that the two reservoirs are in direct contact during the heat transfer process. But this cannot be the case since the temperature at a point can have only one value, and thus it cannot be 800 K on one side of the point of contact and 500 K on the other side. In other words, the temperature function cannot have a jump discontinuity. Therefore, it is reasonable to assume that the two reservoirs are separated by a partition through which the temperature drops from 800 K on one side to 500 K (or 750 K ) on the other. Therefore, the entropy change of the partition should also be considered when evaluating the total entropy change for this process. However, considering that entropy is a property and the values of properties depend on the state of a system, we can argue that the entropy change of the partition is zero since the partition appears to have undergone a steady process and thus experienced no change in its properties at any point. We base this argument on the fact that the temperature on both sides of the partition and thus throughout remains constant during this process. Therefore, we are justified to assume that $\Delta S_{\text {partition }}$ $=0$ since the entropy (as well as the energy) content of the partition remains constant during this process.

The entropy change for each reservoir can be determined from Eq. 7-6 since each reservoir undergoes an internally reversible, isothermal process.
(a) For the heat transfer process to a sink at 500 K :

$$
\begin{gathered}
\Delta S_{\text {source }}=\frac{Q_{\text {soure }}}{T_{\text {source }}}=\frac{-2000 \mathrm{~kJ}}{800 \mathrm{~K}}=-2.5 \mathrm{~kJ} / \mathrm{K} \\
\Delta S_{\text {sink }}=\frac{Q_{\text {sink }}}{T_{\text {sink }}}=\frac{2000 \mathrm{~kJ}}{500 \mathrm{~K}}=+4.0 \mathrm{~kJ} / \mathrm{K}
\end{gathered}
$$

and

$$
S_{\text {gen }}=\Delta S_{\text {total }}=\Delta S_{\text {source }}+\Delta S_{\text {sink }}=(-2.5+4.0) \mathrm{kJ} / \mathrm{K}=\mathbf{1 . 5} \mathrm{kJ} / \mathrm{K}
$$

Therefore, $1.5 \mathrm{~kJ} / \mathrm{K}$ of entropy is generated during this process. Noting that both reservoirs have undergone internally reversible processes, the entire entropy generation took place in the partition.
(b) Repeating the calculations in part (a) for a sink temperature of 750 K , we obtain

$$
\begin{aligned}
\Delta S_{\text {source }} & =-2.5 \mathrm{~kJ} / \mathrm{k} \\
\Delta S_{\text {sink }} & =+2.7 \mathrm{~kJ} / \mathrm{K}
\end{aligned}
$$

and

$$
S_{\mathrm{gen}}=\Delta S_{\text {total }}=(-2.5+2.7) \mathrm{kJ} / \mathrm{K}=0.2 \mathrm{~kJ} / \mathrm{K}
$$

The total entropy change for the process in part $(b)$ is smaller, and therefore it is less irreversible. This is expected since the process in (b) involves a smaller temperature difference and thus a smaller irreversibility.
Discussion The irreversibilities associated with both processes could be eliminated by operating a Carnot heat engine between the source and the sink. For this case it can be shown that $\Delta S_{\text {total }}=0$.

## 7-3 - ENTROPY CHANGE OF PURE SUBSTANCES

Entropy is a property, and thus the value of entropy of a system is fixed once the state of the system is fixed. Specifying two intensive independent properties fixes the state of a simple compressible system, and thus the value of entropy, as well as the values of other properties at that state. Starting with its defining relation, the entropy change of a substance can be expressed in terms of other properties (see Sec. 7-7). But in general, these relations are too complicated and are not practical to use for hand calculations. Therefore, using a suitable reference state, the entropies of substances are evaluated from measurable property data following rather involved computations, and the results are tabulated in the same manner as the other properties such as $v, u$, and $h$ (Fig. 7-10).

The entropy values in the property tables are given relative to an arbitrary reference state. In steam tables the entropy of saturated liquid $s_{f}$ at $0.01^{\circ} \mathrm{C}$ is assigned the value of zero. For refrigerant-134a, the zero value is assigned to saturated liquid at $-40^{\circ} \mathrm{C}$. The entropy values become negative at temperatures below the reference value.

## Interactive

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FIGURE 7-10
The entropy of a pure substance is determined from the tables (like other properties).

The value of entropy at a specified state is determined just like any other property. In the compressed liquid and superheated vapor regions, it can be obtained directly from the tables at the specified state. In the saturated mixture region, it is determined from

$$
s=s_{f}+x s_{f g} \quad(\mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K})
$$

where $x$ is the quality and $s_{f}$ and $s_{f g}$ values are listed in the saturation tables. In the absence of compressed liquid data, the entropy of the compressed liquid can be approximated by the entropy of the saturated liquid at the given temperature:

$$
s_{@ T, P} \cong s_{f @ T} \quad(\mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K})
$$

The entropy change of a specified mass $m$ (a closed system) during a process is simply

$$
\begin{equation*}
\Delta S=m \Delta s=m\left(s_{2}-s_{1}\right) \quad(\mathrm{kJ} / \mathrm{K}) \tag{7-12}
\end{equation*}
$$

which is the difference between the entropy values at the final and initial states.
When studying the second-law aspects of processes, entropy is commonly used as a coordinate on diagrams such as the $T-s$ and $h-s$ diagrams. The general characteristics of the $T-s$ diagram of pure substances are shown in Fig. 7-11 using data for water. Notice from this diagram that the constantvolume lines are steeper than the constant-pressure lines and the constantpressure lines are parallel to the constant-temperature lines in the saturated liquid-vapor mixture region. Also, the constant-pressure lines almost coincide with the saturated liquid line in the compressed liquid region.

FIGURE 7-11
Schematic of the T-s diagram for water.


## EXAMPLE 7-3 Entropy Change of a Substance in a Tank

A rigid tank contains 5 kg of refrigerant-134a initially at $20^{\circ} \mathrm{C}$ and 140 kPa . The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa . Determine the entropy change of the refrigerant during this process.

Solution The refrigerant in a rigid tank is cooled while being stirred. The entropy change of the refrigerant is to be determined.
Assumptions The volume of the tank is constant and thus $v_{2}=v_{1}$.
Analysis We take the refrigerant in the tank as the system (Fig. 7-12). This is a closed system since no mass crosses the system boundary during the process. We note that the change in entropy of a substance during a process is simply the difference between the entropy values at the final and initial states. The initial state of the refrigerant is completely specified.

Recognizing that the specific volume remains constant during this process, the properties of the refrigerant at both states are
State 1: $\left.\quad P_{1}=140 \mathrm{kPa}\right\} \quad s_{1}=1.0624 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$

$$
\left.T_{1}=20^{\circ} \mathrm{C} \quad\right\} \quad v_{1}=0.16544 \mathrm{~m}^{3} / \mathrm{kg}
$$

State 2: $\left.\quad \begin{array}{l}P_{2}=100 \mathrm{kPa} \\ \left(\mathrm{V}_{2}=\mathrm{v}_{1}\right)\end{array}\right\} \quad \begin{aligned} & \mathrm{v}_{f}=0.0007259 \mathrm{~m}^{3} / \mathrm{kg} \\ & \mathrm{v}_{g}=0.19254 \mathrm{~m}^{3} / \mathrm{kg}\end{aligned}$
The refrigerant is a saturated liquid-vapor mixture at the final state since $v_{f}<v_{2}<v_{g}$ at 100 kPa pressure. Therefore, we need to determine the quality first:

$$
x_{2}=\frac{v_{2}-v_{f}}{v_{f g}}=\frac{0.16544-0.0007259}{0.19254-0.0007259}=0.859
$$

Thus,

$$
s_{2}=s_{f}+x_{2} s_{f g}=0.07188+(0.859)(0.87995)=0.8278 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}
$$

Then the entropy change of the refrigerant during this process is

$$
\begin{aligned}
\Delta S & =m\left(s_{2}-s_{1}\right)=(5 \mathrm{~kg})(0.8278-1.0624) \mathrm{kJ} / \mathrm{kg} \cdot \mathrm{~K} \\
& =-\mathbf{1 . 1 7 3} \mathbf{k J} / \mathbf{K}
\end{aligned}
$$

Discussion The negative sign indicates that the entropy of the system is decreasing during this process. This is not a violation of the second law, however, since it is the entropy generation $S_{\text {gen }}$ that cannot be negative.


FIGURE 7-12
Schematic and $T$-s diagram for Example 7-3.

## EXAMPLE 7-4 Entropy Change during a Constant-Pressure Process

A piston-cylinder device initially contains 3 lbm of liquid water at 20 psia and $70^{\circ} \mathrm{F}$. The water is now heated at constant pressure by the addition of 3450 Btu of heat. Determine the entropy change of the water during this process.

Solution Liquid water in a piston-cylinder device is heated at constant pressure. The entropy change of water is to be determined.
Assumptions 1 The tank is stationary and thus the kinetic and potential energy changes are zero, $\Delta \mathrm{KE}=\Delta \mathrm{PE}=0.2$ The process is quasi-equilibrium. 3 The pressure remains constant during the process and thus $P_{2}=P_{1}$. Analysis We take the water in the cylinder as the system (Fig. 7-13). This is a closed system since no mass crosses the system boundary during the process. We note that a piston-cylinder device typically involves a moving boundary and thus boundary work $W_{b}$. Also, heat is transferred to the system.

Water exists as a compressed liquid at the initial state since its pressure is greater than the saturation pressure of 0.3632 psia at $70^{\circ} \mathrm{F}$. By approximating the compressed liquid as a saturated liquid at the given temperature, the properties at the initial state are
$\left.\begin{array}{ll}\text { State 1: } & P_{1}=20 \mathrm{psia} \\ T_{1}=70^{\circ} \mathrm{F}\end{array}\right\} \begin{aligned} & s_{1} \cong s_{f @ 70^{\circ} \mathrm{F}}=0.07459 \mathrm{Btu} / \mathrm{lbm} \cdot \mathrm{R} \\ & h_{1} \cong h_{f @ 70^{\circ} \mathrm{F}}=38.08 \mathrm{Btu} / \mathrm{lbm}\end{aligned}$

$$
\left.T_{1}=70^{\circ} \mathrm{F} \quad\right\} \quad h_{1} \cong h_{f @ 70^{\circ} \mathrm{F}}=38.08 \mathrm{Btu} / \mathrm{lbm}
$$

At the final state, the pressure is still 20 psia, but we need one more property to fix the state. This property is determined from the energy balance,

$$
\begin{aligned}
\underbrace{E_{\text {in }}-E_{\text {out }}}_{\begin{array}{c}
\text { Net energy transfer } \\
\text { by heat, work, and mass }
\end{array}} & =\underbrace{\Delta E_{\text {system }}}_{\begin{array}{c}
\text { Change in internal, kinetic, } \\
\text { potential, etc., energies }
\end{array}} \\
Q_{\text {in }}-W_{b} & =\Delta U \\
Q_{\text {in }} & =\Delta H=m\left(h_{2}-h_{1}\right) \\
3450 \mathrm{Btu} & =(3 \mathrm{lbm})\left(h_{2}-38.08 \mathrm{Btu} / \mathrm{lbm}\right) \\
h_{2} & =1188.1 \mathrm{Btu} / \mathrm{lbm}
\end{aligned}
$$

since $\Delta U+W_{b}=\Delta H$ for a constant-pressure quasi-equilibrium process. Then,
State 2:

$$
\left.\begin{array}{l}
P_{2}=20 \mathrm{psia} \\
h_{2}=1188.1 \mathrm{Btu} / \mathrm{lbm}
\end{array}\right\} \quad \begin{aligned}
& s_{2}=1.7761 \mathrm{Btu} / \mathrm{lbm} \cdot \mathrm{R} \\
& (\text { Table A-6E, interpolation })
\end{aligned}
$$

FIGURE 7-13
Schematic and T-s diagram for Example 7-4.



Therefore, the entropy change of water during this process is

$$
\begin{aligned}
\Delta S=m\left(s_{2}-s_{1}\right) & =(3 \mathrm{lbm})(1.7761-0.07459) \mathrm{Btu} / \mathrm{lbm} \cdot \mathbf{R} \\
& =\mathbf{5 . 1 0 5} \mathbf{B t u} / \mathbf{R}
\end{aligned}
$$

## 7-4 • ISENTROPIC PROCESSES

We mentioned earlier that the entropy of a fixed mass can be changed by (1) heat transfer and (2) irreversibilities. Then it follows that the entropy of a fixed mass does not change during a process that is internally reversible and adiabatic (Fig. 7-14). A process during which the entropy remains constant is called an isentropic process. It is characterized by
Isentropic process:

$$
\begin{equation*}
\Delta s=0 \quad \text { or } \quad s_{2}=s_{1} \tag{7-13}
\end{equation*}
$$

$$
(\mathrm{kJ} / \mathrm{kg} \cdot \mathrm{~K})
$$

That is, a substance will have the same entropy value at the end of the process as it does at the beginning if the process is carried out in an isentropic manner.
Many engineering systems or devices such as pumps, turbines, nozzles, and diffusers are essentially adiabatic in their operation, and they perform best when the irreversibilities, such as the friction associated with the process, are minimized. Therefore, an isentropic process can serve as an appropriate model for actual processes. Also, isentropic processes enable us to define efficiencies for processes to compare the actual performance of these devices to the performance under idealized conditions.
It should be recognized that a reversible adiabatic process is necessarily isentropic ( $s_{2}=s_{1}$ ), but an isentropic process is not necessarily a reversible adiabatic process. (The entropy increase of a substance during a process as a result of irreversibilities may be offset by a decrease in entropy as a result of heat losses, for example.) However, the term isentropic process is customarily used in thermodynamics to imply an internally reversible, adiabatic process.

## EXAMPLE 7-5 Isentropic Expansion of Steam in a Turbine

Steam enters an adiabatic turbine at 5 MPa and $450^{\circ} \mathrm{C}$ and leaves at a pressure of 1.4 MPa . Determine the work output of the turbine per unit mass of steam if the process is reversible.

Solution Steam is expanded in an adiabatic turbine to a specified pressure in a reversible manner. The work output of the turbine is to be determined. Assumptions 1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\mathrm{CV}}=0, \Delta E_{\mathrm{CV}}=0$, and $\Delta S_{\mathrm{CV}}=0.2$ The process is reversible. 3 Kinetic and potential energies are negligible. 4 The turbine is adiabatic and thus there is no heat transfer.
Analysis We take the turbine as the system (Fig. 7-15). This is a control volume since mass crosses the system boundary during the process. We note that there is only one inlet and one exit, and thus $\dot{m}_{1}=\dot{m}_{2}=\dot{m}$.


FIGURE 7-14
During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.
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FIGURE 7-15
Schematic and $T$-s diagram for Example 7-5.

## (2) <br> interactive TUTORIAL

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FIGURE 7-16
On a $T$-S diagram, the area under the process curve represents the heat transfer for internally reversible processes.

The power output of the turbine is determined from the rate form of the energy balance,


$$
\begin{aligned}
\dot{E}_{\text {in }} & =\dot{E}_{\text {out }} \\
\dot{m} h_{1} & =\dot{W}_{\text {out }}+\dot{m} h_{2} \quad(\text { since } \dot{Q}=0, \mathrm{ke} \cong \mathrm{pe} \cong 0) \\
\dot{W}_{\text {out }} & =\dot{m}\left(h_{1}-h_{2}\right)
\end{aligned}
$$

The inlet state is completely specified since two properties are given. But only one property (pressure) is given at the final state, and we need one more property to fix it. The second property comes from the observation that the process is reversible and adiabatic, and thus isentropic. Therefore, $s_{2}=$ $s_{1}$, and

State 1:

$$
\left.\begin{array}{rl}
P_{1} & =5 \mathrm{MPa} \\
T_{1} & =450^{\circ} \mathrm{C}
\end{array}\right\} \quad \begin{aligned}
& h_{1}=3317.2 \mathrm{~kJ} / \mathrm{kg} \\
& s_{1}=6.8210 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}
\end{aligned}
$$

State 2:

$$
\left.\begin{array}{rl}
P_{2} & =1.4 \mathrm{MPa} \\
s_{2} & =s_{1}
\end{array}\right\} \quad h_{2}=2967.4 \mathrm{~kJ} / \mathrm{kg}
$$

Then the work output of the turbine per unit mass of the steam becomes

$$
w_{\text {out }}=h_{1}-h_{2}=3317.2-2967.4=349.8 \mathbf{k J} / \mathbf{k g}
$$

## 7-5 : PROPERTY DIAGRAMS INVOLVING ENTROPY

Property diagrams serve as great visual aids in the thermodynamic analysis of processes. We have used $P-V$ and $T-V$ diagrams extensively in previous chapters in conjunction with the first law of thermodynamics. In the secondlaw analysis, it is very helpful to plot the processes on diagrams for which one of the coordinates is entropy. The two diagrams commonly used in the second-law analysis are the temperature-entropy and the enthalpy-entropy diagrams.

Consider the defining equation of entropy (Eq. 7-4). It can be rearranged as

$$
\begin{equation*}
\delta Q_{\mathrm{intrev}}=T d S \tag{7-14}
\end{equation*}
$$

As shown in Fig. $7-16, \delta Q_{\text {rev int }}$ corresponds to a differential area on a $T-S$ diagram. The total heat transfer during an internally reversible process is determined by integration to be

$$
\begin{equation*}
Q_{\mathrm{int} \mathrm{rev}}=\int_{1}^{2} T d S \quad(\mathrm{~kJ}) \tag{7-15}
\end{equation*}
$$

which corresponds to the area under the process curve on a $T$-S diagram. Therefore, we conclude that the area under the process curve on a T-S diagram represents heat transfer during an internally reversible process. This is somewhat analogous to reversible boundary work being represented by
the area under the process curve on a $P-V$ diagram. Note that the area under the process curve represents heat transfer for processes that are internally (or totally) reversible. The area has no meaning for irreversible processes.

Equations 7-14 and 7-15 can also be expressed on a unit-mass basis as

$$
\begin{equation*}
\delta q_{\mathrm{int} \mathrm{rev}}=T d s \quad(\mathrm{~kJ} / \mathrm{kg}) \tag{7-16}
\end{equation*}
$$

and

$$
\begin{equation*}
q_{\mathrm{intrev}}=\int_{1}^{2} T d s \quad(\mathrm{~kJ} / \mathrm{kg}) \tag{7-17}
\end{equation*}
$$

To perform the integrations in Eqs. 7-15 and 7-17, one needs to know the relationship between $T$ and $s$ during a process. One special case for which these integrations can be performed easily is the internally reversible isothermal process. It yields

$$
\begin{equation*}
Q_{\mathrm{intrev}}=T_{0} \Delta S \tag{kJ}
\end{equation*}
$$

or

$$
\begin{equation*}
q_{\text {int rev }}=T_{0} \Delta s \quad(\mathrm{~kJ} / \mathrm{kg}) \tag{7-19}
\end{equation*}
$$

where $T_{0}$ is the constant temperature and $\Delta S$ is the entropy change of the system during the process.

An isentropic process on a $T-s$ diagram is easily recognized as a verticalline segment. This is expected since an isentropic process involves no heat transfer, and therefore the area under the process path must be zero (Fig. 7-17). The $T-s$ diagrams serve as valuable tools for visualizing the second-law aspects of processes and cycles, and thus they are frequently used in thermodynamics. The $T-s$ diagram of water is given in the appendix in Fig. A-9.

Another diagram commonly used in engineering is the enthalpy-entropy diagram, which is quite valuable in the analysis of steady-flow devices such as turbines, compressors, and nozzles. The coordinates of an $h$-s diagram represent two properties of major interest: enthalpy, which is a primary property in the first-law analysis of the steady-flow devices, and entropy, which is the property that accounts for irreversibilities during adiabatic processes. In analyzing the steady flow of steam through an adiabatic turbine, for example, the vertical distance between the inlet and the exit states $\Delta h$ is a measure of the work output of the turbine, and the horizontal distance $\Delta s$ is a measure of the irreversibilities associated with the process (Fig. 7-18).

The $h$-s diagram is also called a Mollier diagram after the German scientist R. Mollier (1863-1935). An $h-s$ diagram is given in the appendix for steam in Fig. A-10.

EXAMPLE 7-6 The T-S Diagram of the Carnot Cycle
Show the Carnot cycle on a T-S diagram and indicate the areas that represent the heat supplied $Q_{H}$, heat rejected $Q_{L}$, and the net work output $W_{\text {net,out }}$ on this diagram.


FIGURE 7-17
The isentropic process appears as a vertical line segment on a $T$-s diagram.


FIGURE 7-18
For adiabatic steady-flow devices, the vertical distance $\Delta h$ on an $h$-s diagram is a measure of work, and the horizontal distance $\Delta s$ is a measure of irreversibilities.


FIGURE 7-19
The $T$-S diagram of a Carnot cycle (Example 7-6).
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FIGURE 7-20
The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.

Solution The Carnot cycle is to be shown on a $T$-S diagram, and the areas that represent $Q_{H}, Q_{L}$, and $W_{\text {net,out }}$ are to be indicated.
Analysis Recall that the Carnot cycle is made up of two reversible isothermal ( $T=$ constant) processes and two isentropic ( $s=$ constant) processes. These four processes form a rectangle on a $T-S$ diagram, as shown in Fig. 7-19.

On a T-S diagram, the area under the process curve represents the heat transfer for that process. Thus the area $A 12 B$ represents $Q_{H}$, the area $A 43 B$ represents $Q_{L}$, and the difference between these two (the area in color) represents the net work since

$$
W_{\text {net,out }}=Q_{H}-Q_{L}
$$

Therefore, the area enclosed by the path of a cycle (area 1234) on a $T$-S diagram represents the net work. Recall that the area enclosed by the path of a cycle also represents the net work on a $P-V$ diagram.

## 7-6 • WHAT IS ENTROPY?

It is clear from the previous discussion that entropy is a useful property and serves as a valuable tool in the second-law analysis of engineering devices. But this does not mean that we know and understand entropy well. Because we do not. In fact, we cannot even give an adequate answer to the question, What is entropy? Not being able to describe entropy fully, however, does not take anything away from its usefulness. We could not define energy either, but it did not interfere with our understanding of energy transformations and the conservation of energy principle. Granted, entropy is not a household word like energy. But with continued use, our understanding of entropy will deepen, and our appreciation of it will grow. The next discussion should shed some light on the physical meaning of entropy by considering the microscopic nature of matter.
Entropy can be viewed as a measure of molecular disorder, or molecular randomness. As a system becomes more disordered, the positions of the molecules become less predictable and the entropy increases. Thus, it is not surprising that the entropy of a substance is lowest in the solid phase and highest in the gas phase (Fig. 7-20). In the solid phase, the molecules of a substance continually oscillate about their equilibrium positions, but they cannot move relative to each other, and their position at any instant can be predicted with good certainty. In the gas phase, however, the molecules move about at random, collide with each other, and change direction, making it extremely difficult to predict accurately the microscopic state of a system at any instant. Associated with this molecular chaos is a high value of entropy.
When viewed microscopically (from a statistical thermodynamics point of view), an isolated system that appears to be at a state of equilibrium may exhibit a high level of activity because of the continual motion of the molecules. To each state of macroscopic equilibrium there corresponds a large number of possible microscopic states or molecular configurations. The entropy of a system is related to the total number of possible microscopic
states of that system, called thermodynamic probability p, by the Boltzmann relation, expressed as

$$
\begin{equation*}
S=k \ln p \tag{7-20}
\end{equation*}
$$

where $k=1.3806 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ is the Boltzmann constant. Therefore, from a microscopic point of view, the entropy of a system increases whenever the molecular randomness or uncertainty (i.e., molecular probability) of a system increases. Thus, entropy is a measure of molecular disorder, and the molecular disorder of an isolated system increases anytime it undergoes a process.
As mentioned earlier, the molecules of a substance in solid phase continually oscillate, creating an uncertainty about their position. These oscillations, however, fade as the temperature is decreased, and the molecules supposedly become motionless at absolute zero. This represents a state of ultimate molecular order (and minimum energy). Therefore, the entropy of a pure crystalline substance at absolute zero temperature is zero since there is no uncertainty about the state of the molecules at that instant (Fig. 7-21). This statement is known as the third law of thermodynamics. The third law of thermodynamics provides an absolute reference point for the determination of entropy. The entropy determined relative to this point is called absolute entropy, and it is extremely useful in the thermodynamic analysis of chemical reactions. Notice that the entropy of a substance that is not pure crystalline (such as a solid solution) is not zero at absolute zero temperature. This is because more than one molecular configuration exists for such substances, which introduces some uncertainty about the microscopic state of the substance.
Molecules in the gas phase possess a considerable amount of kinetic energy. However, we know that no matter how large their kinetic energies are, the gas molecules do not rotate a paddle wheel inserted into the container and produce work. This is because the gas molecules, and the energy they possess, are disorganized. Probably the number of molecules trying to rotate the wheel in one direction at any instant is equal to the number of molecules that are trying to rotate it in the opposite direction, causing the wheel to remain motionless. Therefore, we cannot extract any useful work directly from disorganized energy (Fig. 7-22).
Now consider a rotating shaft shown in Fig. 7-23. This time the energy of the molecules is completely organized since the molecules of the shaft are rotating in the same direction together. This organized energy can readily be used to perform useful tasks such as raising a weight or generating electricity. Being an organized form of energy, work is free of disorder or randomness and thus free of entropy. There is no entropy transfer associated with energy transfer as work. Therefore, in the absence of any friction, the process of raising a weight by a rotating shaft (or a flywheel) does not produce any entropy. Any process that does not produce a net entropy is reversible, and thus the process just described can be reversed by lowering the weight. Therefore, energy is not degraded during this process, and no potential to do work is lost.

Instead of raising a weight, let us operate the paddle wheel in a container filled with a gas, as shown in Fig. 7-24. The paddle-wheel work in this case


FIGURE 7-21
A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero (the third law of thermodynamics).


FIGURE 7-22
Disorganized energy does not create much useful effect, no matter how large it is.


FIGURE 7-23
In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.


## FIGURE 7-24

The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.


## FIGURE 7-25

During a heat transfer process, the net entropy increases. (The increase in the entropy of the cold body more than offsets the decrease in the entropy of the hot body.)


## FIGURE 7-26

The use of entropy (disorganization, uncertainty) is not limited to thermodynamics.

[^0]is converted to the internal energy of the gas, as evidenced by a rise in gas temperature, creating a higher level of molecular disorder in the container. This process is quite different from raising a weight since the organized paddle-wheel energy is now converted to a highly disorganized form of energy, which cannot be converted back to the paddle wheel as the rotational kinetic energy. Only a portion of this energy can be converted to work by partially reorganizing it through the use of a heat engine. Therefore, energy is degraded during this process, the ability to do work is reduced, molecular disorder is produced, and associated with all this is an increase in entropy.

The quantity of energy is always preserved during an actual process (the first law), but the quality is bound to decrease (the second law). This decrease in quality is always accompanied by an increase in entropy. As an example, consider the transfer of 10 kJ of energy as heat from a hot medium to a cold one. At the end of the process, we still have the 10 kJ of energy, but at a lower temperature and thus at a lower quality.
Heat is, in essence, a form of disorganized energy, and some disorganization (entropy) flows with heat (Fig. 7-25). As a result, the entropy and the level of molecular disorder or randomness of the hot body decreases with the entropy and the level of molecular disorder of the cold body increases. The second law requires that the increase in entropy of the cold body be greater than the decrease in entropy of the hot body, and thus the net entropy of the combined system (the cold body and the hot body) increases. That is, the combined system is at a state of greater disorder at the final state. Thus we can conclude that processes can occur only in the direction of increased overall entropy or molecular disorder. That is, the entire universe is getting more and more chaotic every day.

## Entropy and Entropy Generation in Daily Life

The concept of entropy can also be applied to other areas. Entropy can be viewed as a measure of disorder or disorganization in a system. Likewise, entropy generation can be viewed as a measure of disorder or disorganization generated during a process. The concept of entropy is not used in daily life nearly as extensively as the concept of energy, even though entropy is readily applicable to various aspects of daily life. The extension of the entropy concept to nontechnical fields is not a novel idea. It has been the topic of several articles, and even some books. Next we present several ordinary events and show their relevance to the concept of entropy and entropy generation.
Efficient people lead low-entropy (highly organized) lives. They have a place for everything (minimum uncertainty), and it takes minimum energy for them to locate something. Inefficient people, on the other hand, are disorganized and lead high-entropy lives. It takes them minutes (if not hours) to find something they need, and they are likely to create a bigger disorder as they are searching since they will probably conduct the search in a disorganized manner (Fig. 7-26). People leading high-entropy lifestyles are always on the run, and never seem to catch up.
You probably noticed (with frustration) that some people seem to learn fast and remember well what they learn. We can call this type of learning
organized or low-entropy learning. These people make a conscientious effort to file the new information properly by relating it to their existing knowledge base and creating a solid information network in their minds. On the other hand, people who throw the information into their minds as they study, with no effort to secure it, may think they are learning. They are bound to discover otherwise when they need to locate the information, for example, during a test. It is not easy to retrieve information from a database that is, in a sense, in the gas phase. Students who have blackouts during tests should reexamine their study habits.
A library with a good shelving and indexing system can be viewed as a lowentropy library because of the high level of organization. Likewise, a library with a poor shelving and indexing system can be viewed as a high-entropy library because of the high level of disorganization. A library with no indexing system is like no library, since a book is of no value if it cannot be found.

Consider two identical buildings, each containing one million books. In the first building, the books are piled on top of each other, whereas in the second building they are highly organized, shelved, and indexed for easy reference. There is no doubt about which building a student will prefer to go to for checking out a certain book. Yet, some may argue from the first-law point of view that these two buildings are equivalent since the mass and knowledge content of the two buildings are identical, despite the high level of disorganization (entropy) in the first building. This example illustrates that any realistic comparisons should involve the second-law point of view.
Two textbooks that seem to be identical because both cover basically the same topics and present the same information may actually be very different depending on how they cover the topics. After all, two seemingly identical cars are not so identical if one goes only half as many miles as the other one on the same amount of fuel. Likewise, two seemingly identical books are not so identical if it takes twice as long to learn a topic from one of them as it does from the other. Thus, comparisons made on the basis of the first law only may be highly misleading.
Having a disorganized (high-entropy) army is like having no army at all. It is no coincidence that the command centers of any armed forces are among the primary targets during a war. One army that consists of 10 divisions is 10 times more powerful than 10 armies each consisting of a single division. Likewise, one country that consists of 10 states is more powerful than 10 countries, each consisting of a single state. The United States would not be such a powerful country if there were 50 independent countries in its place instead of a single country with 50 states. The European Union has the potential to be a new economic and political superpower. The old cliché "divide and conquer" can be rephrased as "increase the entropy and conquer."
We know that mechanical friction is always accompanied by entropy generation, and thus reduced performance. We can generalize this to daily life: friction in the workplace with fellow workers is bound to generate entropy, and thus adversely affect performance (Fig. 7-27). It results in reduced productivity.

We also know that unrestrained expansion (or explosion) and uncontrolled electron exchange (chemical reactions) generate entropy and are highly irreversible. Likewise, unrestrained opening of the mouth to scatter angry words


FIGURE 7-27
As in mechanical systems, friction in the workplace is bound to generate entropy and reduce performance.
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FIGURE 7-28
The $T d s$ relations are valid for both reversible and irreversible processes and for both closed and open systems.
is highly irreversible since this generates entropy, and it can cause considerable damage. A person who gets up in anger is bound to sit down at a loss. Hopefully, someday we will be able to come up with some procedures to quantify entropy generated during nontechnical activities, and maybe even pinpoint its primary sources and magnitude.

## 7-7 • THE T ds RELATIONS

Recall that the quantity $(\delta Q / T)_{\text {int rev }}$ corresponds to a differential change in the property entropy. The entropy change for a process, then, can be evaluated by integrating $\delta Q / T$ along some imaginary internally reversible path between the actual end states. For isothermal internally reversible processes, this integration is straightforward. But when the temperature varies during the process, we have to have a relation between $\delta Q$ and $T$ to perform this integration. Finding such relations is what we intend to do in this section.

The differential form of the conservation of energy equation for a closed stationary system (a fixed mass) containing a simple compressible substance can be expressed for an internally reversible process as

$$
\begin{equation*}
\delta Q_{\text {int rev }}-\delta W_{\text {int rev,out }}=d U \tag{7-21}
\end{equation*}
$$

But

$$
\begin{array}{r}
\delta Q_{\text {int rev }}=T d S \\
\delta W_{\text {int rev,out }}=P d V
\end{array}
$$

Thus,

$$
\begin{equation*}
T d S=d U+P d V \quad(\mathrm{~kJ}) \tag{7-22}
\end{equation*}
$$

or

$$
\begin{equation*}
T d s=d u+P d v \quad(\mathrm{~kJ} / \mathrm{kg}) \tag{7-23}
\end{equation*}
$$

This equation is known as the first $T d s$, or Gibbs, equation. Notice that the only type of work interaction a simple compressible system may involve as it undergoes an internally reversible process is the boundary work.

The second $T d s$ equation is obtained by eliminating $d u$ from Eq. 7-23 by using the definition of enthalpy ( $h=u+P \vee$ ):

$$
\left.\begin{array}{lll}
h=u+P v & \longrightarrow & d h=d u+P d v+v d P  \tag{7-24}\\
\text { (Eq. 7-23) } & \longrightarrow & T d s=d u+P d v
\end{array}\right\} T d s=d h-v d P
$$

Equations 7-23 and 7-24 are extremely valuable since they relate entropy changes of a system to the changes in other properties. Unlike Eq. 7-4, they are property relations and therefore are independent of the type of the processes.

These $T d s$ relations are developed with an internally reversible process in mind since the entropy change between two states must be evaluated along a reversible path. However, the results obtained are valid for both reversible and irreversible processes since entropy is a property and the change in a property between two states is independent of the type of process the system undergoes. Equations 7-23 and 7-24 are relations between the properties of a unit mass of a simple compressible system as it undergoes a change of state, and they are applicable whether the change occurs in a closed or an open system (Fig. 7-28).

Explicit relations for differential changes in entropy are obtained by solving for $d s$ in Eqs. 7-23 and 7-24:

$$
\begin{equation*}
d s=\frac{d u}{T}+\frac{P d v}{T} \tag{7-25}
\end{equation*}
$$

and

$$
\begin{equation*}
d s=\frac{d h}{T}-\frac{v d P}{T} \tag{7-26}
\end{equation*}
$$

The entropy change during a process can be determined by integrating either of these equations between the initial and the final states. To perform these integrations, however, we must know the relationship between $d u$ or $d h$ and the temperature (such as $d u=c_{\nu} d T$ and $d h=c_{p} d T$ for ideal gases) as well as the equation of state for the substance (such as the ideal-gas equation of state $P v=R T$ ). For substances for which such relations exist, the integration of Eq. 7-25 or 7-26 is straightforward. For other substances, we have to rely on tabulated data.
The $T d s$ relations for nonsimple systems, that is, systems that involve more than one mode of quasi-equilibrium work, can be obtained in a similar manner by including all the relevant quasi-equilibrium work modes.

## 7-8 : ENTROPY CHANGE OF LIQUIDS AND SOLIDS

Recall that liquids and solids can be approximated as incompressible substances since their specific volumes remain nearly constant during a process. Thus, $d \vee \cong 0$ for liquids and solids, and Eq. $7-25$ for this case reduces to

$$
\begin{equation*}
d s=\frac{d u}{T}=\frac{c d T}{T} \tag{7-27}
\end{equation*}
$$

since $c_{p}=c_{\vee}=c$ and $d u=c d T$ for incompressible substances. Then the entropy change during a process is determined by integration to be
Liquids, solids: $\quad s_{2}-s_{1}=\int_{1}^{2} c(T) \frac{d T}{T} \cong c_{\text {avg }} \ln \frac{T_{2}}{T_{1}} \quad(\mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K})$
where $c_{\text {avg }}$ is the average specific heat of the substance over the given temperature interval. Note that the entropy change of a truly incompressible substance depends on temperature only and is independent of pressure.
Equation 7-28 can be used to determine the entropy changes of solids and liquids with reasonable accuracy. However, for liquids that expand considerably with temperature, it may be necessary to consider the effects of volume change in calculations. This is especially the case when the temperature change is large.
A relation for isentropic processes of liquids and solids is obtained by setting the entropy change relation above equal to zero. It gives

Isentropic:

$$
\begin{equation*}
s_{2}-s_{1}=c_{\text {avg }} \ln \frac{T_{2}}{T_{1}}=0 \quad \rightarrow \quad T_{2}=T_{1} \tag{7-29}
\end{equation*}
$$

That is, the temperature of a truly incompressible substance remains constant during an isentropic process. Therefore, the isentropic process of an incompressible substance is also isothermal. This behavior is closely approximated by liquids and solids.


FIGURE 7-29
Schematic for Example 7-7.

## EXAMPLE 7-7 Effect of Density of a Liquid on Entropy

Liquid methane is commonly used in various cryogenic applications. The critical temperature of methane is 191 K (or $-82^{\circ} \mathrm{C}$ ), and thus methane must be maintained below 191 K to keep it in liquid phase. The properties of liquid methane at various temperatures and pressures are given in Table $7-1$. Determine the entropy change of liquid methane as it undergoes a process from 110 K and 1 MPa to 120 K and 5 MPa (a) using tabulated properties and (b) approximating liquid methane as an incompressible substance. What is the error involved in the latter case?

Solution Liquid methane undergoes a process between two specified states. The entropy change of methane is to be determined by using actual data and by assuming methane to be incompressible.
Analysis (a) We consider a unit mass of liquid methane (Fig. 7-29). The properties of the methane at the initial and final states are

State 1:

$$
\left.\left.\begin{array}{l}
P_{1}=1 \mathrm{MPa} \\
T_{1}=110 \mathrm{~K}
\end{array}\right\} \quad \begin{array}{r}
s_{1}=4.875 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K} \\
c_{p 1}=3.471 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K} \\
P_{2}=5 \mathrm{MPa} \\
T_{2}=120 \mathrm{~K}
\end{array}\right\} \quad \begin{array}{r}
s_{2}=5.145 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K} \\
c_{p 2}=3.486 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}
\end{array}
$$

Therefore,

$$
\Delta s=s_{2}-s_{1}=5.145-4.875=0.270 \mathbf{k J} / \mathbf{k g} \cdot \mathbf{K}
$$

(b) Approximating liquid methane as an incompressible substance, its entropy change is determined to be

$$
\Delta s=c_{\text {avg }} \ln \frac{T_{2}}{T_{1}}=(3.4785 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}) \ln \frac{120 \mathrm{~K}}{110 \mathrm{~K}}=0.303 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}
$$

since

$$
c_{\mathrm{avg}}=\frac{c_{p 1}+c_{p 2}}{2}=\frac{3.471+3.486}{2}=3.4785 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}
$$

## TABLE 7-1

Properties of liquid methane

| Temp., | Pressure, <br> $P, \mathrm{MPa}$ | Density, <br> $\rho, \mathrm{kg} / \mathrm{m}^{3}$ | Enthalpy, <br> $h, \mathrm{~kJ} / \mathrm{kg}$ | Entropy, <br> $s, \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$ | Specific <br> heat, <br> $c_{p}, \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 110 | 0.5 | 425.3 | 208.3 | 4.878 | 3.476 |
|  | 1.0 | 425.8 | 209.0 | 4.875 | 3.471 |
|  | 2.0 | 426.6 | 210.5 | 4.867 | 3.460 |
|  | 5.0 | 429.1 | 215.0 | 4.844 | 3.432 |
| 120 | 0.5 | 410.4 | 243.4 | 5.185 | 3.551 |
|  | 1.0 | 411.0 | 244.1 | 5.180 | 3.543 |
|  | 2.0 | 412.0 | 245.4 | 5.171 | 3.528 |
|  | 5.0 | 415.2 | 249.6 | 5.145 | 3.486 |

Therefore, the error involved in approximating liquid methane as an incompressible substance is

$$
\text { Error }=\frac{\left|\Delta s_{\text {actual }}-\Delta s_{\text {ideal }}\right|}{\Delta s_{\text {actual }}}=\frac{|0.270-0.303|}{0.270}=\mathbf{0 . 1 2 2}(\text { or } \mathbf{1 2 . 2 \%})
$$

Discussion This result is not surprising since the density of liquid methane changes during this process from 425.8 to $415.2 \mathrm{~kg} / \mathrm{m}^{3}$ (about 3 percent), which makes us question the validity of the incompressible substance assumption. Still, this assumption enables us to obtain reasonably accurate results with less effort, which proves to be very convenient in the absence of compressed liquid data.

## EXAMPLE 7-8 Economics of Replacing a Valve by a Turbine

A cryogenic manufacturing facility handles liquid methane at 115 K and 5 MPa at a rate of $0.280 \mathrm{~m}^{3} / \mathrm{s}$. A process requires dropping the pressure of liquid methane to 1 MPa , which is done by throttling the liquid methane by passing it through a flow resistance such as a valve. A recently hired engineer proposes to replace the throttling valve by a turbine in order to produce power while dropping the pressure to 1 MPa . Using data from Table 7-1, determine the maximum amount of power that can be produced by such a turbine. Also, determine how much this turbine will save the facility from electricity usage costs per year if the turbine operates continuously (8760 $\mathrm{h} / \mathrm{yr}$ ) and the facility pays $\$ 0.075 / \mathrm{kWh}$ for electricity.

Solution Liquid methane is expanded in a turbine to a specified pressure at a specified rate. The maximum power that this turbine can produce and the amount of money it can save per year are to be determined.
Assumptions 1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\mathrm{CV}}=0, \Delta E_{\mathrm{CV}}=0$, and $\Delta S_{\mathrm{CV}}=0$. 2 The turbine is adiabatic and thus there is no heat transfer. 3 The process is reversible. 4 Kinetic and potential energies are negligible.
Analysis We take the turbine as the system (Fig. 7-30). This is a control volume since mass crosses the system boundary during the process. We note that there is only one inlet and one exit and thus $\dot{m}_{1}=\dot{m}_{2}=\dot{m}$.

The assumptions above are reasonable since a turbine is normally well insulated and it must involve no irreversibilities for best performance and thus maximum power production. Therefore, the process through the turbine must be reversible adiabatic or isentropic. Then, $s_{2}=s_{1}$ and

State 1:

$$
\left.\begin{array}{l}
P_{1}=5 \mathrm{MPa} \\
T_{1}=115 \mathrm{~K}
\end{array}\right\}\left\{\begin{array}{l}
h_{1}=232.3 \mathrm{~kJ} / \mathrm{kg} \\
s_{1}=4.9945 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K} \\
\rho_{1}=422.15 \mathrm{~kg} / \mathrm{s}
\end{array}\right.
$$

State 2:

$$
\left.\begin{array}{rl}
P_{2} & =1 \mathrm{MPa} \\
s_{2} & =s_{1}
\end{array}\right\} \quad h_{2}=222.8 \mathrm{~kJ} / \mathrm{kg}
$$

Also, the mass flow rate of liquid methane is

$$
\dot{m}=\rho_{1} \dot{V}_{1}=\left(422.15 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(0.280 \mathrm{~m}^{3} / \mathrm{s}\right)=118.2 \mathrm{~kg} / \mathrm{s}
$$



FIGURE 7-30
A 1.0-MW liquified natural gas (LNG) turbine with $95-\mathrm{cm}$ turbine runner diameter being installed in a cryogenic test facility.

[^1]Then the power output of the turbine is determined from the rate form of the energy balance to be


$$
\begin{aligned}
\dot{E}_{\text {in }} & =\dot{E}_{\text {out }} \\
\dot{m} h_{1} & =\dot{W}_{\text {out }}+\dot{m} h_{2} \quad(\text { since } \dot{Q}=0, \mathrm{ke} \cong \text { pe } \cong 0) \\
\dot{W}_{\text {out }} & =\dot{m}\left(h_{1}-h_{2}\right) \\
& =(118.2 \mathrm{~kg} / \mathrm{s})(232.3-222.8) \mathrm{kJ} / \mathrm{kg} \\
& =1123 \mathrm{~kW}
\end{aligned}
$$

For continuous operation ( $365 \times 24=8760$ h), the amount of power produced per year is

$$
\begin{aligned}
\text { Annual power production } & =\dot{W}_{\text {out }} \times \Delta t=(1123 \mathrm{~kW})(8760 \mathrm{~h} / \mathrm{yr}) \\
& =0.9837 \times 10^{7} \mathrm{kWh} / \mathrm{yr}
\end{aligned}
$$

At $\$ 0.075 / \mathrm{kWh}$, the amount of money this turbine can save the facility is
Annual power savings $=($ Annual power production $)($ Unit cost of power $)$

$$
\begin{aligned}
& =\left(0.9837 \times 10^{7} \mathrm{kWh} / \mathrm{yr}\right)(\$ 0.075 / \mathrm{kWh}) \\
& =\$ 737,800 / \mathrm{yr}
\end{aligned}
$$

That is, this turbine can save the facility $\$ 737,800$ a year by simply taking advantage of the potential that is currently being wasted by a throttling valve, and the engineer who made this observation should be rewarded. Discussion This example shows the importance of the property entropy since it enabled us to quantify the work potential that is being wasted. In practice, the turbine will not be isentropic, and thus the power produced will be less. The analysis above gave us the upper limit. An actual turbine-generator assembly can utilize about 80 percent of the potential and produce more than 900 kW of power while saving the facility more than \$600,000 a year.

It can also be shown that the temperature of methane drops to 113.9 K (a drop of 1.1 K ) during the isentropic expansion process in the turbine instead of remaining constant at 115 K as would be the case if methane were assumed to be an incompressible substance. The temperature of methane would rise to 116.6 K (a rise of 1.6 K ) during the throttling process.

## 7-9 • THE ENTROPY CHANGE OF IDEAL GASES

An expression for the entropy change of an ideal gas can be obtained from Eq. $7-25$ or 7-26 by employing the property relations for ideal gases (Fig. $7-31$ ). By substituting $d u=c_{v} d T$ and $P=R T / v$ into Eq. 7-25, the differential entropy change of an ideal gas becomes

$$
\begin{equation*}
d s=c_{v} \frac{d T}{T}+R \frac{d V}{V} \tag{7-30}
\end{equation*}
$$

The entropy change for a process is obtained by integrating this relation between the end states:

$$
\begin{equation*}
s_{2}-s_{1}=\int_{1}^{2} c_{v}(T) \frac{d T}{T}+R \ln \frac{V_{2}}{V_{1}} \tag{7-31}
\end{equation*}
$$

A second relation for the entropy change of an ideal gas is obtained in a similar manner by substituting $d h=c_{p} d T$ and $v=R T / P$ into Eq. 7-26 and integrating. The result is

$$
\begin{equation*}
s_{2}-s_{1}=\int_{1}^{2} c_{p}(T) \frac{d T}{T}-R \ln \frac{P_{2}}{P_{1}} \tag{7-32}
\end{equation*}
$$

The specific heats of ideal gases, with the exception of monatomic gases, depend on temperature, and the integrals in Eqs. 7-31 and 7-32 cannot be performed unless the dependence of $c_{V}$ and $c_{p}$ on temperature is known. Even when the $c_{V}(T)$ and $c_{p}(T)$ functions are available, performing long integrations every time entropy change is calculated is not practical. Then two reasonable choices are left: either perform these integrations by simply assuming constant specific heats or evaluate those integrals once and tabulate the results. Both approaches are presented next.

## Constant Specific Heats (Approximate Analysis)

Assuming constant specific heats for ideal gases is a common approximation, and we used this assumption before on several occasions. It usually simplifies the analysis greatly, and the price we pay for this convenience is some loss in accuracy. The magnitude of the error introduced by this assumption depends on the situation at hand. For example, for monatomic ideal gases such as helium, the specific heats are independent of temperature, and therefore the constant-specific-heat assumption introduces no error. For ideal gases whose specific heats vary almost linearly in the temperature range of interest, the possible error is minimized by using specific heat values evaluated at the average temperature (Fig. 7-32). The results obtained in this way usually are sufficiently accurate if the temperature range is not greater than a few hundred degrees.

The entropy-change relations for ideal gases under the constant-specificheat assumption are easily obtained by replacing $c_{v}(T)$ and $c_{p}(T)$ in Eqs. $7-31$ and $7-32$ by $c_{v, \text { avg }}$ and $c_{p, \text { avg }}$, respectively, and performing the integrations. We obtain

$$
\begin{equation*}
s_{2}-s_{1}=c_{V, \text { avg }} \ln \frac{T_{2}}{T_{1}}+R \ln \frac{V_{2}}{V_{1}} \quad(\mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}) \tag{7-33}
\end{equation*}
$$

and

$$
\begin{equation*}
s_{2}-s_{1}=c_{p, \text { avg }} \ln \frac{T_{2}}{T_{1}}-R \ln \frac{P_{2}}{P_{1}} \quad(\mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}) \tag{7-34}
\end{equation*}
$$

Entropy changes can also be expressed on a unit-mole basis by multiplying these relations by molar mass:

$$
\begin{equation*}
\bar{s}_{2}-\bar{s}_{1}=\bar{c}_{\mathrm{V}, \mathrm{avg}} \ln \frac{T_{2}}{T_{1}}+R_{u} \ln \frac{\mathrm{~V}_{2}}{V_{1}} \quad(\mathrm{~kJ} / \mathrm{kmol} \cdot \mathrm{~K}) \tag{7-35}
\end{equation*}
$$



FIGURE 7-31
A broadcast from channel IG.
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FIGURE 7-32
Under the constant-specific-heat assumption, the specific heat is assumed to be constant at some average value.


FIGURE 7-33
The entropy of an ideal gas depends on both $T$ and $P$. The function $s^{\circ}$ represents only the temperaturedependent part of entropy.
and

$$
\begin{equation*}
\bar{s}_{2}-\bar{s}_{1}=\bar{c}_{p, \text { avg }} \ln \frac{T_{2}}{T_{1}}-R_{u} \ln \frac{P_{2}}{P_{1}} \quad(\mathrm{~kJ} / \mathrm{kmol} \cdot \mathrm{~K}) \tag{7-36}
\end{equation*}
$$

## Variable Specific Heats (Exact Analysis)

When the temperature change during a process is large and the specific heats of the ideal gas vary nonlinearly within the temperature range, the assumption of constant specific heats may lead to considerable errors in entropy-change calculations. For those cases, the variation of specific heats with temperature should be properly accounted for by utilizing accurate relations for the specific heats as a function of temperature. The entropy change during a process is then determined by substituting these $c_{\mathrm{v}}(T)$ or $c_{p}(T)$ relations into Eq. 7-31 or 7-32 and performing the integrations.

Instead of performing these laborious integrals each time we have a new process, it is convenient to perform these integrals once and tabulate the results. For this purpose, we choose absolute zero as the reference temperature and define a function $s^{\circ}$ as

$$
\begin{equation*}
s^{\circ}=\int_{0}^{T} c_{p}(T) \frac{d T}{T} \tag{7-37}
\end{equation*}
$$

Obviously, $s^{\circ}$ is a function of temperature alone, and its value is zero at absolute zero temperature. The values of $s^{\circ}$ are calculated at various temperatures, and the results are tabulated in the appendix as a function of temperature for air. Given this definition, the integral in Eq. 7-32 becomes

$$
\begin{equation*}
\int_{1}^{2} c_{p}(T) \frac{d T}{T}=s_{2}^{\circ}-s_{1}^{\circ} \tag{7-38}
\end{equation*}
$$

where $s_{2}^{\circ}$ is the value of $s^{\circ}$ at $T_{2}$ and $s_{1}^{\circ}$ is the value at $T_{1}$. Thus,

$$
\begin{equation*}
s_{2}-s_{1}=s_{2}^{\circ}-s_{1}^{\circ}-R \ln \frac{P_{2}}{P_{1}} \quad(\mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}) \tag{7-39}
\end{equation*}
$$

It can also be expressed on a unit-mole basis as

$$
\begin{equation*}
\bar{s}_{2}-\bar{s}_{1}=\bar{s}_{2}^{\circ}-\bar{s}_{1}^{\circ}-R_{u} \ln \frac{P_{2}}{P_{1}} \quad(\mathrm{~kJ} / \mathrm{kmol} \cdot \mathrm{~K}) \tag{7-40}
\end{equation*}
$$

Note that unlike internal energy and enthalpy, the entropy of an ideal gas varies with specific volume or pressure as well as the temperature. Therefore, entropy cannot be tabulated as a function of temperature alone. The $s^{\circ}$ values in the tables account for the temperature dependence of entropy (Fig. $7-33$ ). The variation of entropy with pressure is accounted for by the last term in Eq. 7-39. Another relation for entropy change can be developed based on Eq. 7-31, but this would require the definition of another function and tabulation of its values, which is not practical.


## EXAMPLE 7-9 Entropy Change of an Ideal Gas

Air is compressed from an initial state of 100 kPa and $17^{\circ} \mathrm{C}$ to a final state of 600 kPa and $57^{\circ} \mathrm{C}$. Determine the entropy change of air during this compression process by using (a) property values from the air table and (b) average specific heats.

Solution Air is compressed between two specified states. The entropy change of air is to be determined by using tabulated property values and also by using average specific heats.
Assumptions Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. Therefore, entropy change relations developed under the ideal-gas assumption are applicable.
Analysis A sketch of the system and the T-s diagram for the process are given in Fig. 7-34. We note that both the initial and the final states of air are completely specified.
(a) The properties of air are given in the air table (Table A-17). Reading $s^{\circ}$ values at given temperatures and substituting, we find

$$
\begin{aligned}
s_{2}-s_{1} & =s_{2}^{\circ}-s_{1}^{\circ}-R \ln \frac{P_{2}}{P_{1}} \\
& =[(1.79783-1.66802) \mathrm{kJ} / \mathrm{kg} \cdot \mathrm{~K}]-(0.287 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}) \ln \frac{600 \mathrm{kPa}}{100 \mathrm{kPa}} \\
& =-\mathbf{0 . 3 8 4 4} \mathbf{~ k J} / \mathrm{kg} \cdot \mathbf{K}
\end{aligned}
$$

(b) The entropy change of air during this process can also be determined approximately from Eq. 7-34 by using a $c_{p}$ value at the average temperature of $37^{\circ} \mathrm{C}$ (Table A-2b) and treating it as a constant:

$$
\begin{aligned}
s_{2}-s_{1} & =c_{p, \text { avg }} \ln \frac{T_{2}}{T_{1}}-R \ln \frac{P_{2}}{P_{1}} \\
& =(1.006 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}) \ln \frac{330 \mathrm{~K}}{290 \mathrm{~K}}-(0.287 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}) \ln \frac{600 \mathrm{kPa}}{100 \mathrm{kPa}} \\
& =-\mathbf{0 . 3 8 4 2} \mathbf{~ k J} / \mathrm{kg} \cdot \mathbf{K}
\end{aligned}
$$

Discussion The two results above are almost identical since the change in temperature during this process is relatively small (Fig. 7-35). When the temperature change is large, however, they may differ significantly. For those cases, Eq. 7-39 should be used instead of Eq. 7-34 since it accounts for the variation of specific heats with temperature.

FIGURE 7-34
Schematic and T-s diagram for Example 7-9.


FIGURE 7-35
For small temperature differences, the exact and approximate relations for entropy changes of ideal gases give almost identical results.


FIGURE 7-36
The isentropic relations of ideal gases are valid for the isentropic processes of ideal gases only.

## Isentropic Processes of Ideal Gases

Several relations for the isentropic processes of ideal gases can be obtained by setting the entropy-change relations developed previously equal to zero. Again, this is done first for the case of constant specific heats and then for the case of variable specific heats.

## Constant Specific Heats (Approximate Analysis)

When the constant-specific-heat assumption is valid, the isentropic relations for ideal gases are obtained by setting Eqs. 7-33 and 7-34 equal to zero. From Eq. 7-33,

$$
\ln \frac{T_{2}}{T_{1}}=-\frac{R}{c_{v}} \ln \frac{V_{2}}{V_{1}}
$$

which can be rearranged as

$$
\begin{equation*}
\ln \frac{T_{2}}{T_{1}}=\ln \left(\frac{V_{1}}{V_{2}}\right)^{R / c_{v}} \tag{7-41}
\end{equation*}
$$

or

$$
\begin{equation*}
\left(\frac{T_{2}}{T_{1}}\right)_{s=\text { const. }}=\left(\frac{V_{1}}{V_{2}}\right)^{k-1} \quad \text { (ideal gas) } \tag{7-42}
\end{equation*}
$$

since $R=c_{p}-c_{v}, k=c_{p} / c_{v}$, and thus $R / c_{v}=k-1$.
Equation 7-42 is the first isentropic relation for ideal gases under the constant-specific-heat assumption. The second isentropic relation is obtained in a similar manner from Eq. 7-34 with the following result:

$$
\begin{equation*}
\left(\frac{T_{2}}{T_{1}}\right)_{s=\text { const. }}=\left(\frac{P_{2}}{P_{1}}\right)^{(k-1) / k} \quad \text { (ideal gas) } \tag{7-43}
\end{equation*}
$$

The third isentropic relation is obtained by substituting Eq. 7-43 into Eq. 7-42 and simplifying:

$$
\begin{equation*}
\left(\frac{P_{2}}{P_{1}}\right)_{s=\text { const. }}=\left(\frac{V_{1}}{V_{2}}\right)^{k} \quad \text { (ideal gas) } \tag{7-44}
\end{equation*}
$$

Equations 7-42 through 7-44 can also be expressed in a compact form as

$$
\begin{align*}
T \nu^{k-1} & =\text { constant }  \tag{7-45}\\
T P^{(1-k) / k} & =\text { constant } \quad \text { (ideal gas) }  \tag{7-46}\\
P \cup^{k} & =\text { constant } \tag{7-47}
\end{align*}
$$

The specific heat ratio $k$, in general, varies with temperature, and thus an average $k$ value for the given temperature range should be used.
Note that the ideal-gas isentropic relations above, as the name implies, are strictly valid for isentropic processes only when the constant-specific-heat assumption is appropriate (Fig. 7-36).

## Variable Specific Heats (Exact Analysis)

When the constant-specific-heat assumption is not appropriate, the isentropic relations developed previously yields results that are not quite accurate. For such cases, we should use an isentropic relation obtained from Eq. 7-39 that accounts for the variation of specific heats with temperature. Setting this equation equal to zero gives

$$
0=s_{2}^{\circ}-s_{1}^{\circ}-R \ln \frac{P_{2}}{P_{1}}
$$

or

$$
\begin{equation*}
s_{2}^{\circ}=s_{1}^{\circ}+R \ln \frac{P_{2}}{P_{1}} \tag{7-48}
\end{equation*}
$$

where $s_{2}^{\circ}$ is the $s^{\circ}$ value at the end of the isentropic process.

## Relative Pressure and Relative Specific Volume

Equation 7-48 provides an accurate way of evaluating property changes of ideal gases during isentropic processes since it accounts for the variation of specific heats with temperature. However, it involves tedious iterations when the volume ratio is given instead of the pressure ratio. This is quite an inconvenience in optimization studies, which usually require numerous repetitive calculations. To remedy this deficiency, we define two new dimensionless quantities associated with isentropic processes.
The definition of the first is based on Eq. 7-48, which can be rearranged as

$$
\frac{P_{2}}{P_{1}}=\exp \frac{s_{2}^{\circ}-s_{1}^{\circ}}{R}
$$

or

$$
\frac{P_{2}}{P_{1}}=\frac{\exp \left(s_{2}^{\circ} / R\right)}{\exp \left(s_{1}^{\circ} / R\right)}
$$

The quantity $\exp \left(s^{\circ} / R\right)$ is defined as the relative pressure $P_{r}$. With this definition, the last relation becomes

$$
\begin{equation*}
\left(\frac{P_{2}}{P_{1}}\right)_{s=\text { const. }}=\frac{P_{r 2}}{P_{r 1}} \tag{7-49}
\end{equation*}
$$

Note that the relative pressure $P_{r}$ is a dimensionless quantity that is a function of temperature only since $s^{\circ}$ depends on temperature alone. Therefore, values of $P_{r}$ can be tabulated against temperature. This is done for air in Table A-17. The use of $P_{r}$ data is illustrated in Fig. 7-37.

Sometimes specific volume ratios are given instead of pressure ratios. This is particularly the case when automotive engines are analyzed. In such cases, one needs to work with volume ratios. Therefore, we define another quantity related to specific volume ratios for isentropic processes. This is done by utilizing the ideal-gas relation and Eq. 7-49:

$$
\frac{P_{1} \vee_{1}}{T_{1}}=\frac{P_{2} \vee_{2}}{T_{2}} \rightarrow \frac{\vee_{2}}{V_{1}}=\frac{T_{2}}{T_{1}} \frac{P_{1}}{P_{2}}=\frac{T_{2}}{T_{1}} \frac{P_{r 1}}{P_{r 2}}=\frac{T_{2} / P_{r 2}}{T_{1} / P_{r 1}}
$$



FIGURE 7-37
The use of $P_{r}$ data for calculating the final temperature during an isentropic process.

The quantity $T / P_{r}$ is a function of temperature only and is defined as relative specific volume $v_{r}$. Thus,

$$
\begin{equation*}
\left(\frac{V_{2}}{V_{1}}\right)_{s=\text { const. }}=\frac{V_{r 2}}{V_{r 1}} \tag{7-50}
\end{equation*}
$$

Equations 7-49 and 7-50 are strictly valid for isentropic processes of ideal gases only. They account for the variation of specific heats with temperature and therefore give more accurate results than Eqs. 7-42 through 7-47. The values of $P_{r}$ and $v_{r}$ are listed for air in Table A-17.

## EXAMPLE 7-10 Isentropic Compression of Air in a Car Engine

Air is compressed in a car engine from $22^{\circ} \mathrm{C}$ and 95 kPa in a reversible and adiabatic manner. If the compression ratio $V_{1} / V_{2}$ of this engine is 8 , determine the final temperature of the air.

Solution Air is compressed in a car engine isentropically. For a given compression ratio, the final air temperature is to be determined.
Assumptions At specified conditions, air can be treated as an ideal gas. Therefore, the isentropic relations for ideal gases are applicable.
Analysis A sketch of the system and the $T$-s diagram for the process are given in Fig. 7-38.

This process is easily recognized as being isentropic since it is both reversible and adiabatic. The final temperature for this isentropic process can be determined from Eq. 7-50 with the help of relative specific volume data (Table A-17), as illustrated in Fig. 7-39.
For closed systems: $\quad \frac{V_{2}}{V_{1}}=\frac{V_{2}}{V_{1}}$
At $T_{1}=295 \mathrm{~K}: \quad \mathrm{V}_{r 1}=647.9$
From Eq. 7-50: $\quad v_{r 2}=v_{r 1}\left(\frac{v_{2}}{v_{1}}\right)=(647.9)\left(\frac{1}{8}\right)=80.99 \rightarrow T_{2}=662.7 \mathrm{~K}$
Therefore, the temperature of air will increase by $367.7^{\circ} \mathrm{C}$ during this process.

FIGURE 7-38
Schematic and T-s diagram for Example 7-10.


Alternative Solution The final temperature could also be determined from Eq. 7-42 by assuming constant specific heats for air:

$$
\left(\frac{T_{2}}{T_{1}}\right)_{s=\text { const. }}=\left(\frac{V_{1}}{V_{2}}\right)^{k-1}
$$

The specific heat ratio $k$ also varies with temperature, and we need to use the value of $k$ corresponding to the average temperature. However, the final temperature is not given, and so we cannot determine the average temperature in advance. For such cases, calculations can be started with a $k$ value at the initial or the anticipated average temperature. This value could be refined later, if necessary, and the calculations can be repeated. We know that the temperature of the air will rise considerably during this adiabatic compression process, so we guess the average temperature to be about 450 K . The $k$ value at this anticipated average temperature is determined from Table $\mathrm{A}-2 b$ to be 1.391 . Then the final temperature of air becomes

$$
T_{2}=(295 \mathrm{~K})(8)^{1.391-1}=665.2 \mathrm{~K}
$$

This gives an average temperature value of 480.1 K , which is sufficiently close to the assumed value of 450 K . Therefore, it is not necessary to repeat the calculations by using the $k$ value at this average temperature.

The result obtained by assuming constant specific heats for this case is in error by about 0.4 percent, which is rather small. This is not surprising since the temperature change of air is relatively small (only a few hundred degrees) and the specific heats of air vary almost linearly with temperature in this temperature range.

## EXAMPLE 7-11 Isentropic Compression of an Ideal Gas

Helium gas is compressed by an adiabatic compressor from an initial state of 14 psia and $50^{\circ} \mathrm{F}$ to a final temperature of $320^{\circ} \mathrm{F}$ in a reversible manner. Determine the exit pressure of helium.

Solution Helium is compressed from a given state to a specified pressure isentropically. The exit pressure of helium is to be determined.
Assumptions At specified conditions, helium can be treated as an ideal gas. Therefore, the isentropic relations developed earlier for ideal gases are applicable.
Analysis A sketch of the system and the T-s diagram for the process are given in Fig. 7-40.

The specific heat ratio $k$ of helium is 1.667 and is independent of temperature in the region where it behaves as an ideal gas. Thus the final pressure of helium can be determined from Eq. 7-43:

$$
P_{2}=P_{1}\left(\frac{T_{2}}{T_{1}}\right)^{k /(k-1)}=(14 \mathrm{psia})\left(\frac{780 \mathrm{R}}{510 \mathrm{R}}\right)^{1.667 / 0.667}=40.5 \mathrm{psia}
$$



## FIGURE 7-39

The use of $v_{r}$ data for calculating the final temperature during an isentropic process (Example 7-10).

FIGURE 7-40
Schematic and $T-s$ diagram for Example 7-11.
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## 7-10 • REVERSIBLE STEADY-FLOW WORK

The work done during a process depends on the path followed as well as on the properties at the end states. Recall that reversible (quasi-equilibrium) moving boundary work associated with closed systems is expressed in terms of the fluid properties as

$$
W_{b}=\int_{1}^{2} P d V
$$

We mentioned that the quasi-equilibrium work interactions lead to the maximum work output for work-producing devices and the minimum work input for work-consuming devices.

It would also be very insightful to express the work associated with steady-flow devices in terms of fluid properties.
Taking the positive direction of work to be from the system (work output), the energy balance for a steady-flow device undergoing an internally reversible process can be expressed in differential form as

$$
\delta q_{\mathrm{rev}}-\delta w_{\mathrm{rev}}=d h+d \mathrm{ke}+d \mathrm{pe}
$$

But

$$
\left.\begin{array}{ll}
\delta q_{\mathrm{rev}}=T d s & (\text { Eq. 7-16) } \\
T d s=d h-v d P & (\text { Eq. 7-24) }
\end{array}\right\} \quad \delta q_{\mathrm{rev}}=d h-v d P
$$

Substituting this into the relation above and canceling $d h$ yield

$$
-\delta w_{\mathrm{rev}}=v d P+d \mathrm{ke}+d \mathrm{pe}
$$

Integrating, we find

$$
\begin{equation*}
w_{\mathrm{rev}}=-\int_{1}^{2} V d P-\Delta \mathrm{ke}-\Delta \mathrm{pe} \quad(\mathrm{~kJ} / \mathrm{kg}) \tag{7-51}
\end{equation*}
$$

When the changes in kinetic and potential energies are negligible, this equation reduces to

$$
\begin{equation*}
w_{\mathrm{rev}}=-\int_{1}^{2} v d P \quad(\mathrm{~kJ} / \mathrm{kg}) \tag{7-52}
\end{equation*}
$$

Equations 7-51 and 7-52 are relations for the reversible work output associated with an internally reversible process in a steady-flow device. They will
give a negative result when work is done on the system. To avoid the negative sign, Eq. 7-51 can be written for work input to steady-flow devices such as compressors and pumps as

$$
\begin{equation*}
w_{\mathrm{rev}, \mathrm{in}}=\int_{1}^{2} v d P+\Delta \mathrm{ke}+\Delta \mathrm{pe} \tag{7-53}
\end{equation*}
$$

The resemblance between the $v d P$ in these relations and $P d v$ is striking. They should not be confused with each other, however, since $P d \vee$ is associated with reversible boundary work in closed systems (Fig. 7-41).

Obviously, one needs to know $v$ as a function of $P$ for the given process to perform the integration. When the working fluid is incompressible, the specific volume $v$ remains constant during the process and can be taken out of the integration. Then Eq. 7-51 simplifies to

$$
\begin{equation*}
w_{\mathrm{rev}}=-v\left(P_{2}-P_{1}\right)-\Delta \mathrm{ke}-\Delta \mathrm{pe} \quad(\mathrm{~kJ} / \mathrm{kg}) \tag{7-54}
\end{equation*}
$$

For the steady flow of a liquid through a device that involves no work interactions (such as a nozzle or a pipe section), the work term is zero, and the equation above can be expressed as

$$
\begin{equation*}
v\left(P_{2}-P_{1}\right)+\frac{V_{2}^{2}-V_{1}^{1}}{2}+g\left(z_{2}-z_{1}\right)=0 \tag{7-55}
\end{equation*}
$$

which is known as the Bernoulli equation in fluid mechanics. It is developed for an internally reversible process and thus is applicable to incompressible fluids that involve no irreversibilities such as friction or shock waves. This equation can be modified, however, to incorporate these effects.

Equation 7-52 has far-reaching implications in engineering regarding devices that produce or consume work steadily such as turbines, compressors, and pumps. It is obvious from this equation that the reversible steadyflow work is closely associated with the specific volume of the fluid flowing through the device. The larger the specific volume, the larger the reversible work produced or consumed by the steady-flow device (Fig. 7-42). This conclusion is equally valid for actual steady-flow devices. Therefore, every effort should be made to keep the specific volume of a fluid as small as possible during a compression process to minimize the work input and as large as possible during an expansion process to maximize the work output.

In steam or gas power plants, the pressure rise in the pump or compressor is equal to the pressure drop in the turbine if we disregard the pressure losses in various other components. In steam power plants, the pump handles liquid, which has a very small specific volume, and the turbine handles vapor, whose specific volume is many times larger. Therefore, the work output of the turbine is much larger than the work input to the pump. This is one of the reasons for the wide-spread use of steam power plants in electric power generation.

If we were to compress the steam exiting the turbine back to the turbine inlet pressure before cooling it first in the condenser in order to "save" the heat rejected, we would have to supply all the work produced by the turbine back to the compressor. In reality, the required work input would be even greater than the work output of the turbine because of the irreversibilities present in both processes.


FIGURE 7-41
Reversible work relations for steadyflow and closed systems.


FIGURE 7-42
The larger the specific volume, the greater the work produced (or consumed) by a steady-flow device.

FIGURE 7-43
Schematic and T-s diagram for Example 7-12.

In gas power plants, the working fluid (typically air) is compressed in the gas phase, and a considerable portion of the work output of the turbine is consumed by the compressor. As a result, a gas power plant delivers less net work per unit mass of the working fluid.

## EXAMPLE 7-12 Compressing a Substance in the Liquid versus Gas Phases

Determine the compressor work input required to compress steam isentropically from 100 kPa to 1 MPa , assuming that the steam exists as (a) saturated liquid and (b) saturated vapor at the inlet state.

Solution Steam is to be compressed from a given pressure to a specified pressure isentropically. The work input is to be determined for the cases of steam being a saturated liquid and saturated vapor at the inlet.
Assumptions 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible. 3 The process is given to be isentropic.
Analysis We take first the turbine and then the pump as the system. Both are control volumes since mass crosses the boundary. Sketches of the pump and the turbine together with the $T$-s diagram are given in Fig. 7-43.
(a) In this case, steam is a saturated liquid initially, and its specific volume is

$$
v_{1}=v_{f @ 100 \mathrm{kPa}}=0.001043 \mathrm{~m}^{3} / \mathrm{kg} \quad(\text { Table A-5) }
$$

which remains essentially constant during the process. Thus,

$$
\begin{aligned}
w_{\mathrm{rev}, \text { in }} & =\int_{1}^{2} v d P \cong v_{1}\left(P_{2}-P_{1}\right) \\
& =\left(0.001043 \mathrm{~m}^{3} / \mathrm{kg}\right)[(1000-100) \mathrm{kPa}]\left(\frac{1 \mathrm{~kJ}}{1 \mathrm{kPa} \cdot \mathrm{~m}^{3}}\right) \\
& =\mathbf{0 . 9 4} \mathbf{~ k J} / \mathbf{k g}
\end{aligned}
$$

(b) This time, steam is a saturated vapor initially and remains a vapor during the entire compression process. Since the specific volume of a gas changes considerably during a compression process, we need to know how $\vee$ varies with $P$ to perform the integration in Eq. 7-53. This relation, in general, is not readily available. But for an isentropic process, it is easily obtained from the

second $T d s$ relation by setting $d s=0$ :

$$
\left.\begin{array}{rlr}
T d s & =d h-v d P \quad(\text { Eq. 7-24) } \\
d s & =0 \quad \text { (isentropic process) }
\end{array}\right\} \quad v d P=d h
$$

Thus,

$$
w_{\mathrm{rev}, \mathrm{in}}=\int_{1}^{2} \vee d P=\int_{1}^{2} d h=h_{2}-h_{1}
$$

This result could also be obtained from the energy balance relation for an isentropic steady-flow process. Next we determine the enthalpies:
State 1: $\left.\quad P_{1}=100 \mathrm{kPa}\right\} \quad h_{1}=2675.0 \mathrm{~kJ} / \mathrm{kg}$

$$
\left.\begin{array}{l}
P_{1}=100 \mathrm{kPa} \\
(\text { sat. vapor })
\end{array}\right\} \quad \begin{aligned}
& n_{1}=20 / 5.0 \mathrm{~kJ} / \mathrm{kg} \\
& s_{1}=7.3589 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}
\end{aligned} \quad(\text { Table A-5) }
$$

State 2: $\left.\quad \begin{array}{rl}P_{2} & =1 \mathrm{MPa} \\ s_{2} & =s_{1}\end{array}\right\} \quad h_{2}=3194.5 \mathrm{~kJ} / \mathrm{kg} \quad($ Table A-6)
Thus,

$$
w_{\mathrm{rev}, \mathrm{in}}=(3194.5-2675.0) \mathrm{kJ} / \mathrm{kg}=519.5 \mathrm{~kJ} / \mathrm{kg}
$$

Discussion Note that compressing steam in the vapor form would require over 500 times more work than compressing it in the liquid form between the same pressure limits.

## Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

We have shown in Chap. 6 that cyclic devices (heat engines, refrigerators, and heat pumps) deliver the most work and consume the least when reversible processes are used. Now we demonstrate that this is also the case for individual devices such as turbines and compressors in steady operation.

Consider two steady-flow devices, one reversible and the other irreversible, operating between the same inlet and exit states. Again taking heat transfer to the system and work done by the system to be positive quantities, the energy balance for each of these devices can be expressed in the differential form as

Actual:

$$
\delta q_{\text {act }}-\delta w_{\text {act }}=d h+d \mathrm{ke}+d \mathrm{pe}
$$

Reversible:

$$
\delta q_{\mathrm{rev}}-\delta w_{\mathrm{rev}}=d h+d \mathrm{ke}+d \mathrm{pe}
$$

The right-hand sides of these two equations are identical since both devices are operating between the same end states. Thus,

$$
\delta q_{\mathrm{act}}-\delta w_{\mathrm{act}}=\delta q_{\mathrm{rev}}-\delta w_{\mathrm{rev}}
$$

or

$$
\delta w_{\mathrm{rev}}-\delta w_{\mathrm{act}}=\delta q_{\mathrm{rev}}-\delta q_{\mathrm{act}}
$$

However,

$$
\delta q_{\mathrm{rev}}=T d s
$$



FIGURE 7-44
A reversible turbine delivers more work than an irreversible one if both operate between the same end states.

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Substituting this relation into the preceding equation and dividing each term by $T$, we obtain

$$
\frac{\delta w_{\mathrm{rev}}-\delta w_{\mathrm{act}}}{T}=d s-\frac{\delta q_{\mathrm{act}}}{T} \geq 0
$$

since

$$
d s \geq \frac{\delta q_{\mathrm{act}}}{T}
$$

Also, $T$ is the absolute temperature, which is always positive. Thus,

$$
\partial w_{\mathrm{rev}} \geq \partial w_{\mathrm{act}}
$$

or

$$
w_{\mathrm{rev}} \geq w_{\mathrm{act}}
$$

Therefore, work-producing devices such as turbines ( $w$ is positive) deliver more work, and work-consuming devices such as pumps and compressors ( $w$ is negative) require less work when they operate reversibly (Fig. 7-44).

## 7-11 • MINIMIZING THE COMPRESSOR WORK

We have just shown that the work input to a compressor is minimized when the compression process is executed in an internally reversible manner. When the changes in kinetic and potential energies are negligible, the compressor work is given by (Eq. 7-53)

$$
\begin{equation*}
w_{\mathrm{rev}, \mathrm{in}}=\int_{1}^{2} \vee d P \tag{7-56}
\end{equation*}
$$

Obviously one way of minimizing the compressor work is to approximate an internally reversible process as much as possible by minimizing the irreversibilities such as friction, turbulence, and nonquasi-equilibrium compression. The extent to which this can be accomplished is limited by economic considerations. A second (and more practical) way of reducing the compressor work is to keep the specific volume of the gas as small as possible during the compression process. This is done by maintaining the temperature of the gas as low as possible during compression since the specific volume of a gas is proportional to temperature. Therefore, reducing the work input to a compressor requires that the gas be cooled as it is compressed.

To have a better understanding of the effect of cooling during the compression process, we compare the work input requirements for three kinds of processes: an isentropic process (involves no cooling), a polytropic process (involves some cooling), and an isothermal process (involves maximum cooling). Assuming all three processes are executed between the same pressure levels $\left(P_{1}\right.$ and $\left.P_{2}\right)$ in an internally reversible manner and the gas behaves as an ideal gas $(P \vee=R T)$ with constant specific heats, we see that the compression work is determined by performing the integration in Eq. $7-56$ for each case, with the following results:

Isentropic $\left(P V^{k}=\right.$ constant $)$ :

$$
\begin{equation*}
w_{\text {comp,in }}=\frac{k R\left(T_{2}-T_{1}\right)}{k-1}=\frac{k R T_{1}}{k-1}\left[\left(\frac{P_{2}}{P_{1}}\right)^{(k-1) / k}-1\right] \tag{7-57a}
\end{equation*}
$$

Polytropic $\left(P V^{n}=\right.$ constant $)$ :

$$
\begin{equation*}
w_{\text {comp,in }}=\frac{n R\left(T_{2}-T_{1}\right)}{n-1}=\frac{n R T_{1}}{n-1}\left[\left(\frac{P_{2}}{P_{1}}\right)^{(n-1) / n}-1\right] \tag{7-57b}
\end{equation*}
$$

Isothermal ( $P \vee=$ constant $)$ :

$$
\begin{equation*}
w_{\text {comp,in }}=R T \ln \frac{P_{2}}{P_{1}} \tag{7-57c}
\end{equation*}
$$

The three processes are plotted on a $P-V$ diagram in Fig. 7-45 for the same inlet state and exit pressure. On a $P-v$ diagram, the area to the left of the process curve is the integral of $v d P$. Thus it is a measure of the steadyflow compression work. It is interesting to observe from this diagram that of the three internally reversible cases considered, the adiabatic compression $\left(P V^{k}=\right.$ constant $)$ requires the maximum work and the isothermal compression ( $T=$ constant or $P \vee=$ constant) requires the minimum. The work input requirement for the polytropic case $\left(P V^{n}=\right.$ constant $)$ is between these two and decreases as the polytropic exponent $n$ is decreased, by increasing the heat rejection during the compression process. If sufficient heat is removed, the value of $n$ approaches unity and the process becomes isothermal. One common way of cooling the gas during compression is to use cooling jackets around the casing of the compressors.

## Multistage Compression with Intercooling

It is clear from these arguments that cooling a gas as it is compressed is desirable since this reduces the required work input to the compressor. However, often it is not possible to have adequate cooling through the casing of the compressor, and it becomes necessary to use other techniques to achieve effective cooling. One such technique is multistage compression with intercooling, where the gas is compressed in stages and cooled between each stage by passing it through a heat exchanger called an intercooler. Ideally, the cooling process takes place at constant pressure, and the gas is cooled to the initial temperature $T_{1}$ at each intercooler. Multistage compression with intercooling is especially attractive when a gas is to be compressed to very high pressures.

The effect of intercooling on compressor work is graphically illustrated on $P-\vee$ and $T-s$ diagrams in Fig. 7-46 for a two-stage compressor. The gas is compressed in the first stage from $P_{1}$ to an intermediate pressure $P_{x}$, cooled at constant pressure to the initial temperature $T_{1}$, and compressed in the second stage to the final pressure $P_{2}$. The compression processes, in general, can be modeled as polytropic $\left(P \vee^{n}=\right.$ constant) where the value of $n$ varies between $k$ and 1. The colored area on the $P-\vee$ diagram represents the work saved as a result of two-stage compression with intercooling. The process paths for singlestage isothermal and polytropic processes are also shown for comparison.


FIGURE 7-45
$P-\vee$ diagrams of isentropic, polytropic, and isothermal compression processes between the same pressure limits.

FIGURE 7-46
$P-V$ and $T-s$ diagrams for a two-stage steady-flow compression process.



The size of the colored area (the saved work input) varies with the value of the intermediate pressure $P_{x}$, and it is of practical interest to determine the conditions under which this area is maximized. The total work input for a two-stage compressor is the sum of the work inputs for each stage of compression, as determined from Eq. 7-57b:

$$
\begin{align*}
w_{\text {comp, in }} & =w_{\text {comp I,in }}+w_{\text {comp II,in }}  \tag{7-58}\\
& =\frac{n R T_{1}}{n-1}\left[\left(\frac{P_{x}}{P_{1}}\right)^{(n-1) / n}-1\right]+\frac{n R T_{1}}{n-1}\left[\left(\frac{P_{2}}{P_{x}}\right)^{(n-1) / n}-1\right]
\end{align*}
$$

The only variable in this equation is $P_{x}$. The $P_{x}$ value that minimizes the total work is determined by differentiating this expression with respect to $P_{x}$ and setting the resulting expression equal to zero. It yields

$$
\begin{equation*}
P_{x}=\left(P_{1} P_{2}\right)^{1 / 2} \quad \text { or } \quad \frac{P_{x}}{P_{1}}=\frac{P_{2}}{P_{x}} \tag{7-59}
\end{equation*}
$$

That is, to minimize compression work during two-stage compression, the pressure ratio across each stage of the compressor must be the same. When this condition is satisfied, the compression work at each stage becomes identical, that is, $w_{\text {comp I,in }}=w_{\text {comp I,in }}$.

## EXAMPLE 7-13 Work Input for Various Compression Processes

Air is compressed steadily by a reversible compressor from an inlet state of 100 kPa and 300 K to an exit pressure of 900 kPa . Determine the compressor work per unit mass for (a) isentropic compression with $k=1.4$, (b) polytropic compression with $n=1.3$, (c) isothermal compression, and (d) ideal twostage compression with intercooling with a polytropic exponent of 1.3.

Solution Air is compressed reversibly from a specified state to a specified pressure. The compressor work is to be determined for the cases of isentropic, polytropic, isothermal, and two-stage compression.



Assumptions 1 Steady operating conditions exist. 2 At specified conditions, air can be treated as an ideal gas. 3 Kinetic and potential energy changes are negligible.
Analysis We take the compressor to be the system. This is a control volume since mass crosses the boundary. A sketch of the system and the $T$-s diagram for the process are given in Fig. 7-47.

The steady-flow compression work for all these four cases is determined by using the relations developed earlier in this section:
(a) Isentropic compression with $k=1.4$ :

$$
\begin{aligned}
w_{\text {comp,in }} & =\frac{k R T_{1}}{k-1}\left[\left(\frac{P_{2}}{P_{1}}\right)^{(k-1) / k}-1\right] \\
& =\frac{(1.4)(0.287 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K})(300 \mathrm{~K})}{1.4-1}\left[\left(\frac{900 \mathrm{kPa}}{100 \mathrm{kPa}}\right)^{(1.4-1) / 1.4}-1\right] \\
& =\mathbf{2 6 3 . 2} \mathbf{~ k J} / \mathbf{k g}
\end{aligned}
$$

(b) Polytropic compression with $n=1.3$ :

$$
\begin{aligned}
w_{\text {comp,in }} & =\frac{n R T_{1}}{n-1}\left[\left(\frac{P_{2}}{P_{1}}\right)^{(n-1) / n}-1\right] \\
& =\frac{(1.3)(0.287 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K})(300 \mathrm{~K})}{1.3-1}\left[\left(\frac{900 \mathrm{kPa}}{100 \mathrm{kPa}}\right)^{(1.3-1) / 1.3}-1\right] \\
& =\mathbf{2 4 6 . 4} \mathbf{~ k J} / \mathrm{kg}
\end{aligned}
$$

(c) Isothermal compression:

$$
\begin{aligned}
w_{\text {comp,in }} & =R T \ln \frac{P_{2}}{P_{1}}=(0.287 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K})(300 \mathrm{~K}) \ln \frac{900 \mathrm{kPa}}{100 \mathrm{kPa}} \\
& =\mathbf{1 8 9 . 2} \mathbf{~ k J} / \mathbf{k g}
\end{aligned}
$$

(d) Ideal two-stage compression with intercooling ( $n=1.3$ ): In this case, the pressure ratio across each stage is the same, and its value is

$$
P_{x}=\left(P_{1} P_{2}\right)^{1 / 2}=[(100 \mathrm{kPa})(900 \mathrm{kPa})]^{1 / 2}=300 \mathrm{kPa}
$$

FIGURE 7-47
Schematic and $P-\vee$ diagram for Example 7-13.

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FIGURE 7-48
The isentropic process involves no irreversibilities and serves as the ideal process for adiabatic devices.

The compressor work across each stage is also the same. Thus the total compressor work is twice the compression work for a single stage:

$$
\begin{aligned}
w_{\text {comp }, \text { in }} & =2 w_{\text {comp } \mathrm{I}, \text { in }}=2 \frac{n R T_{1}}{n-1}\left[\left(\frac{P_{x}}{P_{1}}\right)^{(n-1) / n}-1\right] \\
& =\frac{2(1.3)(0.287 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K})(300 \mathrm{~K})}{1.3-1}\left[\left(\frac{300 \mathrm{kPa}}{100 \mathrm{kPa}}\right)^{(1.3-1) / 1.3}-1\right] \\
& =215.3 \mathbf{~ k J} / \mathrm{kg}
\end{aligned}
$$

Discussion Of all four cases considered, the isothermal compression requires the minimum work and the isentropic compression the maximum. The compressor work is decreased when two stages of polytropic compression are utilized instead of just one. As the number of compressor stages is increased, the compressor work approaches the value obtained for the isothermal case.

## 7-12 • ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

We mentioned repeatedly that irreversibilities inherently accompany all actual processes and that their effect is always to downgrade the performance of devices. In engineering analysis, it would be very desirable to have some parameters that would enable us to quantify the degree of degradation of energy in these devices. In the last chapter we did this for cyclic devices, such as heat engines and refrigerators, by comparing the actual cycles to the idealized ones, such as the Carnot cycle. A cycle that was composed entirely of reversible processes served as the model cycle to which the actual cycles could be compared. This idealized model cycle enabled us to determine the theoretical limits of performance for cyclic devices under specified conditions and to examine how the performance of actual devices suffered as a result of irreversibilities.

Now we extend the analysis to discrete engineering devices working under steady-flow conditions, such as turbines, compressors, and nozzles, and we examine the degree of degradation of energy in these devices as a result of irreversibilities. However, first we need to define an ideal process that serves as a model for the actual processes.
Although some heat transfer between these devices and the surrounding medium is unavoidable, many steady-flow devices are intended to operate under adiabatic conditions. Therefore, the model process for these devices should be an adiabatic one. Furthermore, an ideal process should involve no irreversibilities since the effect of irreversibilities is always to downgrade the performance of engineering devices. Thus, the ideal process that can serve as a suitable model for adiabatic steady-flow devices is the isentropic process (Fig. 7-48).

The more closely the actual process approximates the idealized isentropic process, the better the device performs. Thus, it would be desirable to have a parameter that expresses quantitatively how efficiently an actual device approximates an idealized one. This parameter is the isentropic or adiabatic efficiency, which is a measure of the deviation of actual processes from the corresponding idealized ones.

Isentropic efficiencies are defined differently for different devices since each device is set up to perform different tasks. Next we define the isentropic efficiencies of turbines, compressors, and nozzles by comparing the actual performance of these devices to their performance under isentropic conditions for the same inlet state and exit pressure.

## Isentropic Efficiency of Turbines

For a turbine under steady operation, the inlet state of the working fluid and the exhaust pressure are fixed. Therefore, the ideal process for an adiabatic turbine is an isentropic process between the inlet state and the exhaust pressure. The desired output of a turbine is the work produced, and the isentropic efficiency of a turbine is defined as the ratio of the actual work output of the turbine to the work output that would be achieved if the process between the inlet state and the exit pressure were isentropic:

$$
\begin{equation*}
\eta_{T}=\frac{\text { Actual turbine work }}{\text { Isentropic turbine work }}=\frac{w_{a}}{w_{s}} \tag{7-60}
\end{equation*}
$$

Usually the changes in kinetic and potential energies associated with a fluid stream flowing through a turbine are small relative to the change in enthalpy and can be neglected. Then the work output of an adiabatic turbine simply becomes the change in enthalpy, and Eq. 7-60 becomes

$$
\begin{equation*}
\eta_{T} \cong \frac{h_{1}-h_{2 a}}{h_{1}-h_{2 s}} \tag{7-61}
\end{equation*}
$$

where $h_{2 a}$ and $h_{2 s}$ are the enthalpy values at the exit state for actual and isentropic processes, respectively (Fig. 7-49).

The value of $\eta_{T}$ greatly depends on the design of the individual components that make up the turbine. Well-designed, large turbines have isentropic efficiencies above 90 percent. For small turbines, however, it may drop even below 70 percent. The value of the isentropic efficiency of a turbine is determined by measuring the actual work output of the turbine and by calculating the isentropic work output for the measured inlet conditions and the exit pressure. This value can then be used conveniently in the design of power plants.

EXAMPLE 7-14 Isentropic Efficiency of a Steam Turbine
Steam enters an adiabatic turbine steadily at 3 MPa and $400^{\circ} \mathrm{C}$ and leaves at 50 kPa and $100^{\circ} \mathrm{C}$. If the power output of the turbine is 2 MW , determine (a) the isentropic efficiency of the turbine and (b) the mass flow rate of the steam flowing through the turbine.

Solution Steam flows steadily in a turbine between inlet and exit states. For a specified power output, the isentropic efficiency and the mass flow rate are to be determined.
Assumptions 1 Steady operating conditions exist. 2 The changes in kinetic and potential energies are negligible.


FIGURE 7-49
The $h-s$ diagram for the actual and isentropic processes of an adiabatic turbine.

## FIGURE 7-50

Schematic and T-s diagram for Example 7-14.



Analysis A sketch of the system and the $T$-s diagram of the process are given in Fig. 7-50.
(a) The enthalpies at various states are

State 1:

$$
\left.\begin{array}{rl}
P_{1} & =3 \mathrm{MPA} \\
T_{1} & =400^{\circ} \mathrm{C}
\end{array}\right\} \quad \begin{aligned}
& h_{1}=3231.7 \mathrm{~kJ} / \mathrm{kg} \\
& s_{1}=6.9235 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}
\end{aligned}
$$

(Table A-6)

State 2a:

$$
\left.\begin{array}{rl}
P_{2 a} & =50 \mathrm{kPa} \\
T_{2 a} & =100^{\circ} \mathrm{C}
\end{array}\right\} \quad h_{2 a}=2682.4 \mathrm{~kJ} / \mathrm{kg}
$$

(Table A-6)
The exit enthalpy of the steam for the isentropic process $h_{2 s}$ is determined from the requirement that the entropy of the steam remain constant ( $s_{2 s}=s_{1}$ ):
State 2s: $\quad \begin{aligned} & P_{2 s}=50 \mathrm{kPa} \\ & \left(s_{2 s}=s_{1}\right)\end{aligned} \rightarrow \quad \begin{aligned} & s_{f}=1.0912 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K} \\ & s_{g}=7.5931 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}\end{aligned}$
(Table A-5)
Obviously, at the end of the isentropic process steam exists as a saturated mixture since $s_{f}<s_{2 s}<s_{g}$. Thus we need to find the quality at state $2 s$ first:

$$
x_{2 s}=\frac{s_{2 s}-s_{f}}{s_{f g}}=\frac{6.9235-1.0912}{6.5019}=0.897
$$

and

$$
h_{2 s}=h_{f}+x_{2 s} h_{f g}=340.54+0.897(2304.7)=2407.9 \mathrm{~kJ} / \mathrm{kg}
$$

By substituting these enthalpy values into Eq. 7-61, the isentropic efficiency of this turbine is determined to be

$$
\eta_{T} \cong \frac{h_{1}-h_{2 a}}{h_{1}-h_{2 s}}=\frac{3231.7-2682.4}{3231.7-2407.9}=0.667, \text { or } 66.7 \%
$$

(b) The mass flow rate of steam through this turbine is determined from the energy balance for steady-flow systems:

$$
\begin{aligned}
\dot{E}_{\text {in }} & =\dot{E}_{\text {out }} \\
\dot{m} h_{1} & =\dot{W}_{a, \text { out }}+\dot{m} h_{2 a} \\
\dot{W}_{a, \text { out }} & =\dot{m}\left(h_{1}-h_{2 a}\right) \\
2 \mathrm{MW}\left(\frac{1000 \mathrm{~kJ} / \mathrm{s}}{1 \mathrm{MW}}\right) & =\dot{m}(3231.7-2682.4) \mathrm{kJ} / \mathrm{kg} \\
\dot{m} & =\mathbf{3 . 6 4} \mathrm{kg} / \mathrm{s}
\end{aligned}
$$

## Isentropic Efficiencies of Compressors and Pumps

The isentropic efficiency of a compressor is defined as the ratio of the work input required to raise the pressure of a gas to a specified value in an isentropic manner to the actual work input:

$$
\begin{equation*}
\eta_{C}=\frac{\text { Isentropic compressor work }}{\text { Actual compressor work }}=\frac{w_{s}}{w_{a}} \tag{7-62}
\end{equation*}
$$

Notice that the isentropic compressor efficiency is defined with the isentropic work input in the numerator instead of in the denominator. This is because $w_{s}$ is a smaller quantity than $w_{a}$, and this definition prevents $\eta_{C}$ from becoming greater than 100 percent, which would falsely imply that the actual compressors performed better than the isentropic ones. Also notice that the inlet conditions and the exit pressure of the gas are the same for both the actual and the isentropic compressor.
When the changes in kinetic and potential energies of the gas being compressed are negligible, the work input to an adiabatic compressor becomes equal to the change in enthalpy, and Eq. 7-62 for this case becomes

$$
\begin{equation*}
\eta_{C} \cong \frac{h_{2 s}-h_{1}}{h_{2 a}-h_{1}} \tag{7-63}
\end{equation*}
$$

where $h_{2 a}$ and $h_{2 s}$ are the enthalpy values at the exit state for actual and isentropic compression processes, respectively, as illustrated in Fig. 7-51. Again, the value of $\eta_{C}$ greatly depends on the design of the compressor. Well-designed compressors have isentropic efficiencies that range from 80 to 90 percent.

When the changes in potential and kinetic energies of a liquid are negligible, the isentropic efficiency of a pump is defined similarly as

$$
\begin{equation*}
\eta_{P}=\frac{w_{s}}{w_{a}}=\frac{v\left(P_{2}-P_{1}\right)}{h_{2 a}-h_{1}} \tag{7-64}
\end{equation*}
$$

When no attempt is made to cool the gas as it is compressed, the actual compression process is nearly adiabatic and the reversible adiabatic (i.e., isentropic) process serves well as the ideal process. However, sometimes compressors are cooled intentionally by utilizing fins or a water jacket placed around the casing to reduce the work input requirements (Fig. 7-52). In this case, the isentropic process is not suitable as the model process since the device is no longer adiabatic and the isentropic compressor efficiency defined above is meaningless. A realistic model process for compressors that are intentionally cooled during the compression process is the reversible isothermal process. Then we can conveniently define an isothermal efficiency for such cases by comparing the actual process to a reversible isothermal one:

$$
\begin{equation*}
\eta_{C}=\frac{w_{t}}{w_{a}} \tag{7-65}
\end{equation*}
$$

where $w_{t}$ and $w_{a}$ are the required work inputs to the compressor for the reversible isothermal and actual cases, respectively.


FIGURE 7-51
The $h$-s diagram of the actual and isentropic processes of an adiabatic compressor.


FIGURE 7-52
Compressors are sometimes intentionally cooled to minimize the work input.

FIGURE 7-53
Schematic and $T-s$ diagram for Example 7-15.

## EXAMPLE 7-15 Effect of Efficiency on Compressor Power Input

Air is compressed by an adiabatic compressor from 100 kPa and $12^{\circ} \mathrm{C}$ to a pressure of 800 kPa at a steady rate of $0.2 \mathrm{~kg} / \mathrm{s}$. If the isentropic efficiency of the compressor is 80 percent, determine (a) the exit temperature of air and (b) the required power input to the compressor.

Solution Air is compressed to a specified pressure at a specified rate. For a given isentropic efficiency, the exit temperature and the power input are to be determined.
Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The changes in kinetic and potential energies are negligible.
Analysis A sketch of the system and the $T$-s diagram of the process are given in Fig. 7-53.
(a) We know only one property (pressure) at the exit state, and we need to know one more to fix the state and thus determine the exit temperature. The property that can be determined with minimal effort in this case is $h_{2 a}$ since the isentropic efficiency of the compressor is given. At the compressor inlet,

$$
\begin{gathered}
T_{1}=285 \mathrm{~K} \quad \rightarrow \quad h_{1}=285.14 \mathrm{~kJ} / \mathrm{kg} \\
\left(P_{r 1}=1.1584\right)
\end{gathered}
$$

(Table A-17)

The enthalpy of the air at the end of the isentropic compression process is determined by using one of the isentropic relations of ideal gases,

$$
P_{r 2}=P_{r 1}\left(\frac{P_{2}}{P_{1}}\right)=1.1584\left(\frac{800 \mathrm{kPa}}{100 \mathrm{kPa}}\right)=9.2672
$$

and

$$
P_{r 2}=9.2672 \quad \rightarrow \quad h_{2 s}=517.05 \mathrm{~kJ} / \mathrm{kg}
$$

Substituting the known quantities into the isentropic efficiency relation, we have

$$
\eta_{C} \cong \frac{h_{2 s}-h_{1}}{h_{2 a}-h_{1}} \quad \rightarrow \quad 0.80=\frac{(517.05-285.14) \mathrm{kJ} / \mathrm{kg}}{\left(h_{2 a}-285.14\right) \mathrm{kJ} / \mathrm{kg}}
$$

Thus,

$$
h_{2 a}=575.03 \mathrm{~kJ} / \mathrm{kg} \quad \rightarrow \quad T_{2 a}=569.5 \mathrm{~K}
$$



(b) The required power input to the compressor is determined from the energy balance for steady-flow devices,

$$
\begin{aligned}
\dot{E}_{\text {in }} & =\dot{E}_{\text {out }} \\
\dot{m} h_{1}+\dot{W}_{a, \text { in }} & =\dot{m} h_{2 a} \\
\dot{W}_{a, \text { in }} & =\dot{m}\left(h_{2 a}-h_{1}\right) \\
& =(0.2 \mathrm{~kg} / \mathrm{s})[(575.03-285.14) \mathrm{kJ} / \mathrm{kg}] \\
& =\mathbf{5 8 . 0} \mathbf{~ k W}
\end{aligned}
$$

Discussion Notice that in determining the power input to the compressor, we used $h_{2 a}$ instead of $h_{2 s}$ since $h_{2 a}$ is the actual enthalpy of the air as it exits the compressor. The quantity $h_{2 s}$ is a hypothetical enthalpy value that the air would have if the process were isentropic.

## Isentropic Efficiency of Nozzles

Nozzles are essentially adiabatic devices and are used to accelerate a fluid. Therefore, the isentropic process serves as a suitable model for nozzles. The isentropic efficiency of a nozzle is defined as the ratio of the actual kinetic energy of the fluid at the nozzle exit to the kinetic energy value at the exit of an isentropic nozzle for the same inlet state and exit pressure. That is,

$$
\begin{equation*}
\eta_{N}=\frac{\text { Actual KE at nozzle exit }}{\text { Isentropic KE at nozzle exit }}=\frac{V_{2 a}^{2}}{V_{2 s}^{2}} \tag{7-66}
\end{equation*}
$$

Note that the exit pressure is the same for both the actual and isentropic processes, but the exit state is different.

Nozzles involve no work interactions, and the fluid experiences little or no change in its potential energy as it flows through the device. If, in addition, the inlet velocity of the fluid is small relative to the exit velocity, the energy balance for this steady-flow device reduces to

$$
h_{1}=h_{2 a}+\frac{V_{2 a}^{2}}{2}
$$

Then the isentropic efficiency of the nozzle can be expressed in terms of enthalpies as

$$
\begin{equation*}
\eta_{N} \cong \frac{h_{1}-h_{2 a}}{h_{1}-h_{2 s}} \tag{7-67}
\end{equation*}
$$

where $h_{2 a}$ and $h_{2 s}$ are the enthalpy values at the nozzle exit for the actual and isentropic processes, respectively (Fig. 7-54). Isentropic efficiencies of nozzles are typically above 90 percent, and nozzle efficiencies above 95 percent are not uncommon.

## EXAMPLE 7-16 Effect of Efficiency on Nozzle Exit Velocity

Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharged at a pressure of 80 kPa . If the isentropic efficiency of the nozzle is 92 percent, determine (a) the maximum possible exit velocity, $(b)$ the exit temperature, and (c) the actual exit velocity of the air. Assume constant specific heats for air.


FIGURE 7-54
The $h$-s diagram of the actual and isentropic processes of an adiabatic nozzle.

FIGURE 7-55
Schematic and $T$-s diagram for Example 7-16.



Solution The acceleration of air in a nozzle is considered. For specified exit pressure and isentropic efficiency, the maximum and actual exit velocities and the exit temperature are to be determined.
Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The inlet kinetic energy is negligible.
Analysis A sketch of the system and the $T$-s diagram of the process are given in Fig. 7-55.
The temperature of air will drop during this acceleration process because some of its internal energy is converted to kinetic energy. This problem can be solved accurately by using property data from the air table. But we will assume constant specific heats (thus sacrifice some accuracy) to demonstrate their use. Let us guess the average temperature of the air to be about 800 K . Then the average values of $c_{p}$ and $k$ at this anticipated average temperature are determined from Table $\mathrm{A}-2 b$ to be $c_{p}=1.099 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$ and $k=1.354$.
(a) The exit velocity of the air will be a maximum when the process in the nozzle involves no irreversibilities. The exit velocity in this case is determined from the steady-flow energy equation. However, first we need to determine the exit temperature. For the isentropic process of an ideal gas we have:

$$
\frac{T_{2 s}}{T_{1}}=\left(\frac{P_{2 s}}{P_{1}}\right)^{(k-1) / k}
$$

or

$$
T_{2 s}=T_{1}\left(\frac{P_{2 s}}{P_{1}}\right)^{(k-1) / k}=(950 \mathrm{~K})\left(\frac{80 \mathrm{kPa}}{200 \mathrm{kPa}}\right)^{0.354 / 1.354}=748 \mathrm{~K}
$$

This gives an average temperature of 849 K , which is somewhat higher than the assumed average temperature ( 800 K ). This result could be refined by reevaluating the $k$ value at 749 K and repeating the calculations, but it is not warranted since the two average temperatures are sufficiently close (doing so would change the temperature by only 1.5 K , which is not significant).

Now we can determine the isentropic exit velocity of the air from the energy balance for this isentropic steady-flow process:

$$
\begin{aligned}
e_{\text {in }} & =e_{\text {out }} \\
h_{1}+\frac{V_{1}^{2}}{2} & =h_{2 s}+\frac{V_{2 s}^{2}}{2}
\end{aligned}
$$

or

$$
\begin{aligned}
V_{2 s} & =\sqrt{2\left(h_{1}-h_{2 s}\right)}=\sqrt{2 c_{p, \text { avg }}\left(T_{1}-T_{2 s}\right)} \\
& =\sqrt{2(1.099 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K})[(950-748) \mathrm{K}]\left(\frac{1000 \mathrm{~m}^{2} / \mathrm{s}^{2}}{1 \mathrm{~kJ} / \mathrm{kg}}\right)} \\
& =666 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) The actual exit temperature of the air is higher than the isentropic exit temperature evaluated above and is determined from

$$
\eta_{N} \cong \frac{h_{1}-h_{2 a}}{h_{1}-h_{2 s}}=\frac{c_{p, \text { avg }}\left(T_{1}-T_{2 a}\right)}{c_{p, \text { avg }}\left(T_{1}-T_{2 s}\right)}
$$

or

$$
0.92=\frac{950-T_{2 a}}{950-748} \quad \rightarrow \quad T_{2 a}=764 \mathrm{~K}
$$

That is, the temperature is 16 K higher at the exit of the actual nozzle as a result of irreversibilities such as friction. It represents a loss since this rise in temperature comes at the expense of kinetic energy (Fig. 7-56).
(c) The actual exit velocity of air can be determined from the definition of isentropic efficiency of a nozzle,

$$
\eta_{N}=\frac{V_{2 a}^{2}}{V_{2 s}^{2}} \quad \rightarrow \quad V_{2 a}=\sqrt{\eta_{N} V_{2 s}^{2}}=\sqrt{0.92(666 \mathrm{~m} / \mathrm{s})^{2}}=639 \mathrm{~m} / \mathrm{s}
$$

## 7-13 • ENTROPY BALANCE

The property entropy is a measure of molecular disorder or randomness of a system, and the second law of thermodynamics states that entropy can be created but it cannot be destroyed. Therefore, the entropy change of a system during a process is greater than the entropy transfer by an amount equal to the entropy generated during the process within the system, and the increase of entropy principle for any system is expressed as (Fig. 7-57)

$$
\left(\begin{array}{c}
\text { Total } \\
\text { entropy } \\
\text { entering }
\end{array}\right)-\left(\begin{array}{c}
\text { Total } \\
\text { entropy } \\
\text { leaving }
\end{array}\right)+\left(\begin{array}{c}
\text { Total } \\
\text { entropy } \\
\text { generated }
\end{array}\right)=\left(\begin{array}{c}
\text { Change in the } \\
\text { total entropy } \\
\text { of the system }
\end{array}\right)
$$

or

$$
\begin{equation*}
S_{\mathrm{in}}-S_{\mathrm{out}}+S_{\mathrm{gen}}=\Delta S_{\mathrm{system}} \tag{7-68}
\end{equation*}
$$

which is a verbal statement of Eq. 7-9. This relation is often referred to as the entropy balance and is applicable to any system undergoing any process. The entropy balance relation above can be stated as: the entropy change of a system during a process is equal to the net entropy transfer through the system boundary and the entropy generated within the system. Next we discuss the various terms in that relation.


## FIGURE 7-56

A substance leaves actual nozzles at a higher temperature (thus a lower velocity) as a result of friction.

## INTERACTIVE TUTORIAL

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FIGURE 7-57
Energy and entropy balances for a system.


FIGURE 7-58
Heat transfer is always accompanied by entropy transfer in the amount of $Q / T$, where $T$ is the boundary temperature.

## Entropy Change of a System, $\Delta \boldsymbol{S}_{\text {system }}$

Despite the reputation of entropy as being vague and abstract and the intimidation associated with it, entropy balance is actually easier to deal with than energy balance since, unlike energy, entropy does not exist in various forms. Therefore, the determination of entropy change of a system during a process involves evaluating entropy of the system at the beginning and at the end of the process and taking their difference. That is,

$$
\text { Entropy change }=\text { Entropy at final state }- \text { Entropy at initial state }
$$

or

$$
\begin{equation*}
\Delta S_{\text {system }}=S_{\text {final }}-S_{\text {initial }}=S_{2}-S_{1} \tag{7-69}
\end{equation*}
$$

Note that entropy is a property, and the value of a property does not change unless the state of the system changes. Therefore, the entropy change of a system is zero if the state of the system does not change during the process. For example, the entropy change of steady-flow devices such as nozzles, compressors, turbines, pumps, and heat exchangers is zero during steady operation.

When the properties of the system are not uniform, the entropy of the system can be determined by integration from

$$
\begin{equation*}
S_{\text {system }}=\int s \delta m=\int_{V} s \rho d V \tag{7-70}
\end{equation*}
$$

where $V$ is the volume of the system and $\rho$ is density.

## Mechanisms of Entropy Transfer, $S_{\text {in }}$ and $S_{\text {out }}$

Entropy can be transferred to or from a system by two mechanisms: heat transfer and mass flow (in contrast, energy is transferred by work also). Entropy transfer is recognized at the system boundary as it crosses the boundary, and it represents the entropy gained or lost by a system during a process. The only form of entropy interaction associated with a fixed mass or closed system is heat transfer, and thus the entropy transfer for an adiabatic closed system is zero.

## 1 Heat Transfer

Heat is, in essence, a form of disorganized energy, and some disorganization (entropy) will flow with heat. Heat transfer to a system increases the entropy of that system and thus the level of molecular disorder or randomness, and heat transfer from a system decreases it. In fact, heat rejection is the only way the entropy of a fixed mass can be decreased. The ratio of the heat transfer $Q$ at a location to the absolute temperature $T$ at that location is called the entropy flow or entropy transfer and is expressed as (Fig. 7-58)

Entropy transfer by heat transfer: $\quad S_{\text {heat }}=\frac{Q}{T} \quad(T=$ constant $)$
The quantity $Q / T$ represents the entropy transfer accompanied by heat transfer, and the direction of entropy transfer is the same as the direction of heat transfer since thermodynamic temperature $T$ is always a positive quantity.

When the temperature $T$ is not constant, the entropy transfer during a process 1-2 can be determined by integration (or by summation if appropriate) as

$$
\begin{equation*}
S_{\text {heat }}=\int_{1}^{2} \frac{\delta Q}{T} \cong \sum \frac{Q_{k}}{T_{k}} \tag{7-72}
\end{equation*}
$$

where $Q_{k}$ is the heat transfer through the boundary at temperature $T_{k}$ at location $k$.

When two systems are in contact, the entropy transfer from the warmer system is equal to the entropy transfer into the cooler one at the point of contact. That is, no entropy can be created or destroyed at the boundary since the boundary has no thickness and occupies no volume.
Note that work is entropy-free, and no entropy is transferred by work. Energy is transferred by both heat and work, whereas entropy is transferred only by heat. That is,

Entropy transfer by work:

$$
\begin{equation*}
S_{\text {work }}=0 \tag{7-73}
\end{equation*}
$$

The first law of thermodynamics makes no distinction between heat transfer and work; it considers them as equals. The distinction between heat transfer and work is brought out by the second law: an energy interaction that is accompanied by entropy transfer is heat transfer, and an energy interaction that is not accompanied by entropy transfer is work. That is, no entropy is exchanged during a work interaction between a system and its surroundings. Thus, only energy is exchanged during work interaction whereas both energy and entropy are exchanged during heat transfer (Fig. 7-59).

## 2 Mass Flow

Mass contains entropy as well as energy, and the entropy and energy contents of a system are proportional to the mass. (When the mass of a system is doubled, so are the entropy and energy contents of the system.) Both entropy and energy are carried into or out of a system by streams of matter, and the rates of entropy and energy transport into or out of a system are proportional to the mass flow rate. Closed systems do not involve any mass flow and thus any entropy transfer by mass. When a mass in the amount of $m$ enters or leaves a system, entropy in the amount of $m s$, where $s$ is the specific entropy (entropy per unit mass entering or leaving), accompanies it (Fig. 7-60). That is,
Entropy transfer by mass flow:

$$
\begin{equation*}
S_{\mathrm{mass}}=m s \tag{7-74}
\end{equation*}
$$

Therefore, the entropy of a system increases by $m s$ when mass in the amount of $m$ enters and decreases by the same amount when the same amount of mass at the same state leaves the system. When the properties of the mass change during the process, the entropy transfer by mass flow can be determined by integration from

$$
\begin{equation*}
\dot{S}_{\text {mass }}=\int_{A_{c}} s \rho V_{n} d A_{c} \quad \text { and } \quad S_{\text {mass }}=\int s \delta m=\int_{\Delta t} \dot{S}_{\text {mass }} d t \tag{7-75}
\end{equation*}
$$

where $A_{c}$ is the cross-sectional area of the flow and $V_{n}$ is the local velocity normal to $d A_{c}$.


FIGURE 7-59
No entropy accompanies work as it crosses the system boundary. But entropy may be generated within the system as work is dissipated into a less useful form of energy.


FIGURE 7-60
Mass contains entropy as well as energy, and thus mass flow into or out of system is always accompanied by energy and entropy transfer.


FIGURE 7-61
Mechanisms of entropy transfer for a general system.


FIGURE 7-62
Entropy generation outside system boundaries can be accounted for by writing an entropy balance on an extended system that includes the system and its immediate surroundings.

## Entropy Generation, $S_{\text {gen }}$

Irreversibilities such as friction, mixing, chemical reactions, heat transfer through a finite temperature difference, unrestrained expansion, nonquasiequilibrium compression, or expansion always cause the entropy of a system to increase, and entropy generation is a measure of the entropy created by such effects during a process.

For a reversible process (a process that involves no irreversibilities), the entropy generation is zero and thus the entropy change of a system is equal to the entropy transfer. Therefore, the entropy balance relation in the reversible case becomes analogous to the energy balance relation, which states that energy change of a system during a process is equal to the energy transfer during that process. However, note that the energy change of a system equals the energy transfer for any process, but the entropy change of a system equals the entropy transfer only for a reversible process.
The entropy transfer by heat $Q / T$ is zero for adiabatic systems, and the entropy transfer by mass $m s$ is zero for systems that involve no mass flow across their boundary (i.e., closed systems).

Entropy balance for any system undergoing any process can be expressed more explicitly as

$$
\underbrace{S_{\text {inss }}-S_{\text {out }}}_{\substack{\text { Net etrnopy }  \tag{7-76}\\
\text { by hatratser and }}}+\underbrace{S_{\text {gen }}}_{\substack{\text { Entropy } \\
\text { generation }}}=\underbrace{\Delta S_{\text {system }}}_{\begin{array}{c}
\text { Change } \\
\text { in entropy }
\end{array}} \quad(\mathrm{kJ} / \mathrm{K})
$$

or, in the rate form, as
where the rates of entropy transfer by heat transferred at a rate of $\dot{Q}$ and mass flowing at a rate of $\dot{m}$ are $\dot{S}_{\text {heat }}=\dot{Q} / T$ and $\dot{S}_{\text {mass }}=\dot{m} s$. The entropy balance can also be expressed on a unit-mass basis as

$$
\begin{equation*}
\left(s_{\text {in }}-s_{\text {out }}\right)+s_{\text {gen }}=\Delta s_{\text {system }} \quad(\mathrm{kJ} / \mathrm{kg} \cdot \mathrm{~K}) \tag{7-78}
\end{equation*}
$$

where all the quantities are expressed per unit mass of the system. Note that for a reversible process, the entropy generation term $S_{\text {gen }}$ drops out from all of the relations above.

The term $S_{\text {gen }}$ represents the entropy generation within the system boundary only (Fig. 7-61), and not the entropy generation that may occur outside the system boundary during the process as a result of external irreversibilities. Therefore, a process for which $S_{\mathrm{gen}}=0$ is internally reversible, but not necessarily totally reversible. The total entropy generated during a process can be determined by applying the entropy balance to an extended system that includes the system itself and its immediate surroundings where external irreversibilities might be occurring (Fig. 7-62). Also, the entropy change in this case is equal to the sum of the entropy change of the system and the entropy change of the immediate surroundings. Note that under steady conditions, the state and thus the entropy of the immediate surroundings (let us call it the "buffer zone") at any point does not change during the process, and the entropy change of the buffer zone is zero. The entropy change of the buffer zone, if any, is usually small relative to the entropy change of the system, and thus it is usually disregarded.

When evaluating the entropy transfer between an extended system and the surroundings, the boundary temperature of the extended system is simply taken to be the environment temperature.

## Closed Systems

A closed system involves no mass flow across its boundaries, and its entropy change is simply the difference between the initial and final entropies of the system. The entropy change of a closed system is due to the entropy transfer accompanying heat transfer and the entropy generation within the system boundaries. Taking the positive direction of heat transfer to be to the system, the general entropy balance relation (Eq. 7-76) can be expressed for a closed system as
Closed system: $\quad \sum \frac{Q_{k}}{T_{k}}+S_{\text {gen }}=\Delta S_{\text {system }}=S_{2}-S_{1} \quad(\mathrm{~kJ} / \mathrm{K})$
The entropy balance relation above can be stated as:
The entropy change of a closed system during a process is equal to the sum of the net entropy transferred through the system boundary by heat transfer and the entropy generated within the system boundaries.

For an adiabatic process ( $Q=0$ ), the entropy transfer term in the above relation drops out and the entropy change of the closed system becomes equal to the entropy generation within the system boundaries. That is,

Adiabatic closed system:

$$
\begin{equation*}
S_{\text {gen }}=\Delta S_{\text {adiabatic system }} \tag{7-80}
\end{equation*}
$$

Noting that any closed system and its surroundings can be treated as an adiabatic system and the total entropy change of a system is equal to the sum of the entropy changes of its parts, the entropy balance for a closed system and its surroundings can be written as
System + Surroundings: $\quad S_{\text {gen }}=\sum \Delta S=\Delta S_{\text {system }}+\Delta S_{\text {surroundings }}$
where $\Delta S_{\text {system }}=m\left(s_{2}-s_{1}\right)$ and the entropy change of the surroundings can be determined from $\Delta S_{\text {surr }}=Q_{\text {surr }} / T_{\text {surr }}$ if its temperature is constant. At initial stages of studying entropy and entropy transfer, it is more instructive to start with the general form of the entropy balance (Eq. 7-76) and to simplify it for the problem under consideration. The specific relations above are convenient to use after a certain degree of intuitive understanding of the material is achieved.

## Control Volumes

The entropy balance relations for control volumes differ from those for closed systems in that they involve one more mechanism of entropy exchange: mass flow across the boundaries. As mentioned earlier, mass possesses entropy as well as energy, and the amounts of these two extensive properties are proportional to the amount of mass (Fig. 7-63).
Taking the positive direction of heat transfer to be to the system, the general entropy balance relations (Eqs. 7-76 and 7-77) can be expressed for control volumes as

$$
\sum \frac{Q_{k}}{T_{k}}+\sum m_{i} s_{i}-\sum m_{e} s_{e}+S_{\mathrm{gen}}=\left(S_{2}-S_{1}\right)_{\mathrm{cV}} \quad(\mathrm{~kJ} / \mathrm{K})
$$

(7-82)


FIGURE 7-63
The entropy of a control volume changes as a result of mass flow as well as heat transfer.


FIGURE 7-64
The entropy of a substance always increases (or remains constant in the case of a reversible process) as it flows through a single-stream, adiabatic, steady-flow device.
or, in the rate form, as

$$
\begin{equation*}
\sum \frac{\dot{Q}_{k}}{T_{k}}+\sum \dot{m}_{i} s_{i}-\sum \dot{m}_{e} s_{e}+\dot{S}_{\mathrm{gen}}=d S_{\mathrm{CV}} / d t \quad(\mathrm{~kW} / \mathrm{K}) \tag{7-83}
\end{equation*}
$$

This entropy balance relation can be stated as:
The rate of entropy change within the control volume during a process is equal to the sum of the rate of entropy transfer through the control volume boundary by heat transfer, the net rate of entropy transfer into the control volume by mass flow, and the rate of entropy generation within the boundaries of the control volume as a result of irreversibilities.

Most control volumes encountered in practice such as turbines, compressors, nozzles, diffusers, heat exchangers, pipes, and ducts operate steadily, and thus they experience no change in their entropy. Therefore, the entropy balance relation for a general steady-flow process can be obtained from Eq. $7-83$ by setting $d S_{\mathrm{CV}} / d t=0$ and rearranging to give

Steady-flow:

$$
\begin{equation*}
\dot{S}_{\text {gen }}=\sum \dot{m}_{e} s_{e}-\sum \dot{m}_{i} s_{i}-\sum \frac{\dot{Q}_{k}}{T_{k}} \tag{7-84}
\end{equation*}
$$

For single-stream (one inlet and one exit) steady-flow devices, the entropy balance relation simplifies to

Steady-flow, single-stream:

$$
\begin{equation*}
\dot{S}_{\text {gen }}=\dot{m}\left(s_{e}-s_{i}\right)-\sum \frac{\dot{Q}_{k}}{T_{k}} \tag{7-85}
\end{equation*}
$$

For the case of an adiabatic single-stream device, the entropy balance relation further simplifies to

Steady-flow, single-stream, adiabatic: $\quad \dot{S}_{\text {gen }}=\dot{m}\left(s_{e}-s_{i}\right)$
which indicates that the specific entropy of the fluid must increase as it flows through an adiabatic device since $\dot{S}_{\text {gen }} \geq 0$ (Fig. 7-64). If the flow through the device is reversible and adiabatic, then the entropy remains constant, $s_{e}=s_{i}$, regardless of the changes in other properties.

## EXAMPLE 7-17 Entropy Generation in a Wall

Consider steady heat transfer through a $5-\mathrm{m} \times 7-\mathrm{m}$ brick wall of a house of thickness 30 cm . On a day when the temperature of the outdoors is $0^{\circ} \mathrm{C}$, the house is maintained at $27^{\circ} \mathrm{C}$. The temperatures of the inner and outer surfaces of the brick wall are measured to be $20^{\circ} \mathrm{C}$ and $5^{\circ} \mathrm{C}$, respectively, and the rate of heat transfer through the wall is 1035 W . Determine the rate of entropy generation in the wall, and the rate of total entropy generation associated with this heat transfer process.

Solution Steady heat transfer through a wall is considered. For specified heat transfer rate, wall temperatures, and environment temperatures, the entropy generation rate within the wall and the total entropy generation rate are to be determined.
Assumptions 1 The process is steady, and thus the rate of heat transfer through the wall is constant. 2 Heat transfer through the wall is onedimensional.

Analysis We first take the wall as the system (Fig. 7-65). This is a closed system since no mass crosses the system boundary during the process. We note that the entropy change of the wall is zero during this process since the state and thus the entropy of the wall do not change anywhere in the wall. Heat and entropy are entering from one side of the wall and leaving from the other side.

The rate form of the entropy balance for the wall simplifies to

$$
\begin{gathered}
\underbrace{\dot{S}_{\text {in }}-\dot{S}_{\text {out }}}_{\begin{array}{c}
\text { Rate of net entropy } \\
\text { transfer by heat } \\
\text { and mass }
\end{array}}+\underbrace{\dot{S}_{\text {gen }}}_{\begin{array}{c}
\text { Rate of entropy } \\
\text { generation }
\end{array}}=\underbrace{d S_{\text {system }} / d t}_{\begin{array}{c}
\text { Rate of change } \\
\text { in entropy }
\end{array}} \\
\quad\left(\frac{\dot{Q}}{T}\right)_{\text {in }}^{0 \text { (steady) }} \\
\\
\frac{1035 \mathrm{~W}}{293 \mathrm{~K}}-\frac{\left(\frac{\dot{Q}}{T}\right)_{\text {out }}}{}+\dot{S}_{\text {gen }}=0 \\
278 \mathrm{~K}
\end{gathered}+\dot{S}_{\text {gen }}=0
$$

Therefore, the rate of entropy generation in the wall is

$$
\dot{S}_{\text {gen,wall }}=0.191 \mathrm{~W} / \mathrm{K}
$$

Note that entropy transfer by heat at any location is $Q / T$ at that location, and the direction of entropy transfer is the same as the direction of heat transfer.

To determine the rate of total entropy generation during this heat transfer process, we extend the system to include the regions on both sides of the wall that experience a temperature change. Then one side of the system boundary becomes room temperature while the other side becomes the temperature of the outdoors. The entropy balance for this extended system (system + immediate surroundings) is the same as that given above, except the two boundary temperatures are now 300 and 273 K instead of 293 and 278 K , respectively. Then the rate of total entropy generation becomes

$$
\frac{1035 \mathrm{~W}}{300 \mathrm{~K}}-\frac{1035 \mathrm{~W}}{273 \mathrm{~K}}+\dot{S}_{\text {gen,total }}=0 \quad \rightarrow \quad \dot{S}_{\text {gen,total }}=0.341 \mathrm{~W} / \mathrm{K}
$$

Discussion Note that the entropy change of this extended system is also zero since the state of air does not change at any point during the process. The differences between the two entropy generations is $0.150 \mathrm{~W} / \mathrm{K}$, and it represents the entropy generated in the air layers on both sides of the wall. The entropy generation in this case is entirely due to irreversible heat transfer through a finite temperature difference.

## EXAMPLE 7-18 Entropy Generation during a Throttling Process

Steam at 7 MPa and $450^{\circ} \mathrm{C}$ is throttled in a valve to a pressure of 3 MPa during a steady-flow process. Determine the entropy generated during this process and check if the increase of entropy principle is satisfied.

Solution Steam is throttled to a specified pressure. The entropy generated during this process is to be determined, and the validity of the increase of entropy principle is to be verified.
Assumptions 1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\mathrm{CV}}=0, \Delta E_{\mathrm{CV}}=0$, and $\Delta S_{\mathrm{CV}}=0.2$ Heat transfer to or from the valve is negligible. 3 The kinetic and potential energy changes are negligible, $\Delta \mathrm{ke}=\Delta \mathrm{pe}=0$.


FIGURE 7-65
Schematic for Example 7-17.

## FIGURE 7-66

Schematic and T-s diagram for Example 7-18.


Analysis We take the throttling valve as the system (Fig. 7-66). This is a control volume since mass crosses the system boundary during the process. We note that there is only one inlet and one exit and thus $\dot{m}_{1}=\dot{m}_{2}=\dot{m}$. Also, the enthalpy of a fluid remains nearly constant during a throttling process and thus $h_{2} \cong h_{1}$.

The entropy of the steam at the inlet and the exit states is determined from the steam tables to be

State 1:

$$
\left.\begin{array}{rl}
P_{1} & =7 \mathrm{MPa} \\
T_{1} & =450^{\circ} \mathrm{C}
\end{array}\right\} \quad \begin{aligned}
& h_{1}=3288.3 \mathrm{~kJ} / \mathrm{kg} \\
& s_{1}=6.6353 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}
\end{aligned}
$$

$$
\left.\begin{array}{rl}
P_{2} & =3 \mathrm{MPa} \\
h_{2} & =h_{1}
\end{array}\right\} \quad s_{2}=7.0046 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}
$$

Then the entropy generation per unit mass of the steam is determined from the entropy balance applied to the throttling valve,

$$
\begin{gathered}
\underbrace{\dot{S}_{\text {in }}-\dot{S}_{\text {out }}}_{\begin{array}{c}
\text { ate of net entropy } \\
\text { transfer by heat } \\
\text { and mass }
\end{array}}+\underbrace{\dot{S}_{\text {gen }}}_{\begin{array}{c}
\text { Rate of entropy } \\
\text { generation }
\end{array}}=\underbrace{d S_{\text {system }} / d t}_{\begin{array}{c}
\text { Rate of change } \\
\text { in entropy }
\end{array}} \\
\dot{m} s_{1}-\dot{m} s_{2}+\dot{S}_{\text {gen }}=0 \\
\dot{S}_{\text {gen }}=\dot{m}\left(s_{2}-s_{1}\right)
\end{gathered}
$$

Dividing by mass flow rate and substituting gives

$$
s_{\mathrm{gen}}=s_{2}-s_{1}=7.0046-6.6353=0.3693 \mathbf{k J} / \mathbf{k g} \cdot \mathbf{K}
$$

This is the amount of entropy generated per unit mass of steam as it is throttled from the inlet state to the final pressure, and it is caused by unrestrained expansion. The increase of entropy principle is obviously satisfied during this process since the entropy generation is positive.

## EXAMPLE 7-19 Entropy Generated when a Hot Block Is Dropped in a Lake

A 50-kg block of iron casting at 500 K is thrown into a large lake that is at a temperature of 285 K . The iron block eventually reaches thermal equilibrium with the lake water. Assuming an average specific heat of $0.45 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$ for the iron, determine (a) the entropy change of the iron block, ( $b$ ) the entropy change of the lake water, and (c) the entropy generated during this process.

Solution A hot iron block is thrown into a lake, and cools to the lake temperature. The entropy changes of the iron and of the lake as well as the entropy generated during this process are to be determined.
Assumptions 1 Both the water and the iron block are incompressible substances. 2 Constant specific heats can be used for the water and the iron. 3 The kinetic and potential energy changes of the iron are negligible, $\Delta \mathrm{KE}=$ $\Delta \mathrm{PE}=0$ and thus $\Delta E=\Delta U$.
Properties The specific heat of the iron is $0.45 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$ (Table A-3).
Analysis We take the iron casting as the system (Fig. 7-67). This is a closed system since no mass crosses the system boundary during the process.

To determine the entropy change for the iron block and for the lake, first we need to know the final equilibrium temperature. Given that the thermal energy capacity of the lake is very large relative to that of the iron block, the lake will absorb all the heat rejected by the iron block without experiencing any change in its temperature. Therefore, the iron block will cool to 285 K during this process while the lake temperature remains constant at 285 K .
(a) The entropy change of the iron block can be determined from

$$
\begin{aligned}
\Delta S_{\text {iron }} & =m\left(s_{2}-s_{1}\right)=m c_{\text {avg }} \ln \frac{T_{2}}{T_{1}} \\
& =(50 \mathrm{~kg})(0.45 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K}) \ln \frac{285 \mathrm{~K}}{500 \mathrm{~K}} \\
& =-\mathbf{1 2 . 6 5} \mathbf{~ k J} / \mathrm{K}
\end{aligned}
$$

(b) The temperature of the lake water remains constant during this process at 285 K . Also, the amount of heat transfer from the iron block to the lake is determined from an energy balance on the iron block to be

$$
\begin{gathered}
\underbrace{E_{\text {in }}-E_{\text {out }}}_{\begin{array}{c}
\text { Net energy transfer } \\
\text { by heat, work, and mass }
\end{array}}=\underbrace{\Delta E_{\text {system }}}_{\begin{array}{c}
\text { Change in internal, kinetic, } \\
\text { potential, etc., energies }
\end{array}} \\
-Q_{\text {out }}=\Delta U=m c_{\text {avg }}\left(T_{2}-T_{1}\right)
\end{gathered}
$$

or
$Q_{\text {out }}=m c_{\text {avg }}\left(T_{1}-T_{2}\right)=(50 \mathrm{~kg})(0.45 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K})(500-285) \mathrm{K}=4838 \mathrm{~kJ}$
Then the entropy change of the lake becomes

$$
\Delta S_{\text {lake }}=\frac{Q_{\text {lake }}}{T_{\text {lake }}}=\frac{+4838 \mathrm{~kJ}}{285 \mathrm{~K}}=16.97 \mathrm{~kJ} / \mathrm{K}
$$



FIGURE 7-68
Schematic for Example 7-20.
(c) The entropy generated during this process can be determined by applying an entropy balance on an extended system that includes the iron block and its immediate surroundings so that the boundary temperature of the extended system is at 285 K at all times:

$$
\begin{array}{r}
\underbrace{S_{\text {in }}-S_{\text {out }}}_{\substack{\text { Net entopy tranter } \\
\text { by heat and mass }}}+\underbrace{S_{\text {gen }}}_{\substack{\text { Enntopy } \\
\text { generation }}}=\underbrace{\Delta S_{\text {sytem }}}_{\begin{array}{c}
\text { Channe } \\
\text { in entropy }
\end{array}} \\
-\frac{Q_{\text {out }}}{T_{b}}+S_{\text {gen }}=\Delta S_{\text {system }}
\end{array}
$$

or

$$
S_{\text {gen }}=\frac{Q_{\text {out }}}{T_{b}}+\Delta S_{\text {system }}=\frac{4838 \mathrm{~kJ}}{285 \mathrm{~K}}-(12.65 \mathrm{~kJ} / \mathrm{K})=4.32 \mathrm{~kJ} / \mathrm{K}
$$

Discussion The entropy generated can also be determined by taking the iron block and the entire lake as the system, which is an isolated system, and applying an entropy balance. An isolated system involves no heat or entropy transfer, and thus the entropy generation in this case becomes equal to the total entropy change,

$$
S_{\text {gen }}=\Delta S_{\text {total }}=\Delta S_{\text {system }}+\Delta S_{\text {lake }}=-12.65+16.97=4.32 \mathrm{~kJ} / \mathrm{K}
$$

which is the same result obtained above.

## EXAMPLE 7-20 Entropy Generation in a Mixing Chamber

Water at 20 psia and $50^{\circ} \mathrm{F}$ enters a mixing chamber at a rate of $300 \mathrm{Ibm} / \mathrm{min}$ where it is mixed steadily with steam entering at 20 psia and $240^{\circ}$. The mixture leaves the chamber at 20 psia and $130^{\circ} \mathrm{F}$, and heat is lost to the surrounding air at $70^{\circ} \mathrm{F}$ at a rate of $180 \mathrm{Btu} / \mathrm{min}$. Neglecting the changes in kinetic and potential energies, determine the rate of entropy generation during this process.

Solution Water and steam are mixed in a chamber that is losing heat at a specified rate. The rate of entropy generation during this process is to be

## determined.

Assumptions 1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\mathrm{cv}}=0, \Delta E_{\mathrm{cv}}=0$, and $\Delta S_{\mathrm{cv}}=0.2$ There are no work interactions involved. 3 The kinetic and potential energies are negligible, $\mathrm{ke} \cong \mathrm{pe} \cong 0$.
Analysis We take the mixing chamber as the system (Fig. 7-68). This is a control volume since mass crosses the system boundary during the process. We note that there are two inlets and one exit.

Under the stated assumptions and observations, the mass and energy balances for this steady-flow system can be expressed in the rate form as follows:

Mass balance:

$$
\dot{m}_{\text {in }}-\dot{m}_{\text {out }}=d m_{\text {system }}^{0(\text { steady })} / d t=0 \rightarrow \dot{m}_{1}+\dot{m}_{2}=\dot{m}_{3}
$$

Energy balance:

$$
\underbrace{\dot{E}_{\text {in }}-\dot{E}_{\text {out }}}_{\begin{array}{c}
\text { Rate of net energy transfer } \\
\text { by heat, work, and mass }
\end{array}}=\underbrace{d E_{\text {system }}^{0} / d t=0}_{\begin{array}{c}
\text { Rate of change in internal, kinetic, } \\
\text { potential, etc., energies }
\end{array}}
$$

$$
\dot{E}_{\mathrm{in}}=\dot{E}_{\mathrm{out}}
$$

$$
\dot{m}_{1} h_{1}+\dot{m}_{2} h_{2}=\dot{m}_{3} h_{3}+\dot{Q}_{\text {out }} \quad(\text { since } \dot{W}=0, \mathrm{ke} \cong \mathrm{pe} \cong 0)
$$

Combining the mass and energy balances gives

$$
\dot{Q}_{\text {out }}=\dot{m}_{1} h_{1}+\dot{m}_{2} h_{2}-\left(\dot{m}_{1}+\dot{m}_{2}\right) h_{3}
$$

The desired properties at the specified states are determined from the steam tables to be

State 1:

$$
\left.\begin{array}{rl}
P_{1} & =20 \mathrm{psia} \\
T_{1} & =50^{\circ} \mathrm{F}
\end{array}\right\} \quad \begin{aligned}
h_{1} & =h_{f @ 50^{\circ} \mathrm{F}}=18.07 \mathrm{Btu} / \mathrm{lbm} \\
s_{1} & =s_{f @ 50^{\circ} \mathrm{F}}=0.03609 \mathrm{Btu} / \mathrm{lbm} \cdot \mathrm{R}
\end{aligned}
$$

State 2:

$$
\left.\begin{array}{rl}
P_{2} & =20 \mathrm{psia} \\
T_{2} & =240^{\circ} \mathrm{F}
\end{array}\right\} \quad \begin{aligned}
h_{2} & =1162.3 \mathrm{Btu} / \mathrm{lbm} \\
s_{2} & =1.7406 \mathrm{Btu} / \mathrm{lbm} \cdot \mathrm{R}
\end{aligned}
$$

State 3:

$$
\left.\begin{array}{rl}
P_{3} & =20 \mathrm{psia} \\
T_{3} & =130^{\circ} \mathrm{F}
\end{array}\right\} \quad \begin{array}{r}
h_{3}=h_{f @ 130^{\circ} \mathrm{F}}=97.99 \mathrm{Btu} / \mathrm{lbm} \\
s_{3}=s_{f @ 130^{\circ} \mathrm{F}}=0.18174 \mathrm{Btu} / \mathrm{lbm} \cdot \mathrm{R}
\end{array}
$$

Substituting,
$180 \mathrm{Btu} / \mathrm{min}=\left[300 \times 18.07+\dot{m}_{2} \times 1162.3-\left(300+\dot{m}_{2}\right) \times 97.99\right] \mathrm{Btu} / \mathrm{min}$ which gives

$$
\dot{m}_{2}=22.7 \mathrm{lbm} / \mathrm{min}
$$

The rate of entropy generation during this process can be determined by applying the rate form of the entropy balance on an extended system that includes the mixing chamber and its immediate surroundings so that the boundary temperature of the extended system is $70^{\circ} \mathrm{F}=530 \mathrm{R}$ :

$$
\begin{aligned}
& \underbrace{\dot{S}_{\text {in }}-\dot{S}_{\text {out }}}_{\begin{array}{c}
\text { Rate of net entropy } \\
\text { transfer by heat } \\
\text { and mass }
\end{array}}+\underbrace{\dot{S}_{\text {gen }}}_{\begin{array}{c}
\text { Rate of entropy } \\
\text { generation }
\end{array}}=\underbrace{d S_{\text {system }} / d t}_{\begin{array}{c}
\text { Rate of change } \\
\text { in entropy }
\end{array}} \\
& \dot{m}_{1} s_{1}+\dot{m}_{2} s_{2}-\dot{m}_{3} s_{3}-\frac{\dot{Q}_{\text {out }}}{T_{b}}+\dot{S}_{\text {gen }}=0
\end{aligned}
$$

Substituting, the rate of entropy generation is determined to be

$$
\begin{aligned}
\dot{S}_{\mathrm{gen}}= & \dot{m}_{3} s_{3}-\dot{m}_{1} s_{1}-\dot{m}_{2} s_{2}+\frac{\dot{Q}_{\text {out }}}{T_{b}} \\
= & (322.7 \times 0.18174-300 \times 0.03609-22.7 \times 1.7406) \mathrm{Btu} / \mathrm{min} \cdot \mathrm{R} \\
& +\frac{180 \mathrm{Btu} / \mathrm{min}}{530 \mathrm{R}} \\
= & \mathbf{8 . 6 5} \mathrm{Btu} / \mathrm{min} \cdot \mathbf{R}
\end{aligned}
$$

Discussion Note that entropy is generated during this process at a rate of 8.65 Btu/min . R. This entropy generation is caused by the mixing of two fluid streams (an irreversible process) and the heat transfer between the mixing chamber and the surroundings through a finite temperature difference (another irreversible process).

## EXAMPLE 7-21 Entropy Generation Associated with Heat Transfer

A frictionless piston-cylinder device contains a saturated liquid-vapor mixture of water at $100^{\circ} \mathrm{C}$. During a constant-pressure process, 600 kJ of heat is transferred to the surrounding air at $25^{\circ} \mathrm{C}$. As a result, part of the water vapor contained in the cylinder condenses. Determine (a) the entropy change of the water and (b) the total entropy generation during this heat transfer process.

Solution Saturated liquid-vapor mixture of water loses heat to its surroundings, and some of the vapor condenses. The entropy change of the water and the total entropy generation are to be determined.
Assumptions 1 There are no irreversibilities involved within the system boundaries, and thus the process is internally reversible. 2 The water temperature remains constant at $100^{\circ} \mathrm{C}$ everywhere, including the boundaries.
Analysis We first take the water in the cylinder as the system (Fig. 7-69). This is a closed system since no mass crosses the system boundary during the process. We note that the pressure and thus the temperature of water in the cylinder remain constant during this process. Also, the entropy of the system decreases during the process because of heat loss.
(a) Noting that water undergoes an internally reversible isothermal process, its entropy change can be determined from

$$
\Delta S_{\text {system }}=\frac{Q}{T_{\text {system }}}=\frac{-600 \mathrm{~kJ}}{(100+273 \mathrm{~K})}=-\mathbf{1 . 6 1} \mathrm{kJ} / \mathrm{K}
$$

(b) To determine the total entropy generation during this process, we consider the extended system, which includes the water, the piston-cylinder device, and the region immediately outside the system that experiences a
temperature change so that the entire boundary of the extended system is at the surrounding temperature of $25^{\circ} \mathrm{C}$. The entropy balance for this extended system (system + immediate surroundings) yields

$$
\underbrace{S_{\text {in }}-S_{\text {out }}}_{\begin{array}{c}
\text { Net entropy transfer } \\
\text { by heat and mass }
\end{array}}+\underbrace{S_{\text {gen }}}_{\begin{array}{c}
\text { Entropy } \\
\text { generation }
\end{array}}=\underbrace{\Delta S_{\text {system }}}_{\begin{array}{c}
\text { Change } \\
\text { in entropy }
\end{array}}
$$

$$
-\frac{Q_{\mathrm{out}}}{T_{b}}+S_{\mathrm{gen}}=\Delta S_{\mathrm{sysem}}
$$

or

$$
S_{\mathrm{gen}}=\frac{Q_{\mathrm{out}}}{T_{b}}+\Delta S_{\mathrm{system}}=\frac{600 \mathrm{~kJ}}{(25+273) \mathrm{K}}+(-1.61 \mathrm{~kJ} / \mathrm{K})=0.40 \mathrm{~kJ} / \mathrm{K}
$$

The entropy generation in this case is entirely due to irreversible heat transfer through a finite temperature difference.

Note that the entropy change of this extended system is equivalent to the entropy change of water since the piston-cylinder device and the immediate surroundings do not experience any change of state at any point, and thus any change in any property, including entropy.
Discussion For the sake of argument, consider the reverse process (i.e., the transfer of 600 kJ of heat from the surrounding air at $25^{\circ} \mathrm{C}$ to saturated water at $100^{\circ} \mathrm{C}$ ) and see if the increase of entropy principle can detect the impossibility of this process. This time, heat transfer will be to the water (heat gain instead of heat loss), and thus the entropy change of water will be $+1.61 \mathrm{~kJ} / \mathrm{K}$. Also, the entropy transfer at the boundary of the extended system will have the same magnitude but opposite direction. This will result in an entropy generation of $-0.4 \mathrm{~kJ} / \mathrm{K}$. The negative sign for the entropy generation indicates that the reverse process is impossible.

To complete the discussion, let us consider the case where the surrounding air temperature is a differential amount below $100^{\circ} \mathrm{C}$ (say $99.999 \ldots 9^{\circ} \mathrm{C}$ ) instead of being $25^{\circ} \mathrm{C}$. This time, heat transfer from the saturated water to the surrounding air will take place through a differential temperature difference rendering this process reversible. It can be shown that $S_{\text {gen }}=0$ for this process.

Remember that reversible processes are idealized processes, and they can be approached but never reached in reality.

## Entropy Generation Associated with a Heat Transfer Process

In Example 7-21 it is determined that $0.4 \mathrm{~kJ} / \mathrm{K}$ of entropy is generated during the heat transfer process, but it is not clear where exactly the entropy generation takes place, and how. To pinpoint the location of entropy generation, we need to be more precise about the description of the system, its surroundings, and the system boundary.

In that example, we assumed both the system and the surrounding air to be isothermal at $100^{\circ} \mathrm{C}$ and $25^{\circ} \mathrm{C}$, respectively. This assumption is reasonable if both fluids are well mixed. The inner surface of the wall must also be
at $100^{\circ} \mathrm{C}$ while the outer surface is at $25^{\circ} \mathrm{C}$ since two bodies in physical contact must have the same temperature at the point of contact. Considering that entropy transfer with heat transfer $Q$ through a surface at constant temperature $T$ is $Q / T$, the entropy transfer from the water into the wall is $Q / T_{\text {sys }}=1.61$ $\mathrm{kJ} / \mathrm{K}$. Likewise, entropy transfer from the outer surface of the wall into the surrounding air is $Q / T_{\text {surr }}=2.01 \mathrm{~kJ} / \mathrm{K}$. Obviously, entropy in the amount of $2.01-1.61=0.4 \mathrm{~kJ} / \mathrm{K}$ is generated in the wall, as illustrated in Fig. 7-70b.

Identifying the location of entropy generation enables us to determine whether a process is internally reversible. A process is internally reversible if no entropy is generated within the system boundaries. Therefore, the heat transfer process discussed in Example 7-21 is internally reversible if the inner surface of the wall is taken as the system boundary, and thus the system excludes the container wall. If the system boundary is taken to be the outer surface of the container wall, then the process is no longer internally reversible since the wall, which is the site of entropy generation, is now part of the system.

For thin walls, it is very tempting to ignore the mass of the wall and to regard the wall as the boundary between the system and the surroundings. This seemingly harmless choice hides the site of the entropy generation from view and is a source of confusion. The temperature in this case drops suddenly from $T_{\text {sys }}$ to $T_{\text {surr }}$ at the boundary surface, and confusion arises as to which temperature to use in the relation $Q / T$ for entropy transfer at the boundary.

Note that if the system and the surrounding air are not isothermal as a result of insufficient mixing, then part of the entropy generation will occur in both the system and the surrounding air in the vicinity of the wall, as shown in Fig. 7-70c.


FIGURE 7-70
Graphical representation of entropy generation during a heat transfer process through a finite temperature difference.

## TOPIC OF SPECIAL INTEREST* <br> Reducing the Cost of Compressed Air

Compressed air at gage pressures of 550 to 1000 kPa ( 80 to 150 psig ) is commonly used in industrial facilities to perform a wide variety of tasks such as cleaning, operating pneumatic equipment, and even refrigeration. It is often referred to as the fourth utility after electricity, water, and natural gas or oil. In production facilities, there is a widespread waste of energy associated with compressed-air systems and a general lack of awareness about the opportunities to conserve energy. A considerable portion of the energy waste associated with compressed-air systems can be avoided by following some commonsense measures. In this section we discuss the energy losses associated with compressed-air systems and their costs to manufacturers. We also show how to reduce the cost of compressed air in existing facilities by making some modifications with attractive payback periods. With the exception of a few compressors that are driven by natural gas engines, all compressors are driven by electric motors (Fig. 7-71).

Some primitive methods of producing an air blast to keep the fire in furnaces alive, such as air-threading bags and the Chinese wind box, date back at least to 2000 вс. The water trompe that compresses air by the fall of water in a tube to blow forges (metal heat shops) is believed to have been in use by 150 вс. In 1650, Otto van Guericke made great improvements in both the compressor and vacuum pump. In 1683, Papin proposed using compressed air to transmit power over long distances. In 1829, William Mann received a patent for multistage compression of air. In 1830, Thilorier was recognized for compressing gases to high pressures in stages. In 1890, Edward Rix transmitted power with air several miles to operate lifting machines in the North Star mine near Grass Valley, California, by using a compressor driven by Pelton wheels. In 1872, cooling was adapted to increase efficiency by spraying water directly into the cylinder through the air inlet valves. This "wet compression" was abandoned later because of the problems it caused. The cooling then was accomplished externally by water jacketing the cylinders. The first large-scale compressor used in the United States was a four-cylinder unit built in 1866 for use in the Hoosac tunnel. The cooling was first accomplished by water injection into the cylinder, and later by running a stream of water over the cylinder. Major advances in recent compressor technology are due to Burleigh, Ingersoll, Sergeant, Rand, and Clayton, among others.

The compressors used range from a few horsepower to more than 10,000 hp in size, and they are among the major energy-consuming equipment in most manufacturing facilities. Manufacturers are quick to identify energy (and thus money) losses from hot surfaces and to insulate those surfaces. However, somehow they are not so sensitive when it comes to saving compressed air since they view air as being free, and the only time the air leaks and dirty air filters get some attention is when the air and pressure losses interfere with the normal operation of the plant. However, paying attention to the compressed-air system and practicing some simple conservation measures can result in considerable energy and cost savings for the plants.

[^2]

FIGURE 7-71
A 1250-hp compressor assembly.
Courtesy of Dresser Rand Company, Painted Post, NY.


FIGURE 7-72
The cost of electricity to operate a compressor for one year can exceed the purchase price of the compressor.


FIGURE 7-73
Air leaks commonly occur at joints and connections.

The hissing of air leaks can sometimes be heard even in high-noise manufacturing facilities. Pressure drops at end-use points in the order of 40 percent of the compressor-discharged pressure are not uncommon. Yet a common response to such a problem is the installation of a larger compressor instead of checking the system and finding out what the problem is. The latter corrective action is usually taken only after the larger compressor also fails to eliminate the problem. The energy wasted in compressed-air systems because of poor installation and maintenance can account for up to 50 percent of the energy consumed by the compressor, and about half of this amount can be saved by simple measures.

The cost of electricity to operate a compressor for one year can exceed the purchase price of the compressor. This is especially the case for larger compressors operating two or three shifts. For example, operating a $125-\mathrm{hp}$ compressor powered by a 90 -percent efficient electric motor at full load for 6000 hours a year at $\$ 0.085 / \mathrm{kWh}$ will cost $\$ 52,820$ a year in electricity cost, which greatly exceeds the purchase and installation cost of a typical unit (Fig. 7-72).

Below we describe some procedures to reduce the cost of compressed air in industrial facilities and quantify the energy and cost savings associated with them. Once the compressor power wasted is determined, the annual energy (usually electricity) and cost savings can be determined from

$$
\begin{equation*}
\text { Energy savings }=(\text { Power saved })(\text { Operating hours }) / \eta_{\text {motor }} \tag{7-87}
\end{equation*}
$$

and

$$
\begin{equation*}
\text { Cost savings }=(\text { Energy savings })(\text { Unit cost of energy }) \tag{7-88}
\end{equation*}
$$

where $\eta_{\text {motor }}$ is the efficiency of the motor driving the compressor and the unit cost of energy is usually expressed in dollars per kilowatt hour ( $1 \mathrm{kWh}=$ 3600 kJ ).

## 1 Repairing Air Leaks on Compressed-Air Lines

Air leaks are the greatest single cause of energy loss in manufacturing facilities associated with compressed-air systems. It takes energy to compress the air, and thus the loss of compressed air is a loss of energy for the facility. A compressor must work harder and longer to make up for the lost air and must use more energy in the process. Several studies at plants have revealed that up to 40 percent of the compressed air is lost through leaks. Eliminating the air leaks totally is impractical, and a leakage rate of 10 percent is considered acceptable.

Air leaks, in general, occur at the joints, flange connections, elbows, reducing bushes, sudden expansions, valve systems, filters, hoses, check valves, relief valves, extensions, and the equipment connected to the compressed-air lines (Fig. 7-73). Expansion and contraction as a result of thermal cycling and vibration are common causes of loosening at the joints, and thus air leaks. Therefore, it is a good practice to check the joints for tightness and to tighten them periodically. Air leaks also commonly occur at the points of end use or where the compressed-air lines are connected to the equipment that operates on compressed air. Because of the frequent opening and closing of the compressed-air lines at these points, the gaskets wear out quickly, and they need to be replaced periodically.

There are many ways of detecting air leaks in a compressed-air system. Perhaps the simplest way of detecting a large air leak is to listen for it. The high velocity of the air escaping the line produces a hissing sound that is difficult not to notice except in environments with a high noise level. Another way of detecting air leaks, especially small ones, is to test the suspected area with soap water and to watch for soap bubbles. This method is obviously not practical for a large system with many connections. A modern way of checking for air leaks is to use an acoustic leak detector, which consists of a directional microphone, amplifiers, audio filters, and digital indicators.
A practical way of quantifying the air leaks in a production facility in its entirety is to conduct a pressure drop test. The test is conducted by stopping all the operations that use compressed air and by shutting down the compressors and closing the pressure relief valve, which relieves pressure automatically if the compressor is equipped with one. This way, any pressure drop in the compressed-air lines is due to the cumulative effects of air leaks. The drop in pressure in the system with time is observed, and the test is conducted until the pressure drops by an amount that can be measured accurately, usually 0.5 atm. The time it takes for the pressure to drop by this amount is measured, and the decay of pressure as a function of time is recorded. The total volume of the compressed-air system, including the compressed-air tanks, the headers, accumulators, and the primary compressed-air lines, is calculated. Ignoring the small lines will make the job easier and will cause the result to be more conservative. The rate of air leak can be determined using the ideal gas equation of state.

The amount of mechanical energy wasted as a unit mass of air escapes through the leaks is equivalent to the actual amount of energy it takes to compress it, and is determined from Eq. 7-57, modified as (Fig. 7-74)

$$
\begin{equation*}
w_{\text {comp,in }}=\frac{w_{\text {reversible comp,in }}}{\eta_{\text {comp }}}=\frac{n R T_{1}}{\eta_{\text {comp }}(n-1)}\left[\left(\frac{P_{2}}{P_{1}}\right)^{(n-1) / n}-1\right] \tag{7-89}
\end{equation*}
$$

where $n$ is the polytropic compression exponent ( $n=1.4$ when the compression is isentropic and $1<n<1.4$ when there is intercooling) and $\eta_{\text {comp }}$ is the compressor efficiency, whose value usually ranges between 0.7 and 0.9 .

Using compressible-flow theory (see Chap. 17), it can be shown that whenever the line pressure is above 2 atm , which is usually the case, the velocity of air at the leak site must be equal to the local speed of sound.


FIGURE 7-74
The energy wasted as compressed air escapes through the leaks is equivalent to the energy it takes to compress it.

Then the mass flow rate of air through a leak of minimum cross-sectional area $A$ becomes

$$
\begin{equation*}
\dot{m}_{\text {air }}=C_{\text {discharge }}\left(\frac{2}{k+1}\right)^{1 /(k-1)} \frac{P_{\text {line }}}{R T_{\text {line }}} A \sqrt{k R\left(\frac{2}{k+1}\right) T_{\text {line }}} \tag{7-90}
\end{equation*}
$$

where $k$ is the specific heat ratio ( $k=1.4$ for air) and $C_{\text {discharge }}$ is a discharge (or loss) coefficient that accounts for imperfections in flow at the leak site. Its value ranges from about 0.60 for an orifice with sharp edges to 0.97 for a well-rounded circular hole. The air-leak sites are imperfect in shape, and thus the discharge coefficient can be taken to be 0.65 in the absence of actual data. Also, $T_{\text {line }}$ and $P_{\text {line }}$ are the temperature and pressure in the compressedair line, respectively.

Once $\dot{m}_{\text {air }}$ and $w_{\text {comp,in }}$ are available, the power wasted by the leaking compressed air (or the power saved by repairing the leak) is determined from

$$
\begin{equation*}
\text { Power saved }=\text { Power wasted }=\dot{m}_{\text {air }} w_{\text {comp,in }} \tag{7-91}
\end{equation*}
$$

## EXAMPLE 7-22 Energy and Cost Savings by Fixing Air Leaks

The compressors of a production facility maintain the compressed-air lines at a (gauge) pressure of 700 kPa at sea level where the atmospheric pressure is 101 kPa (Fig. 7-75). The average temperature of air is $20^{\circ} \mathrm{C}$ at the compressor inlet and $24^{\circ} \mathrm{C}$ in the compressed-air lines. The facility operates 4200 hours a year, and the average price of electricity is $\$ 0.078 / \mathrm{kWh}$. Taking the compressor efficiency to be 0.8 , the motor efficiency to be 0.92 , and the discharge coefficient to be 0.65 , determine the energy and money saved per year by sealing a leak equivalent to a 3-mm-diameter hole on the compressedair line.

Solution An air leak in the compressed air lines of a facility is considered. The energy and money saved per year by sealing the leak are to be determined.
Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 Pressure losses in the compressed air lines are negligible.
Analysis We note that the absolute pressure is the sum of the gauge and atmospheric pressures.

FIGURE 7-75
Schematic for Example 7-22.


The work needed to compress a unit mass of air at $20^{\circ} \mathrm{C}$ from the atmospheric pressure of 101 kPa to $700+101=801 \mathrm{kPa}$ is

$$
\begin{aligned}
w_{\text {comp }, \text { in }} & =\frac{n R T_{1}}{\eta_{\text {comp }}(n-1)}\left[\left(\frac{P_{2}}{P_{1}}\right)^{(n-1) / n}-1\right] \\
& =\frac{(1.4)(0.287 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K})(293 \mathrm{~K})}{(0.8)(1.4-1)}\left[\left(\frac{801 \mathrm{kPa}}{101 \mathrm{kPa}}\right)^{0.4 / 1.4}-1\right]=296.9 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

The cross-sectional area of the 3-mm-diameter hole is

$$
A=\pi D^{2} / 4=\pi\left(3 \times 10^{-3} \mathrm{~m}\right)^{2} / 4=7.069 \times 10^{-6} \mathrm{~m}^{2}
$$

Noting that the line conditions are 297 K and 801 kPa , the mass flow rate of the air leaking through the hole is determined to be

$$
\begin{aligned}
\dot{m}_{\text {air }}= & C_{\text {discharge }}\left(\frac{2}{k+1}\right)^{1 /(k-1)} \frac{P_{\text {line }}}{R T_{\text {line }}} A \sqrt{k R\left(\frac{2}{k+1}\right) T_{\text {line }}} \\
= & (0.65)\left(\frac{2}{1.4+1}\right)^{1 /(1.4-1)} \frac{801 \mathrm{kPa}}{\left(0.287 \mathrm{kPa} \cdot \mathrm{~m}^{3} / \mathrm{kg} \cdot \mathrm{~K}\right)(297 \mathrm{~K})}\left(7.069 \times 10^{-6} \mathrm{~m}^{2}\right) \\
& \times \sqrt{(1.4)(0.287 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{~K})\left(\frac{1000 \mathrm{~m}^{2} / \mathrm{s}^{2}}{1 \mathrm{~kJ} / \mathrm{kg}}\right)\left(\frac{2}{1.4+1}\right)(297 \mathrm{~K})} \\
= & 0.008632 \mathrm{~kg} / \mathrm{s}
\end{aligned}
$$

Then the power wasted by the leaking compressed air becomes

$$
\begin{aligned}
\text { Power wasted } & =\dot{m}_{\mathrm{air}} w_{\text {comp,in }} \\
& =(0.008632 \mathrm{~kg} / \mathrm{s})(296.9 \mathrm{~kJ} / \mathrm{kg}) \\
& =2.563 \mathrm{~kW}
\end{aligned}
$$

The compressor operates $4200 \mathrm{~h} / \mathrm{yr}$, and the motor efficiency is 0.92 . Then the annual energy and cost savings resulting from repairing this leak are determined to be

$$
\begin{aligned}
\text { Energy savings } & =(\text { Power saved })(\text { Operating hours }) / \eta_{\text {motor }} \\
& =(2.563 \mathrm{~kW})(4200 \mathrm{~h} / \mathrm{yr}) / 0.92 \\
& =\mathbf{1 1 , 7 0 0} \mathbf{k W h} / \mathbf{y r} \\
\text { Cost savings } & =(\text { Energy savings })(\text { Unit cost of energy }) \\
& =(11,700 \mathrm{kWh} / \mathrm{yr})(\$ 0.078 / \mathrm{kWh}) \\
& =\$ 913 / \mathbf{y r}
\end{aligned}
$$

Discussion Note that the facility will save $11,700 \mathrm{kWh}$ of electricity worth $\$ 913$ a year when this air leak is fixed. This is a substantial amount for a single leak whose equivalent diameter is 3 mm .

## 2 Installing High-Efficiency Motors

Practically all compressors are powered by electric motors, and the electrical energy a motor draws for a specified power output is inversely proportional to its efficiency. Electric motors cannot convert the electrical energy they consume into mechanical energy completely, and the ratio of the mechanical


| Motor <br> efficiency <br> $\eta_{\text {motor }}$ | Electrical power <br> consumed per kW of <br> mechanical (shaft) <br> power output, |
| :---: | :---: |
|  | $\dot{W}_{\text {electric }}=\dot{W}_{\text {shaft }} / \eta_{\text {motor }}$ |
| $100 \%$ | 1.00 kW |
| 90 | 1.11 |
| 80 | 1.25 |
| 70 | 1.43 |
| 60 | 1.67 |
| 50 | 2.00 |
| 40 | 2.50 |
| 30 | 3.33 |
| 20 | 5.00 |
| 10 | 10.00 |

## FIGURE 7-76

The electrical energy consumed by a motor is inversely proportional to its efficiency.


FIGURE 7-77
The efficiency of an electric motor decreases at part load.
power supplied to the electrical power consumed during operation is called the motor efficiency, $\eta_{\text {motor }}$. Therefore, the electric power consumed by the motor and the mechanical (shaft) power supplied to the compressor are related to each other by (Fig. 7-76)

$$
\begin{equation*}
\dot{W}_{\text {electric }}=\dot{W}_{\text {comp }} / \eta_{\text {motor }} \tag{7-92}
\end{equation*}
$$

For example, assuming no transmission losses, a motor that is 80 percent efficient will draw $1 / 0.8=1.25 \mathrm{~kW}$ of electric power for each kW of shaft power it delivers to the compressor, whereas a motor that is 95 percent efficient will draw only $1 / 0.95=1.05 \mathrm{~kW}$ to deliver 1 kW . Therefore, highefficiency motors cost less to operate than their standard counterparts, but they also usually cost more to purchase. However, the energy savings usually make up for the price differential during the first few years. This is especially true for large compressors that operate more than one regular shift. The electric power saved by replacing the existing standard motor of efficiency $\eta_{\text {standard }}$ by a high-efficiency one of efficiency $\eta_{\text {efficient }}$ is determined from

$$
\begin{align*}
\dot{W}_{\text {electric,saved }} & =\dot{W}_{\text {electric,standard }}-\dot{W}_{\text {electric,efficient }} \\
& =\dot{W}_{\text {comp }}\left(1 / \eta_{\text {standard }}-1 / \eta_{\text {efficicent }}\right)  \tag{7-93}\\
& =(\text { Rated power })(\text { Load factor })\left(1 / \eta_{\text {standard }}-1 / \eta_{\text {effficient }}\right)
\end{align*}
$$

where rated power is the nominal power of the motor listed on its label (the power the motor delivers at full load) and the load factor is the fraction of the rated power at which the motor normally operates. Then the annual energy savings as a result of replacing a motor by a high-efficiency motor instead of a comparable standard one is

$$
\begin{equation*}
\text { Energy savings }=\dot{W}_{\text {electric,saved }} \times \text { Annual operating hours } \tag{7-94}
\end{equation*}
$$

The efficiencies of motors used to power compressors usually range from about 70 percent to over 96 percent. The portion of electric energy not converted to mechanical energy is converted to heat. The amount of heat generated by the motors may reach high levels, especially at part load, and it may cause overheating if not dissipated effectively. It may also cause the air temperature in the compressor room to rise to undesirable levels. For example, a 90 -percent-efficient $100-\mathrm{kW}$ motor generates as much heat as a $10-\mathrm{kW}$ resistance heater in the confined space of the compressor room, and it contributes greatly to the heating of the air in the room. If this heated air is not vented properly, and the air into the compressor is drawn from inside the compressor room, the performance of the compressor will also decline, as explained later.
Important considerations in the selection of a motor for a compressor are the operating profile of the compressor (i.e., the variation of the load with time), and the efficiency of the motor at part-load conditions. The part-load efficiency of a motor is as important as the full-load efficiency if the compressor is expected to operate at part load during a significant portion of the total operating time. A typical motor has a nearly flat efficiency curve between half load and full load, and peak efficiency is usually at about $75 \%$ load. Efficiency falls off pretty steeply below half load, and thus operation below $50 \%$ load should be avoided as much as possible. For example, the efficiency of a motor may drop from 90 percent at full load to 87 percent at half load and 80 percent at quarter load (Fig. 7-77). The efficiency of another motor of similar
specifications, on the other hand, may drop from 91 percent at full load to 75 percent at quarter load. The first motor is obviously better suited for a situation in which a compressor is expected to operate at quarter load during a significant portion of the time. The efficiency at part-load conditions can be improved greatly by installing variable voltage controllers if it is economical to do so. Also, oversizing a motor just to be on the safe side and to have some excess power just in case is a bad practice since this will cause the motor to operate almost always at part load and thus at a lower efficiency. Besides, oversized motors have a higher initial cost. However, oversized motors waste little energy as long as they operate at loads above $50 \%$ of design.

## Using a Smaller Motor at High Capacity

We tend to purchase larger equipment than needed for reasons like having a safety margin or anticipated future expansion, and compressors are no exception. The uncertainties in plant operation are partially responsible for opting for a larger compressor, since it is preferred to have an oversized compressor than an undersized one. Sometimes compressors that have several times the required capacity are purchased with the perception that the extra capacity may be needed some day. The result is a compressor that runs intermittently at full load, or one that runs continuously at part load.

A compressor that operates at part load also causes the motor to operate less efficiently since the efficiency of an electric motor decreases as the point of operation shifts down from its rated power, as explained above. The result is a motor that consumes more electricity per unit power delivered, and thus a more expensive operation. The operating costs can be reduced by switching to a smaller motor that runs at rated power and thus at a higher efficiency.

## 3 Using Outside Air for Compressor Intake

We have pointed out earlier that the power consumed by a compressor is proportional to the specific volume, which is proportional to the absolute temperature of the gas at a given pressure. It is also clear from Eq. 7-89 that the compressor work is directly proportional to the inlet temperature of air. Therefore, the lower the inlet temperature of the air, the smaller the compressor work. Then the power reduction factor, which is the fraction of compressor power reduced as a result of taking intake air from the outside, becomes

$$
\begin{equation*}
f_{\text {reduction }}=\frac{W_{\text {comp,inside }}-W_{\text {comp,outside }}}{W_{\text {comp,inside }}}=\frac{T_{\text {inside }}-T_{\text {outside }}}{T_{\text {inside }}}=1-\frac{T_{\text {outside }}}{T_{\text {inside }}} \tag{7-95}
\end{equation*}
$$

where $T_{\text {inside }}$ and $T_{\text {outside }}$ are the absolute temperatures (in K or R ) of the ambient air inside and outside the facility, respectively. Thus, reducing the absolute inlet temperature by 5 percent, for example, will reduce the compressor power input by 5 percent. As a rule of thumb, for a specified amount of compressed air, the power consumption of the compressor decreases (or, for a fixed power input, the amount of compressed air increases) by 1 percent for each $3^{\circ} \mathrm{C}$ drop in the temperature of the inlet air to the compressor.

Compressors are usually located inside the production facilities or in adjacent shelters specifically built outside these facilities. The intake air is


FIGURE 7-78
The power consumption of a compressor can be reduced by taking in air from the outside.


FIGURE 7-79
Schematic for Example 7-23.
normally drawn from inside the building or the shelter. However, in many locations the air temperature in the building is higher than the outside air temperature, because of space heaters in the winter and the heat given up by a large number of mechanical and electrical equipment as well as the furnaces year round. The temperature rise in the shelter is also due to the heat dissipation from the compressor and its motor. The outside air is generally cooler and thus denser than the air in the compressor room even on hot summer days. Therefore, it is advisable to install an intake duct to the compressor inlet so that the air is supplied directly from the outside of the building instead of the inside, as shown in Fig. 7-78. This will reduce the energy consumption of the compressor since it takes less energy to compress a specified amount of cool air than the same amount of warm air. Compressing the warm air in a building in winter also wastes the energy used to heat the air.

## 4 Reducing the Air Pressure Setting

Another source of energy waste in compressed-air systems is compressing the air to a higher pressure than required by the air-driven equipment since it takes more energy to compress air to a higher pressure. In such cases considerable energy savings can be realized by determining the minimum required pressure and then reducing the air pressure control setting on the compressor accordingly. This can be done on both screw-type and reciprocating compressors by simply adjusting the pressure setting to match the needs.

The amount of energy it takes to compress a unit mass of air is determined from Eq. 7-89. We note from that relation that the higher the pressure $P_{2}$ at the compressor exit, the larger the work required for compression. Reducing the exit pressure of the compressor to $P_{2 \text {,reduced }}$ will reduce the power input requirements of the compressor by a factor of

$$
\begin{equation*}
f_{\text {reduction }}=\frac{w_{\text {comp,current }}-w_{\text {comp,reduced }}}{w_{\text {comp,current }}}=1-\frac{\left(P_{2, \text { reduced }} / P_{1}\right)^{(n-1) / n}-1}{\left(P_{2} / P_{1}\right)^{(n-1) / n}-1} \tag{7-96}
\end{equation*}
$$

A power reduction (or savings) factor of $f_{\text {reduction }}=0.08$, for example, indicates that the power consumption of the compressor is reduced by 8 percent as a result of reducing the pressure setting.
Some applications require slightly compressed air. In such cases, the need can be met by a blower instead of a compressor. Considerable energy can be saved in this manner since a blower requires a small fraction of the power needed by a compressor for a specified mass flow rate.

## EXAMPLE 7-23 Reducing the Pressure Setting to Reduce Cost

The compressed-air requirements of a plant located at $1400-\mathrm{m}$ elevation is being met by a 75 -hp compressor that takes in air at the local atmospheric pressure of 85.6 kPa and the average temperature of $15^{\circ} \mathrm{C}$ and compresses it to 900 kPa gauge (Fig. 7-79). The plant is currently paying \$12,000 a year in electricity costs to run the compressor. An investigation of the compressedair system and the equipment using the compressed air reveals that compressing the air to 800 kPa is sufficient for this plant. Determine how
much money will be saved as a result of reducing the pressure of the compressed air.

Solution It is observed that the compressor of a facility compresses the air to much higher pressures than needed. The cost savings associated with pressure reduction of the compressor are to be determined.
Assumptions 1 Air is an ideal gas. 2 Compression process is isentropic, and thus $n=k=1.4$.
Analysis The fraction of energy saved as a result of reducing the pressure setting of the compressor is

$$
\begin{aligned}
f_{\text {reduction }} & =1-\frac{\left(P_{2, \text { reduced }} / P_{1}\right)^{(n-1) / n}-1}{\left(P_{2} / P_{1}\right)^{(n-1) / n}-1} \\
& =1-\frac{(885.6 / 85.6)^{(1.4-1) / 1.4}-1}{(985.6 / 85.6)^{(1.4-1) / 1.4}-1}=0.060
\end{aligned}
$$

That is, reducing the pressure setting will reduce the energy consumed by the compressor by about 6 percent. Then,

Cost savings $=($ Current cost $) f_{\text {reduction }}=(\$ 12,000 / \mathrm{yr})(0.06)=\$ 720 / \mathbf{y r}$
Therefore, reducing the pressure setting by 100 kPa will result in annual savings of $\$ 720$ in this case.

There are also other ways to reduce the cost of compressed air in industrial facilities. An obvious way is turning the compressor off during nonproduction periods such as lunch hours, nights, and even weekends. A considerable amount of power can be wasted during this stand-by mode. This is especially the case for screw-type compressors since they consume up to 85 percent of their rated power in this mode. The reciprocating compressors are not immune from this deficiency, however, since they also must cycle on and off because of the air leaks present in the compressed-air lines. The system can be shut down manually during nonproduction periods to save energy, but installing a timer (with manual override) is preferred to do this automatically since it is human nature to put things off when the benefits are not obvious or immediate.

The compressed air is sometimes cooled considerably below its dew point in refrigerated dryers in order to condense and remove a large fraction of the water vapor in the air as well as other noncondensable gases such as oil vapors. The temperature of air rises considerably as it is compressed, sometimes exceeding $250^{\circ} \mathrm{C}$ at compressor exit when compressed adiabatically to just 700 kPa . Therefore, it is desirable to cool air after compression in order to minimize the amount of power consumed by the refrigeration system, just as it is desirable to let the hot food in a pan cool to the ambient temperature before putting it into the refrigerator. The cooling can be done by either ambient air or water, and the heat picked up by the cooling medium can be used for space heating, feedwater heating, or process-related heating.

Compressors are commonly cooled directly by air or by circulating a liquid such as oil or water through them in order to minimize the power consumption. The heat picked up by the oil or water is usually rejected to the


FIGURE 7-80
Waste heat from a compressor can be used to heat a building in winter.
ambient in a liquid-to-air heat exchanger. This heat rejected usually amounts to 60 to 90 percent of the power input, and thus it represents a huge amount of energy that can be used for a useful purpose such as space heating in winter, preheating the air or water in a furnace, or other process-related purposes (Fig. 7-80). For example, assuming 80 percent of the power input is converted to heat, a $150-\mathrm{hp}$ compressor can reject as much heat as a $90-\mathrm{kW}$ electric resistance heater or a $400,000-\mathrm{Btu} / \mathrm{h}$ natural gas heater when operating at full load. Thus, the proper utilization of the waste heat from a compressor can result in significant energy and cost savings.

## SUMMARY

The second law of thermodynamics leads to the definition of a new property called entropy, which is a quantitative measure of microscopic disorder for a system. Any quantity whose cyclic integral is zero is a property, and entropy is defined as

$$
d S=\left(\frac{d Q}{T}\right)_{\mathrm{int} \mathrm{rev}}
$$

For the special case of an internally reversible, isothermal process, it gives

$$
\Delta S=\frac{Q}{T_{0}}
$$

The inequality part of the Clausius inequality combined with the definition of entropy yields an inequality known as the increase of entropy principle, expressed as

$$
S_{\text {gen }} \geq 0
$$

where $S_{\text {gen }}$ is the entropy generated during the process. Entropy change is caused by heat transfer, mass flow, and irreversibilities. Heat transfer to a system increases the entropy, and heat transfer from a system decreases it. The effect of irreversibilities is always to increase the entropy.

The entropy-change and isentropic relations for a process can be summarized as follows:

1. Pure substances:

Any process:

$$
\begin{aligned}
\Delta s & =s_{2}-s_{1} \\
s_{2} & =s_{1}
\end{aligned}
$$

Isentropic process:

## 2. Incompressible substances:

Any process:

$$
s_{2}-s_{1}=c_{\mathrm{avg}} \ln \frac{T_{2}}{T_{1}}
$$

Isentropic process:

$$
T_{2}=T_{1}
$$

3. Ideal gases:
a. Constant specific heats (approximate treatment):

Any process:

$$
\begin{aligned}
& s_{2}-s_{1}=c_{V, \text { avg }} \ln \frac{T_{2}}{T_{1}}+R \ln \frac{V_{2}}{V_{1}} \\
& s_{2}-s_{1}=c_{p, \text { avg }} \ln \frac{T_{2}}{T_{1}}-R \ln \frac{P_{2}}{P_{1}}
\end{aligned}
$$

Isentropic process:

$$
\begin{aligned}
& \left(\frac{T_{2}}{T_{1}}\right)_{s=\text { const. }}=\left(\frac{V_{1}}{V_{2}}\right)^{k-1} \\
& \left(\frac{T_{2}}{T_{1}}\right)_{s=\text { const. }}=\left(\frac{P_{2}}{P_{1}}\right)^{(k-1) / k} \\
& \left(\frac{P_{2}}{P_{1}}\right)_{s=\text { const. }}=\left(\frac{V_{1}}{V_{2}}\right)^{k}
\end{aligned}
$$

b. Variable specific heats (exact treatment):

Any process:

$$
s_{2}-s_{1}=s_{2}^{\circ}-s_{1}^{\circ}-R \ln \frac{P_{2}}{P_{1}}
$$

Isentropic process:

$$
\begin{aligned}
s_{2}^{\circ} & =s_{1}^{\circ}+R \ln \frac{P_{2}}{P_{1}} \\
\left(\frac{P_{2}}{P_{1}}\right)_{s=\text { const. }} & =\frac{P_{r 2}}{P_{r 1}} \\
\left(\frac{V_{2}}{V_{1}}\right)_{s=\text { const. }} & =\frac{V_{r 2}}{V_{r 1}}
\end{aligned}
$$

where $P_{r}$ is the relative pressure and $\nu_{r}$ is the relative specific volume. The function $s^{\circ}$ depends on temperature only.

The steady-flow work for a reversible process can be expressed in terms of the fluid properties as

$$
w_{\mathrm{rev}}=-\int_{1}^{2} v d P-\Delta \mathrm{ke}-\Delta \mathrm{pe}
$$

For incompressible substances ( $V=$ constant $)$ it simplifies to

$$
w_{\mathrm{rev}}=-v\left(P_{2}-P_{1}\right)-\Delta \mathrm{ke}-\Delta \mathrm{pe}
$$

The work done during a steady-flow process is proportional to the specific volume. Therefore, $v$ should be kept as small as possible during a compression process to minimize the work input and as large as possible during an expansion process to maximize the work output.

The reversible work inputs to a compressor compressing an ideal gas from $T_{1}, P_{1}$ to $P_{2}$ in an isentropic ( $P V^{k}=$ constant $)$, polytropic $\left(P \vee^{n}=\right.$ constant $)$, or isothermal ( $P \vee=$ constant $)$ manner, are determined by integration for each case with the following results:
Isentropic: $\quad w_{\text {comp,in }}=\frac{k R\left(T_{2}-T_{1}\right)}{k-1}=\frac{k R T_{1}}{k-1}\left[\left(\frac{P_{2}}{P_{1}}\right)^{(k-1) / k}-1\right]$
Polytropic: $\quad w_{\text {comp, in }}=\frac{n R\left(T_{2}-T_{1}\right)}{n-1}=\frac{n R T_{1}}{n-1}\left[\left(\frac{P_{2}}{P_{1}}\right)^{(n-1) / n}-1\right]$
Isothermal: $w_{\text {comp,in }}=R T \ln \frac{P_{2}}{P_{1}}$

The work input to a compressor can be reduced by using multistage compression with intercooling. For maximum savings from the work input, the pressure ratio across each stage of the compressor must be the same.

Most steady-flow devices operate under adiabatic conditions, and the ideal process for these devices is the isentropic process. The parameter that describes how efficiently a device approximates a corresponding isentropic device is called isentropic or adiabatic efficiency. It is expressed for turbines, compressors, and nozzles as follows:

$$
\begin{aligned}
\eta_{T} & =\frac{\text { Actual turbine work }}{\text { Isentropic turbine work }}=\frac{w_{a}}{w_{s}} \cong \frac{h_{1}-h_{2 a}}{h_{1}-h_{2 s}} \\
\eta_{C} & =\frac{\text { Isentropic compressor work }}{\text { Actual compressor work }}=\frac{w_{s}}{w_{a}} \cong \frac{h_{2 s}-h_{1}}{h_{2 a}-h_{1}} \\
\eta_{N} & =\frac{\text { Actual KE at nozzle exit }}{\text { Isentropic KE at nozzle exit }}=\frac{V_{2 a}^{2}}{V_{2 s}^{2}} \cong \frac{h_{1}-h_{2 a}}{h_{1}-h_{2 s}}
\end{aligned}
$$

In the relations above, $h_{2 a}$ and $h_{2 s}$ are the enthalpy values at the exit state for actual and isentropic processes, respectively.

The entropy balance for any system undergoing any process can be expressed in the general form as

$$
\underbrace{S_{\text {in }}-S_{\text {out }}}_{\begin{array}{c}
\text { Net entropy transfer } \\
\text { by heat and mass }
\end{array}}+\underbrace{S_{\text {gen }}}_{\begin{array}{c}
\text { Entropy } \\
\text { generation }
\end{array}}=\underbrace{\Delta S_{\text {system }}}_{\begin{array}{c}
\text { Change } \\
\text { in entropy }
\end{array}}
$$

or, in the rate form, as


For a general steady-flow process it simplifies to

$$
\dot{S}_{\text {gen }}=\sum \dot{m}_{e} s_{e}-\sum \dot{m}_{i} s_{i}-\sum \frac{\dot{Q}_{k}}{T_{k}}
$$

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## PROBLEMS*

## Entropy and the Increase of Entropy Principle

7-1C Does the temperature in the Clausius inequality relation have to be absolute temperature? Why?
7-2C Does a cycle for which $\oint \delta Q>0$ violate the Clausius inequality? Why?
7-3C Is a quantity whose cyclic integral is zero necessarily a property?

7-4C Does the cyclic integral of heat have to be zero (i.e., does a system have to reject as much heat as it receives to complete a cycle)? Explain.
7-5C Does the cyclic integral of work have to be zero (i.e., does a system have to produce as much work as it consumes to complete a cycle)? Explain.
7-6C A system undergoes a process between two fixed states first in a reversible manner and then in an irreversible manner. For which case is the entropy change greater? Why?
$7-7 \mathrm{C}$ Is the value of the integral $\int_{1}^{2} \delta Q / T$ the same for all processes between states 1 and 2? Explain.
$7-8 \mathrm{C}$ Is the value of the integral $\int_{1}^{2} \delta Q / T$ the same for all reversible processes between states 1 and 2? Why?

7-9C To determine the entropy change for an irreversible process between states 1 and 2 , should the integral $\int_{1}^{2} \delta Q / T$ be performed along the actual process path or an imaginary reversible path? Explain.
7-10C Is an isothermal process necessarily internally reversible? Explain your answer with an example.
7-11C How do the values of the integral $\int_{1}^{2} \delta Q / T$ compare for a reversible and irreversible process between the same end states?

7-12C The entropy of a hot baked potato decreases as it cools. Is this a violation of the increase of entropy principle? Explain.

7-13C Is it possible to create entropy? Is it possible to destroy it?
7-14C A piston-cylinder device contains helium gas. During a reversible, isothermal process, the entropy of the helium will (never, sometimes, always) increase.

[^3]7-15C A piston-cylinder device contains nitrogen gas. During a reversible, adiabatic process, the entropy of the nitrogen will (never, sometimes, always) increase.
7-16C A piston-cylinder device contains superheated steam. During an actual adiabatic process, the entropy of the steam will (never, sometimes, always) increase.
7-17C The entropy of steam will (increase, decrease, remain the same) as it flows through an actual adiabatic turbine.

7-18C The entropy of the working fluid of the ideal Carnot cycle (increases, decreases, remains the same) during the isothermal heat addition process.

7-19C The entropy of the working fluid of the ideal Carnot cycle (increases, decreases, remains the same) during the isothermal heat rejection process.
7-20C During a heat transfer process, the entropy of a system (always, sometimes, never) increases.
7-21C Is it possible for the entropy change of a closed system to be zero during an irreversible process? Explain.

7-22C What three different mechanisms can cause the entropy of a control volume to change?
7-23C Steam is accelerated as it flows through an actual adiabatic nozzle. The entropy of the steam at the nozzle exit will be (greater than, equal to, less than) the entropy at the nozzle inlet.
7-24 A rigid tank contains an ideal gas at $40^{\circ} \mathrm{C}$ that is being stirred by a paddle wheel. The paddle wheel does 200 kJ of work on the ideal gas. It is observed that the temperature of the ideal gas remains constant during this process as a result of heat transfer between the system and the surroundings at $30^{\circ} \mathrm{C}$. Determine the entropy change of the ideal gas.


FIGURE P7-24

7-25 Air is compressed by a $12-\mathrm{kW}$ compressor from $P_{1}$ to $P_{2}$. The air temperature is maintained constant at $25^{\circ} \mathrm{C}$ during this process as a result of heat transfer to the surrounding medium at $10^{\circ} \mathrm{C}$. Determine the rate of entropy change of the air. State the assumptions made in solving this problem. Answer: -0.0403 kW/K

7-26 During the isothermal heat addition process of a Carnot cycle, 900 kJ of heat is added to the working fluid from a source at $400^{\circ} \mathrm{C}$. Determine (a) the entropy change of the working fluid, (b) the entropy change of the source, and (c) the total entropy change for the process.

7-27 e૯S Reconsider Prob. 7-26. Using EES (or other) software, study the effects of the varying heat added to the working fluid and the source temperature on the entropy change of the working fluid, the entropy change of the source, and the total entropy change for the process. Let the source temperature vary from 100 to $1000^{\circ} \mathrm{C}$. Plot the entropy changes of the source and of the working fluid against the source temperature for heat transfer amounts of $500 \mathrm{~kJ}, 900 \mathrm{~kJ}$, and 1300 kJ , and discuss the results.
7-28E During the isothermal heat rejection process of a Carnot cycle, the working fluid experiences an entropy change of $-0.7 \mathrm{Btu} / \mathrm{R}$. If the temperature of the heat sink is $95^{\circ} \mathrm{F}$, determine (a) the amount of heat transfer, (b) the entropy change of the sink, and (c) the total entropy change for this process. Answers: (a) 388.5 Btu , (b) $0.7 \mathrm{Btu} / \mathrm{R}$, (c) 0


FIGURE P7-28E

7-29 Refrigerant-134a enters the coils of the evaporator of a refrigeration system as a saturated liquid-vapor mixture at a pressure of 160 kPa . The refrigerant absorbs 180 kJ of heat from the cooled space, which is maintained at $-5^{\circ} \mathrm{C}$, and leaves as saturated vapor at the same pressure. Determine (a) the entropy change of the refrigerant, (b) the entropy change of the cooled space, and (c) the total entropy change for this process.

## Entropy Changes of Pure Substances

7-30C Is a process that is internally reversible and adiabatic necessarily isentropic? Explain.
7-31 The radiator of a steam heating system has a volume of 20 L and is filled with superheated water vapor at 200 kPa and $150^{\circ} \mathrm{C}$. At this moment both the inlet and the exit valves to the radiator are closed. After a while the temperature of the steam drops to $40^{\circ} \mathrm{C}$ as a result of heat transfer to the room air. Determine the entropy change of the steam during this process. Answer: $-0.132 \mathrm{~kJ} / \mathrm{K}$
7-32 A $0.5-\mathrm{m}^{3}$ rigid tank contains refrigerant-134a initially at 200 kPa and 40 percent quality. Heat is transferred now to the refrigerant from a source at $35^{\circ} \mathrm{C}$ until the pressure rises to 400 kPa . Determine (a) the entropy change of the refriger-
ant, (b) the entropy change of the heat source, and (c) the total entropy change for this process.
Answers: (a) $3.880 \mathrm{~kJ} / \mathrm{K}$, (b) $-3.439 \mathrm{~kJ} / \mathrm{K}$, (c) $0.441 \mathrm{~kJ} / \mathrm{K}$
7-33 © Reconsider Prob. 7-32. Using EES (or other) software, investigate the effects of the source temperature and final pressure on the total entropy change for the process. Let the source temperature vary from 30 to $210^{\circ} \mathrm{C}$, and the final pressure vary from 250 to 500 kPa . Plot the total entropy change for the process as a function of the source temperature for final pressures of $250 \mathrm{kPa}, 400 \mathrm{kPa}$, and 500 kPa , and discuss the results.
7-34 A well-insulated rigid tank contains 2 kg of a saturated liquid-vapor mixture of water at 100 kPa . Initially, three-quarters of the mass is in the liquid phase. An electric resistance heater placed in the tank is now turned on and kept on until all the liquid in the tank is vaporized. Determine the entropy change of the steam during this process. Answer: 8.10 kJ/K


FIGURE P7-34
7-35
A rigid tank is divided into two equal parts by a partition. One part of the tank contains 1.5 kg of compressed liquid water at 300 kPa and $60^{\circ} \mathrm{C}$ while the other part is evacuated. The partition is now removed, and the water expands to fill the entire tank. Determine the entropy change of water during this process, if the final pressure in the tank is 15 kPa . Answer: $-0.114 \mathrm{~kJ} / \mathrm{K}$


FIGURE P7-35
 Reconsider Prob. 7-35. Using EES (or other) software, evaluate and plot the entropy generated as a function of surrounding temperature, and determine the values of the surrounding temperatures that are valid for
this problem. Let the surrounding temperature vary from 0 to $100^{\circ} \mathrm{C}$. Discuss your results.
7-37E A piston-cylinder device contains 2 lbm of refriger-ant-134a at 120 psia and $100^{\circ} \mathrm{F}$. The refrigerant is now cooled at constant pressure until it exists as a liquid at $50^{\circ} \mathrm{F}$. Determine the entropy change of the refrigerant during this process.
7-38 An insulated piston-cylinder device contains 5 L of saturated liquid water at a constant pressure of 150 kPa . An electric resistance heater inside the cylinder is now turned on, and 2200 kJ of energy is transferred to the steam. Determine the entropy change of the water during this process.
Answer: 5.72 kJ/K
7-39 An insulated piston-cylinder device contains $0.05 \mathrm{~m}^{3}$ of saturated refrigerant-134a vapor at $0.8-\mathrm{MPa}$ pressure. The refrigerant is now allowed to expand in a reversible manner until the pressure drops to 0.4 MPa . Determine (a) the final temperature in the cylinder and (b) the work done by the refrigerant.


FIGURE P7-39
 Reconsider Prob. 7-39. Using EES (or other) software, evaluate and plot the work done by the refrigerant as a function of final pressure as it varies from 0.8 to 0.4 MPa . Compare the work done for this process to one for which the temperature is constant over the same pressure range. Discuss your results.
7-41 Refrigerant-134a enters an adiabatic compressor as saturated vapor at 160 kPa at a rate of $2 \mathrm{~m}^{3} / \mathrm{min}$ and is compressed to a pressure of 900 kPa . Determine the minimum power that must be supplied to the compressor.

7-42E Steam enters an adiabatic turbine at 800 psia and $900^{\circ} \mathrm{F}$ and leaves at a pressure of 40 psia. Determine the maximum amount of work that can be delivered by this turbine.

7-43E © $\in S$ Reconsider Prob. 7-42E. Using EES (or other) software, evaluate and plot the work done by the steam as a function of final pressure as it varies from 800 to 40 psia . Also investigate the effect of varying the turbine inlet temperature from the saturation temperature at 800 psia to $900^{\circ} \mathrm{F}$ on the turbine work.

7-44 A heavily insulated piston-cylinder device contains $0.05 \mathrm{~m}^{3}$ of steam at 300 kPa and $150^{\circ} \mathrm{C}$. Steam is now compressed in a reversible manner to a pressure of 1 MPa . Determine the work done on the steam during this process.
7-45 e $\in S$ Reconsider Prob. 7-44. Using EES (or other) software, evaluate and plot the work done on the steam as a function of final pressure as the pressure varies from 300 kPa to 1 MPa .

7-46 A piston-cylinder device contains 1.2 kg of saturated water vapor at $200^{\circ} \mathrm{C}$. Heat is now transferred to steam, and steam expands reversibly and isothermally to a final pressure of 800 kPa . Determine the heat transferred and the work done during this process.
7-47 e $\in S$ Reconsider Prob. 7-46. Using EES (or other) software, evaluate and plot the heat transferred to the steam and the work done as a function of final pressure as the pressure varies from the initial value to the final value of 800 kPa .

7-48 A piston-cylinder device contains 5 kg of steam at $100^{\circ} \mathrm{C}$ with a quality of 50 percent. This steam undergoes two processes as follows:
1-2 Heat is transferred to the steam in a reversible manner while the temperature is held constant until the steam exists as a saturated vapor.
2-3 The steam expands in an adiabatic, reversible process until the pressure is 15 kPa .
(a) Sketch these processes with respect to the saturation lines on a single $T-s$ diagram.
(b) Determine the heat added to the steam in process 1-2, in kJ .
(c) Determine the work done by the steam in process 2-3, in kJ .

7-49 A rigid tank contains 5 kg of saturated vapor steam at $100^{\circ} \mathrm{C}$. The steam is cooled to the ambient temperature of $25^{\circ} \mathrm{C}$.
(a) Sketch the process with respect to the saturation lines on a $T-v$ diagram.
(b) Determine the entropy change of the steam, in $\mathrm{kJ} / \mathrm{K}$.
(c) For the steam and its surroundings, determine the total entropy change or $S_{\text {gen }}$ associated with this process, in $\mathrm{kJ} / \mathrm{K}$.

7-50 Steam at 6000 kPa and $500^{\circ} \mathrm{C}$ enters a steady-flow turbine. The steam expands in the turbine while doing work until the pressure is 1000 kPa . When the pressure is 1000 $\mathrm{kPa}, 10$ percent of the steam is removed from the turbine for other uses. The remaining 90 percent of the steam continues to expand through the turbine while doing work and leaves the turbine at 10 kPa . The entire expansion process by the steam through the turbine is reversible and adiabatic.
(a) Sketch the process on a T-s diagram with respect to the saturation lines. Be sure to label the data states and the lines of constant pressure.
(b) If the turbine has an isentropic efficiency of 85 percent, what is the work done by the steam as it flows through the turbine per unit mass of steam flowing into the turbine, in $\mathrm{kJ} / \mathrm{kg}$ ?

7-51E A $1.2-\mathrm{ft}^{3}$ well-insulated rigid can initially contains refrigerant-134a at 140 psia and $70^{\circ} \mathrm{F}$. Now a crack develops in the can, and the refrigerant starts to leak out slowly, Assuming the refrigerant remaining in the can has undergone a reversible, adiabatic process, determine the final mass in the can when the pressure drops to 20 psia.


FIGURE P7-51E

## Entropy Change of Incompressible Substances

7-52C Consider two solid blocks, one hot and the other cold, brought into contact in an adiabatic container. After a while, thermal equilibrium is established in the container as a result of heat transfer. The first law requires that the amount of energy lost by the hot solid be equal to the amount of energy gained by the cold one. Does the second law require that the decrease in entropy of the hot solid be equal to the increase in entropy of the cold one?
7-53 A $50-\mathrm{kg}$ copper block initially at $80^{\circ} \mathrm{C}$ is dropped into an insulated tank that contains 120 L of water at $25^{\circ} \mathrm{C}$. Determine the final equilibrium temperature and the total entropy change for this process.


FIGURE P7-53

7-54 A $25-\mathrm{kg}$ iron block initially at $350^{\circ} \mathrm{C}$ is quenched in an insulated tank that contains 100 kg of water at $18^{\circ} \mathrm{C}$. Assuming the water that vaporizes during the process condenses back in the tank, determine the total entropy change during this process.
7-55 A $20-\mathrm{kg}$ aluminum block initially at $200^{\circ} \mathrm{C}$ is brought into contact with a $20-\mathrm{kg}$ block of iron at $100^{\circ} \mathrm{C}$ in an insulated enclosure. Determine the final equilibrium temperature
and the total entropy change for this process. Answers: $168.4^{\circ} \mathrm{C}, 0.169 \mathrm{~kJ} / \mathrm{K}$
7-56
 Reconsider Prob. 7-55. Using EES (or other) software, study the effect of the mass of the iron block on the final equilibrium temperature and the total entropy change for the process. Let the mass of the iron vary from 1 to 10 kg . Plot the equilibrium temperature and the total entropy change as a function of iron mass, and discuss the results.

7-57 A $50-\mathrm{kg}$ iron block and a $20-\mathrm{kg}$ copper block, both initially at $80^{\circ} \mathrm{C}$, are dropped into a large lake at $15^{\circ} \mathrm{C}$. Thermal equilibrium is established after a while as a result of heat transfer between the blocks and the lake water. Determine the total entropy change for this process.


FIGURE P7-57
7-58 An adiabatic pump is to be used to compress saturated liquid water at 10 kPa to a pressure to 15 MPa in a reversible manner. Determine the work input using (a) entropy data from the compressed liquid table, $(b)$ inlet specific volume and pressure values, (c) average specific volume and pressure values. Also, determine the errors involved in parts $(b)$ and $(c)$.


FIGURE P7-58

## Entropy Change of Ideal Gases

7-59C Prove that the two relations for entropy change of ideal gases under the constant-specific-heat assumption (Eqs. $7-33$ and 7-34) are equivalent.

7-60C Starting with the second $T d s$ relation (Eq. 7-26), obtain Eq. 7-34 for the entropy change of ideal gases under the constant-specific-heat assumption.
7-61C Some properties of ideal gases such as internal energy and enthalpy vary with temperature only [that is, $u=$ $u(T)$ and $h=h(T)]$. Is this also the case for entropy?

7-62C Starting with Eq. 7-34, obtain Eq. 7-43.
7-63C What are $P_{r}$ and $v_{r}$ called? Is their use limited to isentropic processes? Explain.
7-64C Can the entropy of an ideal gas change during an isothermal process?
7-65C An ideal gas undergoes a process between two specified temperatures, first at constant pressure and then at constant volume. For which case will the ideal gas experience a larger entropy change? Explain.
7-66 Oxygen gas is compressed in a piston-cylinder device from an initial state of $0.8 \mathrm{~m}^{3} / \mathrm{kg}$ and $25^{\circ} \mathrm{C}$ to a final state of $0.1 \mathrm{~m}^{3} / \mathrm{kg}$ and $287^{\circ} \mathrm{C}$. Determine the entropy change of the oxygen during this process. Assume constant specific heats.
7-67 A $1.5-\mathrm{m}^{3}$ insulated rigid tank contains 2.7 kg of carbon dioxide at 100 kPa . Now paddle-wheel work is done on the system until the pressure in the tank rises to 150 kPa . Determine the entropy change of carbon dioxide during this process. Assume constant specific heats. Answer: $0.719 \mathrm{~kJ} / \mathrm{K}$


FIGURE P7-67
7-68 An insulated piston-cylinder device initially contains 300 L of air at 120 kPa and $17^{\circ} \mathrm{C}$. Air is now heated for 15 min by a $200-\mathrm{W}$ resistance heater placed inside the cylinder. The pressure of air is maintained constant during this process. Determine the entropy change of air, assuming (a) constant specific heats and (b) variable specific heats.

7-69 A piston-cylinder device contains 1.2 kg of nitrogen gas at 120 kPa and $27^{\circ} \mathrm{C}$. The gas is now compressed slowly in a polytropic process during which $P \bigvee^{1.3}=$ constant. The process ends when the volume is reduced by one-half. Determine the entropy change of nitrogen during this process. Answer: - $0.0617 \mathrm{~kJ} / \mathrm{K}$

7-70 (efs Reconsider Prob. 7-69. Using EES (or other) software, investigate the effect of varying the polytropic exponent from 1 to 1.4 on the entropy change of the nitrogen. Show the processes on a common $P-\vee$ diagram.
7-71E A mass of 15 lbm of helium undergoes a process from an initial state of $50 \mathrm{ft}^{3} / \mathrm{lbm}$ and $80^{\circ} \mathrm{F}$ to a final state of $10 \mathrm{ft}^{3} / \mathrm{lbm}$ and $200^{\circ} \mathrm{F}$. Determine the entropy change of
helium during this process, assuming (a) the process is reversible and (b) the process is irreversible.
7-72 Air is compressed in a piston-cylinder device from 90 kPa and $20^{\circ} \mathrm{C}$ to 400 kPa in a reversible isothermal process. Determine $(a)$ the entropy change of air and $(b)$ the work done.

7-73 Air is compressed steadily by a $5-\mathrm{kW}$ compressor from 100 kPa and $17^{\circ} \mathrm{C}$ to 600 kPa and $167^{\circ} \mathrm{C}$ at a rate of 1.6 $\mathrm{kg} / \mathrm{min}$. During this process, some heat transfer takes place between the compressor and the surrounding medium at $17^{\circ} \mathrm{C}$. Determine the rate of entropy change of air during this process. Answer: $-0.0025 \mathrm{~kW} / \mathrm{K}$


FIGURE P7-73
7-74 An insulated rigid tank is divided into two equal parts by a partition. Initially, one part contains 5 kmol of an ideal gas at 250 kPa and $40^{\circ} \mathrm{C}$, and the other side is evacuated. The partition is now removed, and the gas fills the entire tank. Determine the total entropy change during this process. Answer: $28.81 \mathrm{~kJ} / \mathrm{K}$
7-75 Air is compressed in a piston-cylinder device from 100 kPa and $17^{\circ} \mathrm{C}$ to 800 kPa in a reversible, adiabatic process. Determine the final temperature and the work done during this process, assuming (a) constant specific heats and (b) variable specific heats for air. Answers: (a) 525.3 K , $171.1 \mathrm{~kJ} / \mathrm{kg}$, (b) $522.4 \mathrm{~K}, 169.3 \mathrm{~kJ} / \mathrm{kg}$
7-76 (EGS Reconsider Prob. 7-75. Using EES (or other) software, evaluate and plot the work done and final temperature during the compression process as functions of the final pressure for the two cases as the final pressure varies from 100 to 800 kPa .

7-77 Helium gas is compressed from 90 kPa and $30^{\circ} \mathrm{C}$ to 450 kPa in a reversible, adiabatic process. Determine the final temperature and the work done, assuming the process takes place $(a)$ in a piston-cylinder device and $(b)$ in a steady-flow compressor.

7-78 An insulated rigid tank contains 4 kg of argon gas at 450 kPa and $30^{\circ} \mathrm{C}$. A valve is now opened, and argon is allowed to escape until the pressure inside drops to 200 kPa .

Assuming the argon remaining inside the tank has undergone a reversible, adiabatic process, determine the final mass in the tank. Answer: 2.46 kg


FIGURE P7-78
7-79 e氏S
Reconsider Prob. 7-78. Using EES (or other) software, investigate the effect of the final pressure on the final mass in the tank as the pressure varies from 450 to 150 kPa , and plot the results.
$7-80 \mathrm{E}$ Air enters an adiabatic nozzle at $60 \mathrm{psia}, 540^{\circ} \mathrm{F}$, and $200 \mathrm{ft} / \mathrm{s}$ and exits at 12 psia . Assuming air to be an ideal gas with variable specific heats and disregarding any irreversibilities, determine the exit velocity of the air.
7-81 Air enters a nozzle steadily at 280 kPa and $77^{\circ} \mathrm{C}$ with a velocity of $50 \mathrm{~m} / \mathrm{s}$ and exits at 85 kPa and $320 \mathrm{~m} / \mathrm{s}$. The heat losses from the nozzle to the surrounding medium at $20^{\circ} \mathrm{C}$ are estimated to be $3.2 \mathrm{~kJ} / \mathrm{kg}$. Determine (a) the exit temperature and $(b)$ the total entropy change for this process.

7-82 e $\in S$ Reconsider Prob. 7-81. Using EES (or other) software, study the effect of varying the surrounding medium temperature from 10 to $40^{\circ} \mathrm{C}$ on the exit temperature and the total entropy change for this process, and plot the results.
7-83 A container filled with 45 kg of liquid water at $95^{\circ} \mathrm{C}$ is placed in a $90-\mathrm{m}^{3}$ room that is initially at $12^{\circ} \mathrm{C}$. Thermal equilibrium is established after a while as a result of heat transfer between the water and the air in the room. Using constant specific heats, determine (a) the final equilibrium temperature, ( $b$ ) the amount of heat transfer between the


FIGURE P7-83
water and the air in the room, and (c) the entropy generation. Assume the room is well sealed and heavily insulated.
7-84 Air at 800 kPa and $400^{\circ} \mathrm{C}$ enters a steady-flow nozzle with a low velocity and leaves at 100 kPa . If the air undergoes an adiabatic expansion process through the nozzle, what is the maximum velocity of the air at the nozzle exit, in $\mathrm{m} / \mathrm{s}$ ?
7-85 An ideal gas at 100 kPa and $27^{\circ} \mathrm{C}$ enters a steady-flow compressor. The gas is compressed to 400 kPa , and 10 percent of the mass that entered the compressor is removed for some other use. The remaining 90 percent of the inlet gas is compressed to 600 kPa before leaving the compressor. The entire compression process is assumed to be reversible and adiabatic. The power supplied to the compressor is measured to be 32 kW . If the ideal gas has constant specific heats such that $c_{v}=$ $0.8 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$ and $c_{p}=1.1 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$, (a) sketch the compression process on a $T-s$ diagram, (b) determine the temperature of the gas at the two compressor exits, in K , and (c) determine the mass flow rate of the gas into the compressor, in $\mathrm{kg} / \mathrm{s}$.
7-86 A constant-volume tank contains 5 kg of air at 100 kPa and $327^{\circ} \mathrm{C}$. The air is cooled to the surroundings temperature of $27^{\circ} \mathrm{C}$. Assume constant specific heats at 300 K . (a) Determine the entropy change of the air in the tank during the process, in $\mathrm{kJ} / \mathrm{K}$, (b) determine the net entropy change of the universe due to this process, in $\mathrm{kJ} / \mathrm{K}$, and (c) sketch the processes for the air in the tank and the surroundings on a single $T$-s diagram. Be sure to label the initial and final states for both processes.

## Reversible Steady-Flow Work

7-87C In large compressors, the gas is frequently cooled while being compressed to reduce the power consumed by the compressor. Explain how cooling the gas during a compression process reduces the power consumption.

7-88C The turbines in steam power plants operate essentially under adiabatic conditions. A plant engineer suggests to end this practice. She proposes to run cooling water through the outer surface of the casing to cool the steam as it flows through the turbine. This way, she reasons, the entropy of the steam will decrease, the performance of the turbine will improve, and as a result the work output of the turbine will increase. How would you evaluate this proposal?
7-89C It is well known that the power consumed by a compressor can be reduced by cooling the gas during compression. Inspired by this, somebody proposes to cool the liquid as it flows through a pump, in order to reduce the power consumption of the pump. Would you support this proposal? Explain.

7-90 Water enters the pump of a steam power plant as saturated liquid at 20 kPa at a rate of $45 \mathrm{~kg} / \mathrm{s}$ and exits at 6 MPa . Neglecting the changes in kinetic and potential energies and assuming the process to be reversible, determine the power input to the pump.

7-91 Liquid water enters a $25-\mathrm{kW}$ pump at $100-\mathrm{kPa}$ pressure at a rate of $5 \mathrm{~kg} / \mathrm{s}$. Determine the highest pressure the liquid water can have at the exit of the pump. Neglect the kinetic and potential energy changes of water, and take the specific volume of water to be $0.001 \mathrm{~m}^{3} / \mathrm{kg}$. Answer: 5100 kPa


FIGURE P7-91
7-92E Saturated refrigerant-134a vapor at 15 psia is compressed reversibly in an adiabatic compressor to 80 psia. Determine the work input to the compressor. What would your answer be if the refrigerant were first condensed at constant pressure before it was compressed?

7-93 Consider a steam power plant that operates between the pressure limits of 10 MPa and 20 kPa . Steam enters the pump as saturated liquid and leaves the turbine as saturated vapor. Determine the ratio of the work delivered by the turbine to the work consumed by the pump. Assume the entire cycle to be reversible and the heat losses from the pump and the turbine to be negligible.
7-94 e૯S Reconsider Prob. 7-93. Using EES (or other) software, investigate the effect of the quality of the steam at the turbine exit on the net work output. Vary the quality from 0.5 to 1.0 , and plot the net work output as a function of this quality.
7-95 Liquid water at 120 kPa enters a $7-\mathrm{kW}$ pump where its pressure is raised to 5 MPa . If the elevation difference between the exit and the inlet levels is 10 m , determine the highest mass flow rate of liquid water this pump can handle. Neglect the kinetic energy change of water, and take the specific volume of water to be $0.001 \mathrm{~m}^{3} / \mathrm{kg}$.
$7-96 \mathrm{E}$ Helium gas is compressed from 14 psia and $70^{\circ} \mathrm{F}$ to 120 psia at a rate of $5 \mathrm{ft}^{3} / \mathrm{s}$. Determine the power input to the compressor, assuming the compression process to be (a) isentropic, (b) polytropic with $n=1.2$, (c) isothermal, and (d) ideal two-stage polytropic with $n=1.2$.

7-97E
Reconsider Prob. 7-96E. Using EES (or other) software, evaluate and plot the work of compression and entropy change of the helium as functions of the polytropic exponent as it varies from 1 to 1.667. Discuss your results.

7-98 Nitrogen gas is compressed from 80 kPa and $27^{\circ} \mathrm{C}$ to 480 kPa by a $10-\mathrm{kW}$ compressor. Determine the mass flow rate of nitrogen through the compressor, assuming the com-
pression process to be ( $a$ ) isentropic, (b) polytropic with $n=$ 1.3, (c) isothermal, and (d) ideal two-stage polytropic with $n$ $=1.3$. Answers: (a) $0.048 \mathrm{~kg} / \mathrm{s}$, (b) $0.051 \mathrm{~kg} / \mathrm{s}$, (c) $0.063 \mathrm{~kg} / \mathrm{s}$, (d) $0.056 \mathrm{~kg} / \mathrm{s}$

7-99 The compression stages in the axial compressor of the industrial gas turbine are close coupled, making intercooling very impractical. To cool the air in such compressors and to reduce the compression power, it is proposed to spray water mist with drop size on the order of 5 microns into the air stream as it is compressed and to cool the air continuously as the water evaporates. Although the collision of water droplets with turbine blades is a concern, experience with steam turbines indicates that they can cope with water droplet concentrations of up to 14 percent. Assuming air is compressed isentropically at a rate of $2 \mathrm{~kg} / \mathrm{s}$ from 300 K and 100 kPa to 1200 kPa and the water is injected at a temperature of $20^{\circ} \mathrm{C}$ at a rate of $0.2 \mathrm{~kg} / \mathrm{s}$, determine the reduction in the exit temperature of the compressed air and the compressor power saved. Assume the water vaporizes completely before leaving the compressor, and assume an average mass flow rate of 2.1 $\mathrm{kg} / \mathrm{s}$ throughout the compressor.

7-100 Reconsider Prob. 7-99. The water-injected compressor is used in a gas turbine power plant. It is claimed that the power output of a gas turbine will increase because of the increase in the mass flow rate of the gas (air + water vapor) through the turbine. Do you agree?

## Isentropic Efficiencies of Steady-Flow Devices

7-101C Describe the ideal process for an (a) adiabatic turbine, (b) adiabatic compressor, and (c) adiabatic nozzle, and define the isentropic efficiency for each device.

7-102C Is the isentropic process a suitable model for compressors that are cooled intentionally? Explain.

7-103C On a $T-s$ diagram, does the actual exit state (state 2) of an adiabatic turbine have to be on the right-hand side of the isentropic exit state (state $2 s$ )? Why?
7-104 Steam enters an adiabatic turbine at 8 MPa and $500^{\circ} \mathrm{C}$ with a mass flow rate of $3 \mathrm{~kg} / \mathrm{s}$ and leaves at 30 kPa . The isentropic efficiency of the turbine is 0.90 . Neglecting


FIGURE P7-104
the kinetic energy change of the steam, determine (a) the temperature at the turbine exit and $(b)$ the power output of the turbine. Answers: (a) $69.1^{\circ} \mathrm{C}$, (b) 3054 kW
7-105 $\in \in S$ Reconsider Prob. 7-104. Using EES (or other) software, study the effect of varying the turbine isentropic efficiency from 0.75 to 1.0 on both the work done and the exit temperature of the steam, and plot your results.
7-106 Steam enters an adiabatic turbine at $7 \mathrm{MPa}, 600^{\circ} \mathrm{C}$, and $80 \mathrm{~m} / \mathrm{s}$ and leaves at $50 \mathrm{kPa}, 150^{\circ} \mathrm{C}$, and $140 \mathrm{~m} / \mathrm{s}$. If the power output of the turbine is 6 MW , determine $(a)$ the mass flow rate of the steam flowing through the turbine and (b) the isentropic efficiency of the turbine. Answers: (a) $6.95 \mathrm{~kg} / \mathrm{s}$, (b) 73.4 percent

7-107 Argon gas enters an adiabatic turbine at $800^{\circ} \mathrm{C}$ and 1.5 MPa at a rate of $80 \mathrm{~kg} / \mathrm{min}$ and exhausts at 200 kPa . If the power output of the turbine is 370 kW , determine the isentropic efficiency of the turbine.
7-108E Combustion gases enter an adiabatic gas turbine at $1540^{\circ} \mathrm{F}$ and 120 psia and leave at 60 psia with a low velocity. Treating the combustion gases as air and assuming an isentropic efficiency of 82 percent, determine the work output of the turbine. Answer: 71.7 Btu/lbm

7-109 Refrigerant-134a enters an adiabatic compressor as saturated vapor at 120 kPa at a rate of $0.3 \mathrm{~m}^{3} / \mathrm{min}$ and exits at $1-\mathrm{MPa}$ pressure. If the isentropic efficiency of the compressor is 80 percent, determine (a) the temperature of the refrigerant at the exit of the compressor and $(b)$ the power input, in kW . Also, show the process on a $T$-s diagram with respect to saturation lines.


FIGURE P7-109 Reconsider Prob. 7-109. Using EES (or other) software, redo the problem by including the effects of the kinetic energy of the flow by assuming an inlet-to-exit area ratio of 1.5 for the compressor when the compressor exit pipe inside diameter is 2 cm .

7-111 Air enters an adiabatic compressor at 100 kPa and $17^{\circ} \mathrm{C}$ at a rate of $2.4 \mathrm{~m}^{3} / \mathrm{s}$, and it exits at $257^{\circ} \mathrm{C}$. The compressor has an isentropic efficiency of 84 percent. Neglecting the changes in kinetic and potential energies, determine (a) the
exit pressure of air and (b) the power required to drive the compressor.
7-112 Air is compressed by an adiabatic compressor from 95 kPa and $27^{\circ} \mathrm{C}$ to 600 kPa and $277^{\circ} \mathrm{C}$. Assuming variable specific heats and neglecting the changes in kinetic and potential energies, determine (a) the isentropic efficiency of the compressor and (b) the exit temperature of air if the process were reversible. Answers: (a) 81.9 percent, (b) 505.5 K

7-113E Argon gas enters an adiabatic compressor at 20 psia and $90^{\circ} \mathrm{F}$ with a velocity of $60 \mathrm{ft} / \mathrm{s}$, and it exits at 200 psia and $240 \mathrm{ft} / \mathrm{s}$. If the isentropic efficiency of the compressor is 80 percent, determine (a) the exit temperature of the argon and (b) the work input to the compressor.
7-114 Carbon dioxide enters an adiabatic compressor at 100 kPa and 300 K at a rate of $1.8 \mathrm{~kg} / \mathrm{s}$ and exits at 600 kPa and 450 K . Neglecting the kinetic energy changes, determine the isentropic efficiency of the compressor.

7-115E Air enters an adiabatic nozzle at 60 psia and $1020^{\circ} \mathrm{F}$ with low velocity and exits at $800 \mathrm{ft} / \mathrm{s}$. If the isentropic efficiency of the nozzle is 90 percent, determine the exit temperature and pressure of the air.
7-116E 飞eS Reconsider Prob. 7-115E. Using EES (or other) software, study the effect of varying the nozzle isentropic efficiency from 0.8 to 1.0 on both the exit temperature and pressure of the air, and plot the results.
7-117 Hot combustion gases enter the nozzle of a turbojet engine at $260 \mathrm{kPa}, 747^{\circ} \mathrm{C}$, and $80 \mathrm{~m} / \mathrm{s}$, and they exit at a pressure of 85 kPa . Assuming an isentropic efficiency of 92 percent and treating the combustion gases as air, determine (a) the exit velocity and (b) the exit temperature. Answers: (a) 728.2 m/s, (b) 786.3 K


FIGURE P7-117

## Entropy Balance

7-118 Refrigerant-134a is throttled from 900 kPa and $35^{\circ} \mathrm{C}$ to 200 kPa . Heat is lost from the refrigerant in the amount of $0.8 \mathrm{~kJ} / \mathrm{kg}$ to the surroundings at $25^{\circ} \mathrm{C}$. Determine (a) the exit


FIGURE P7-118
temperature of the refrigerant and $(b)$ the entropy generation during this process.
7-119 Steam enters an adiabatic turbine steadily at 7 MPa , $500^{\circ} \mathrm{C}$, and $45 \mathrm{~m} / \mathrm{s}$, and leaves at 100 kPa and $75 \mathrm{~m} / \mathrm{s}$. If the power output of the turbine is 5 MW and the isentropic efficiency is 77 percent, determine (a) the mass flow rate of steam through the turbine, $(b)$ the temperature at the turbine exit, and $(c)$ the rate of entropy generation during this process.


FIGURE P7-119
7-120 Air enters a compressor steadily at the ambient conditions of 100 kPa and $22^{\circ} \mathrm{C}$ and leaves at 800 kPa . Heat is lost from the compressor in the amount of $120 \mathrm{~kJ} / \mathrm{kg}$ and the air experiences an entropy decrease of $0.40 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$. Using constant specific heats, determine (a) the exit temperature of the air, (b) the work input to the compressor, and (c) the entropy generation during this process.
7-121 A rigid tank contains 7.5 kg of saturated water mixture at 400 kPa . A valve at the bottom of the tank is now opened, and liquid is withdrawn from the tank. Heat is transferred to the steam such that the pressure inside the tank remains constant. The valve is closed when no liquid is left in the tank. If it is estimated that a total of 5 kJ of heat is transferred to the tank, determine (a) the quality of steam in the tank at the initial state, (b) the amount of mass that has escaped, and (c) the entropy generation during this process if heat is supplied to the tank from a source at $500^{\circ} \mathrm{C}$.
7-122 Consider a family of four, with each person taking a 5 -min shower every morning. The average flow rate through the shower head is $12 \mathrm{~L} / \mathrm{min}$. City water at $15^{\circ} \mathrm{C}$ is heated to $55^{\circ} \mathrm{C}$ in an electric water heater and tempered to $42^{\circ} \mathrm{C}$ by cold water at the T-elbow of the shower before being routed to the shower head. Determine the amount of entropy generated by this family per year as a result of taking daily showers.
7-123 Steam is to be condensed in the condenser of a steam power plant at a temperature of $60^{\circ} \mathrm{C}$ with cooling water from a nearby lake, which enters the tubes of the condenser at $18^{\circ} \mathrm{C}$ at a rate of $75 \mathrm{~kg} / \mathrm{s}$ and leaves at $27^{\circ} \mathrm{C}$. Assuming the condenser to be perfectly insulated, determine (a) the rate of condensation of the steam and (b) the rate of
entropy generation in the condenser. Answers: (a) $1.20 \mathrm{~kg} / \mathrm{s}$, (b) $1.06 \mathrm{~kW} / \mathrm{K}$

7-124 A well-insulated heat exchanger is to heat water ( $c_{p}=$ $4.18 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$ ) from 25 to $60^{\circ} \mathrm{C}$ at a rate of $0.50 \mathrm{~kg} / \mathrm{s}$. The heating is to be accomplished by geothermal water ( $c_{p}=$ $4.31 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$ ) available at $140^{\circ} \mathrm{C}$ at a mass flow rate of 0.75 $\mathrm{kg} / \mathrm{s}$. Determine (a) the rate of heat transfer and (b) the rate of entropy generation in the heat exchanger.


FIGURE P7-124

7-125 An adiabatic heat exchanger is to cool ethylene gly$\operatorname{col}\left(c_{p}=2.56 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)$ flowing at a rate of $2 \mathrm{~kg} / \mathrm{s}$ from 80 to $40^{\circ} \mathrm{C}$ by water $\left(c_{p}=4.18 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)$ that enters at $20^{\circ} \mathrm{C}$ and leaves at $55^{\circ} \mathrm{C}$. Determine (a) the rate of heat transfer and (b) the rate of entropy generation in the heat exchanger.
7-126 A well-insulated, thin-walled, double-pipe, counterflow heat exchanger is to be used to cool oil ( $c_{p}=2.20 \mathrm{~kJ} / \mathrm{kg}$ - ${ }^{\circ} \mathrm{C}$ ) from $150^{\circ} \mathrm{C}$ to $40^{\circ} \mathrm{C}$ at a rate of $2 \mathrm{~kg} / \mathrm{s}$ by water ( $c_{p}=$ $4.18 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$ ) that enters at $22^{\circ} \mathrm{C}$ at a rate of $1.5 \mathrm{~kg} / \mathrm{s}$. Determine ( $a$ ) the rate of heat transfer and $(b)$ the rate of entropy generation in the heat exchanger.
7-127 Cold water ( $c_{p}=4.18 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$ ) leading to a shower enters a well-insulated, thin-walled, double-pipe, counter-flow heat exchanger at $15^{\circ} \mathrm{C}$ at a rate of $0.25 \mathrm{~kg} / \mathrm{s}$ and is heated to $45^{\circ} \mathrm{C}$ by hot water ( $c_{p}=4.19 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$ ) that enters at $100^{\circ} \mathrm{C}$ at a rate of $3 \mathrm{~kg} / \mathrm{s}$. Determine (a) the rate of heat transfer and (b) the rate of entropy generation in the heat exchanger.


FIGURE P7-127

7-128 Air $\left(c_{p}=1.005 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)$ is to be preheated by hot exhaust gases in a cross-flow heat exchanger before it enters the furnace. Air enters the heat exchanger at 95 kPa and $20^{\circ} \mathrm{C}$ at a rate of $1.6 \mathrm{~m}^{3} / \mathrm{s}$. The combustion gases $\left(c_{p}=1.10 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)$
enter at $180^{\circ} \mathrm{C}$ at a rate of $2.2 \mathrm{~kg} / \mathrm{s}$ and leave at $95^{\circ} \mathrm{C}$. Determine (a) the rate of heat transfer to the air, (b) the outlet temperature of the air, and (c) the rate of entropy generation.
7-129 A well-insulated, shell-and-tube heat exchanger is used to heat water $\left(c_{p}=4.18 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)$ in the tubes from 20 to $70^{\circ} \mathrm{C}$ at a rate of $4.5 \mathrm{~kg} / \mathrm{s}$. Heat is supplied by hot oil $\left(c_{p}=2.30 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)$ that enters the shell side at $170^{\circ} \mathrm{C}$ at a rate of $10 \mathrm{~kg} / \mathrm{s}$. Disregarding any heat loss from the heat exchanger, determine (a) the exit temperature of the oil and (b) the rate of entropy generation in the heat exchanger.


FIGURE P7-129
7-130E Steam is to be condensed on the shell side of a heat exchanger at $120^{\circ} \mathrm{F}$. Cooling water enters the tubes at $60^{\circ} \mathrm{F}$ at a rate of $92 \mathrm{lbm} / \mathrm{s}$ and leaves at $73^{\circ} \mathrm{F}$. Assuming the heat exchanger to be well-insulated, determine (a) the rate of heat transfer in the heat exchanger and $(b)$ the rate of entropy generation in the heat exchanger.

7-131 Chickens with an average mass of 2.2 kg and average specific heat of $3.54 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$ are to be cooled by chilled water that enters a continuous-flow-type immersion chiller at $0.5^{\circ} \mathrm{C}$ and leaves at $2.5^{\circ} \mathrm{C}$. Chickens are dropped into the chiller at a uniform temperature of $15^{\circ} \mathrm{C}$ at a rate of 250 chickens per hour and are cooled to an average temperature of $3^{\circ} \mathrm{C}$ before they are taken out. The chiller gains heat from the surroundings at $25^{\circ} \mathrm{C}$ at a rate of $150 \mathrm{~kJ} / \mathrm{h}$. Determine (a) the rate of heat removal from the chickens, in kW , and (b) the rate of entropy generation during this chilling process.
7-132 In a dairy plant, milk at $4^{\circ} \mathrm{C}$ is pasteurized continuously at $72^{\circ} \mathrm{C}$ at a rate of $12 \mathrm{~L} / \mathrm{s}$ for 24 hours a day and 365 days a year. The milk is heated to the pasteurizing temperature by hot water heated in a natural-gas-fired boiler that has


FIGURE P7-132
an efficiency of 82 percent. The pasteurized milk is then cooled by cold water at $18^{\circ} \mathrm{C}$ before it is finally refrigerated back to $4^{\circ} \mathrm{C}$. To save energy and money, the plant installs a regenerator that has an effectiveness of 82 percent. If the cost of natural gas is $\$ 1.04 /$ therm ( 1 therm $=105,500 \mathrm{~kJ}$ ), determine how much energy and money the regenerator will save this company per year and the annual reduction in entropy generation.
7-133 Stainless-steel ball bearings ( $\rho=8085 \mathrm{~kg} / \mathrm{m}^{3}$ and $c_{p}$ $=0.480 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$ ) having a diameter of 1.2 cm are to be quenched in water at a rate of 1400 per minute. The balls leave the oven at a uniform temperature of $900^{\circ} \mathrm{C}$ and are exposed to air at $30^{\circ} \mathrm{C}$ for a while before they are dropped into the water. If the temperature of the balls drops to $850^{\circ} \mathrm{C}$ prior to quenching, determine (a) the rate of heat transfer from the balls to the air and $(b)$ the rate of entropy generation due to heat loss from the balls to the air.

7-134 Carbon-steel balls $\left(\rho=7833 \mathrm{~kg} / \mathrm{m}^{3}\right.$ and $c_{p}=0.465$ $\left.\mathrm{kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right) 8 \mathrm{~mm}$ in diameter are annealed by heating them first to $900^{\circ} \mathrm{C}$ in a furnace and then allowing them to cool slowly to $100^{\circ} \mathrm{C}$ in ambient air at $35^{\circ} \mathrm{C}$. If 2500 balls are to be annealed per hour, determine (a) the rate of heat transfer from the balls to the air and $(b)$ the rate of entropy generation due to heat loss from the balls to the air. Answers: (a) 542 W , (b) $0.986 \mathrm{~W} / \mathrm{K}$


FIGURE P7-134

7-135 An ordinary egg can be approximated as a $5.5-\mathrm{cm}-$ diameter sphere. The egg is initially at a uniform temperature of $8^{\circ} \mathrm{C}$ and is dropped into boiling water at $97^{\circ} \mathrm{C}$. Taking the properties of the egg to be $\rho=1020 \mathrm{~kg} / \mathrm{m}^{3}$ and $c_{p}=3.32$ $\mathrm{kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$, determine (a) how much heat is transferred to the egg by the time the average temperature of the egg rises to $70^{\circ} \mathrm{C}$ and (b) the amount of entropy generation associated with this heat transfer process.


FIGURE P7-135

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7-136E In a production facility, 1.2-in.-thick, $2-\mathrm{ft} \times 2-\mathrm{ft}$ square brass plates ( $\rho=532.5 \mathrm{lbm} / \mathrm{ft}^{3}$ and $c_{p}=0.091$ $\mathrm{Btu} / \mathrm{lbm} \cdot{ }^{\circ} \mathrm{F}$ ) that are initially at a uniform temperature of $75^{\circ} \mathrm{F}$ are heated by passing them through an oven at $1300^{\circ} \mathrm{F}$ at a rate of 450 per minute. If the plates remain in the oven until their average temperature rises to $1000^{\circ} \mathrm{F}$, determine (a) the rate of heat transfer to the plates in the furnace and (b) the rate of entropy generation associated with this heat transfer process.

7-137 Long cylindrical steel rods ( $\rho=7833 \mathrm{~kg} / \mathrm{m}^{3}$ and $c_{p}$ $=0.465 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$ ) of $10-\mathrm{cm}$ diameter are heat treated by drawing them at a velocity of $3 \mathrm{~m} / \mathrm{min}$ through a $7-\mathrm{m}$-long oven maintained at $900^{\circ} \mathrm{C}$. If the rods enter the oven at $30^{\circ} \mathrm{C}$ and leave at $700^{\circ} \mathrm{C}$, determine (a) the rate of heat transfer to the rods in the oven and $(b)$ the rate of entropy generation associated with this heat transfer process.


FIGURE P7-137
7-138 The inner and outer surfaces of a $5-\mathrm{m} \times 7-\mathrm{m}$ brick wall of thickness 20 cm are maintained at temperatures of $20^{\circ} \mathrm{C}$ and $5^{\circ} \mathrm{C}$, respectively. If the rate of heat transfer through the wall is 1515 W , determine the rate of entropy generation within the wall.
7-139 For heat transfer purposes, a standing man can be modeled as a $30-\mathrm{cm}$-diameter, $170-\mathrm{cm}$-long vertical cylinder with both the top and bottom surfaces insulated and with the side surface at an average temperature of $34^{\circ} \mathrm{C}$. If the rate of heat loss from this man to the environment at $20^{\circ} \mathrm{C}$ is 336 W , determine the rate of entropy transfer from the body of this person accompanying heat transfer, in W/K.
7-140 A 1000-W iron is left on the ironing board with its base exposed to the air at $20^{\circ} \mathrm{C}$. If the surface temperature is $400^{\circ} \mathrm{C}$, determine the rate of entropy generation during this process in steady operation. How much of this entropy generation occurs within the iron?

7-141E A frictionless piston-cylinder device contains saturated liquid water at 25 -psia pressure. Now 400 Btu of heat is transferred to water from a source at $900^{\circ} \mathrm{F}$, and part of the liquid vaporizes at constant pressure. Determine the total entropy generated during this process, in Btu/R.

7-142E Steam enters a diffuser at 20 psia and $240^{\circ} \mathrm{F}$ with a velocity of $900 \mathrm{ft} / \mathrm{s}$ and exits as saturated vapor at $240^{\circ} \mathrm{F}$ and
$100 \mathrm{ft} / \mathrm{s}$. The exit area of the diffuser is $1 \mathrm{ft}^{2}$. Determine (a) the mass flow rate of the steam and (b) the rate of entropy generation during this process. Assume an ambient temperature of $77^{\circ} \mathrm{F}$.
7-143 Steam expands in a turbine steadily at a rate of $25,000 \mathrm{~kg} / \mathrm{h}$, entering at 6 MPa and $450^{\circ} \mathrm{C}$ and leaving at 20 kPa as saturated vapor. If the power generated by the turbine is 4 MW , determine the rate of entropy generation for this process. Assume the surrounding medium is at $25^{\circ} \mathrm{C}$.
Answer: 11.0 kW/K


FIGURE P7-143
7-144 A hot-water stream at $70^{\circ} \mathrm{C}$ enters an adiabatic mixing chamber with a mass flow rate of $3.6 \mathrm{~kg} / \mathrm{s}$, where it is mixed with a stream of cold water at $20^{\circ} \mathrm{C}$. If the mixture leaves the chamber at $42^{\circ} \mathrm{C}$, determine (a) the mass flow rate of the cold water and (b) the rate of entropy generation during this adiabatic mixing process. Assume all the streams are at a pressure of 200 kPa .

7-145 Liquid water at 200 kPa and $20^{\circ} \mathrm{C}$ is heated in a chamber by mixing it with superheated steam at 200 kPa and $150^{\circ} \mathrm{C}$. Liquid water enters the mixing chamber at a rate of $2.5 \mathrm{~kg} / \mathrm{s}$, and the chamber is estimated to lose heat to the surrounding air at $25^{\circ} \mathrm{C}$ at a rate of $1200 \mathrm{~kJ} / \mathrm{min}$. If the mixture leaves the mixing chamber at 200 kPa and $60^{\circ} \mathrm{C}$, determine (a) the mass flow rate of the superheated steam and (b) the rate of entropy generation during this mixing process.
Answers: (a) 0.166 kg/s, (b) 0.333 kW/K


FIGURE P7-145
7-146 A $0.3-\mathrm{m}^{3}$ rigid tank is filled with saturated liquid water at $150^{\circ} \mathrm{C}$. A valve at the bottom of the tank is now
opened, and one-half of the total mass is withdrawn from the tank in the liquid form. Heat is transferred to water from a source at $200^{\circ} \mathrm{C}$ so that the temperature in the tank remains constant. Determine (a) the amount of heat transfer and (b) the total entropy generation for this process.
$7-147 \mathrm{E}$ An iron block of unknown mass at $185^{\circ} \mathrm{F}$ is dropped into an insulated tank that contains $0.8 \mathrm{ft}^{3}$ of water at $70^{\circ} \mathrm{F}$. At the same time, a paddle wheel driven by a $200-\mathrm{W}$ motor is activated to stir the water. Thermal equilibrium is established after 10 min with a final temperature of $75^{\circ} \mathrm{F}$. Determine ( $a$ ) the mass of the iron block and $(b)$ the entropy generated during this process.

7-148E Air enters a compressor at ambient conditions of 15 psia and $60^{\circ} \mathrm{F}$ with a low velocity and exits at 150 psia , $620^{\circ} \mathrm{F}$, and $350 \mathrm{ft} / \mathrm{s}$. The compressor is cooled by the ambient air at $60^{\circ} \mathrm{F}$ at a rate of $1500 \mathrm{Btu} / \mathrm{min}$. The power input to the compressor is 400 hp . Determine (a) the mass flow rate of air and (b) the rate of entropy generation.
7-149 Steam enters an adiabatic nozzle at 4 MPa and $450^{\circ} \mathrm{C}$ with a velocity of $70 \mathrm{~m} / \mathrm{s}$ and exits at 3 MPa and 320 $\mathrm{m} / \mathrm{s}$. If the nozzle has an inlet area of $7 \mathrm{~cm}^{2}$, determine (a) the exit temperature and $(b)$ the rate of entropy generation for this process. Answers: (a) $422.3^{\circ} \mathrm{C}$, (b) $0.0361 \mathrm{~kW} / \mathrm{K}$

## Special Topic: Reducing the Cost of Compressed Air

7-150 Compressed air is one of the key utilities in manufacturing facilities, and the total installed power of compressedair systems in the United States is estimated to be about 20 million horsepower. Assuming the compressors to operate at full load during one-third of the time on average and the average motor efficiency to be 85 percent, determine how much energy and money will be saved per year if the energy consumed by compressors is reduced by 5 percent as a result of implementing some conservation measures. Take the unit cost of electricity to be $\$ 0.07 / \mathrm{kWh}$.
7-151 The energy used to compress air in the United States is estimated to exceed one-half quadrillion $\left(0.5 \times 10^{15}\right) \mathrm{kJ}$ per year. It is also estimated that 10 to 40 percent of the compressed air is lost through leaks. Assuming, on average, 20 percent of the compressed air is lost through air leaks and the unit cost of electricity is $\$ 0.07 / \mathrm{kWh}$, determine the amount and cost of electricity wasted per year due to air leaks.
7-152 The compressed-air requirements of a plant at sea level are being met by a $125-\mathrm{hp}$ compressor that takes in air at the local atmospheric pressure of 101.3 kPa and the average temperature of $15^{\circ} \mathrm{C}$ and compresses it to 900 kPa . An investigation of the compressed-air system and the equipment using the compressed air reveals that compressing the air to 750 kPa is sufficient for this plant. The compressor operates $3500 \mathrm{~h} / \mathrm{yr}$ at 75 percent of the rated load and is driven by an electric motor that has an efficiency of 88 percent. Taking the price of electricity to be $\$ 0.085 / \mathrm{kWh}$, determine the amount of energy and money saved as a result of reducing the pressure of the compressed air.

7-153 A 150-hp compressor in an industrial facility is housed inside the production area where the average temperature during operating hours is $25^{\circ} \mathrm{C}$. The average temperature outdoors during the same hours is $10^{\circ} \mathrm{C}$. The compressor operates $4500 \mathrm{~h} / \mathrm{yr}$ at 85 percent of rated load and is driven by an electric motor that has an efficiency of 90 percent. Taking the price of electricity to be $\$ 0.07 / \mathrm{kWh}$, determine the amount of energy and money saved as a result of drawing outside air to the compressor instead of using the inside air.
7-154 The compressed-air requirements of a plant are being met by a $100-\mathrm{hp}$ screw compressor that runs at full load during 40 percent of the time and idles the rest of the time during operating hours. The compressor consumes 35 percent of the rated power when idling and 90 percent of the power when compressing air. The annual operating hours of the facility are 3800 h , and the unit cost of electricity is $\$ 0.075 / \mathrm{kWh}$.

It is determined that the compressed-air requirements of the facility during 60 percent of the time can be met by a $25-$ hp reciprocating compressor that consumes 95 percent of the rated power when compressing air and no power when not compressing air. It is estimated that the $25-\mathrm{hp}$ compressor runs 85 percent of the time. The efficiencies of the motors of the large and the small compressors at or near full load are 0.90 and 0.88 , respectively. The efficiency of the large motor at 35 percent load is 0.82 . Determine the amount of energy and money saved as a result of switching to the $25-\mathrm{hp}$ compressor during 60 percent of the time.
7-155 The compressed-air requirements of a plant are being met by a $125-\mathrm{hp}$ screw compressor. The facility stops production for one hour every day, including weekends, for lunch break, but the compressor is kept operating. The compressor consumes 35 percent of the rated power when idling, and the unit cost of electricity is $\$ 0.09 / \mathrm{kWh}$. Determine the amount of energy and money saved per year as a result of turning the compressor off during lunch break. Take the efficiency of the motor at part load to be 84 percent.
7-156 The compressed-air requirements of a plant are met by a $150-\mathrm{hp}$ compressor equipped with an intercooler, an aftercooler, and a refrigerated dryer. The plant operates 4800 $\mathrm{h} / \mathrm{yr}$, but the compressor is estimated to be compressing air during only one-third of the operating hours, that is, 1600 hours a year. The compressor is either idling or is shut off the rest of the time. Temperature measurements and calculations indicate that 40 percent of the energy input to the compressor is removed from the compressed air as heat in the aftercooler. The COP of the refrigeration unit is 3.5 , and the cost of electricity is $\$ 0.06 / \mathrm{kWh}$. Determine the amount of the energy and money saved per year as a result of cooling the compressed air before it enters the refrigerated dryer.
7-157 The $1800-\mathrm{rpm}, 150-\mathrm{hp}$ motor of a compressor is burned out and is to be replaced by either a standard motor that has a full-load efficiency of 93.0 percent and costs $\$ 9031$ or a high-efficiency motor that has an efficiency of 96.2 percent and costs $\$ 10,942$. The compressor operates $4368 \mathrm{~h} / \mathrm{yr}$ at
full load, and its operation at part load is negligible. If the cost of electricity is $\$ 0.075 / \mathrm{kWh}$, determine the amount of energy and money this facility will save by purchasing the highefficiency motor instead of the standard motor. Also, determine if the savings from the high-efficiency motor justify the price differential if the expected life of the motor is 10 years. Ignore any possible rebates from the local power company.

7-158 The space heating of a facility is accomplished by natural gas heaters that are 80 percent efficient. The compressed air needs of the facility are met by a large liquid-cooled compressor. The coolant of the compressor is cooled by air in a liquid-to-air heat exchanger whose airflow section is $1.0-\mathrm{m}$ high and $1.0-\mathrm{m}$ wide. During typical operation, the air is heated from 20 to $52^{\circ} \mathrm{C}$ as it flows through the heat exchanger. The average velocity of air on the inlet side is measured to be $3 \mathrm{~m} / \mathrm{s}$. The compressor operates 20 hours a day and 5 days a week throughout the year. Taking the heating season to be 6 months ( 26 weeks) and the cost of the natural gas to be $\$ 1.00 /$ therm ( 1 therm $=100,000 \mathrm{Btu}=105,500 \mathrm{~kJ}$ ), determine how much money will be saved by diverting the compressor waste heat into the facility during the heating season.
7-159 The compressors of a production facility maintain the compressed-air lines at a (gage) pressure of 850 kPa at $1400-$ m elevation, where the atmospheric pressure is 85.6 kPa . The average temperature of air is $15^{\circ} \mathrm{C}$ at the compressor inlet and $25^{\circ} \mathrm{C}$ in the compressed-air lines. The facility operates 4200 $\mathrm{h} / \mathrm{yr}$, and the average price of electricity is $\$ 0.07 / \mathrm{kWh}$. Taking the compressor efficiency to be 0.8 , the motor efficiency to be 0.93 , and the discharge coefficient to be 0.65 , determine the energy and money saved per year by sealing a leak equivalent to a 5-mm-diameter hole on the compressed-air line.

## Review Problems

7-160 A piston-cylinder device contains steam that undergoes a reversible thermodynamic cycle. Initially the steam is at 400 kPa and $350^{\circ} \mathrm{C}$ with a volume of $0.3 \mathrm{~m}^{3}$. The steam is first expanded isothermally to 150 kPa , then compressed adiabatically to the initial pressure, and finally compressed at the constant pressure to the initial state. Determine the net work and heat transfer for the cycle after you calculate the work and heat interaction for each process.

7-161 Determine the work input and entropy generation during the compression of steam from 100 kPa to 1 MPa in


FIGURE P7-161
(a) an adiabatic pump and (b) an adiabatic compressor if the inlet state is saturated liquid in the pump and saturated vapor in the compressor and the isentropic efficiency is 85 percent for both devices.
7-162 A rigid tank contains 1.5 kg of water at $120^{\circ} \mathrm{C}$ and 500 kPa . Now 22 kJ of shaft work is done on the system and the final temperature in the tank is $95^{\circ} \mathrm{C}$. If the entropy change of water is zero and the surroundings are at $15^{\circ} \mathrm{C}$, determine (a) the final pressure in the tank, (b) the amount of heat transfer between the tank and the surroundings, and (c) the entropy generation during this process. Answers: (a) 84.6 kPa , (b) 38.5 kJ , (c) $0.134 \mathrm{~kJ} / \mathrm{K}$

7-163 A horizontal cylinder is separated into two compartments by an adiabatic, frictionless piston. One side contains $0.2 \mathrm{~m}^{3}$ of nitrogen and the other side contains 0.1 kg of helium, both initially at $20^{\circ} \mathrm{C}$ and 95 kPa . The sides of the cylinder and the helium end are insulated. Now heat is added to the nitrogen side from a reservoir at $500^{\circ} \mathrm{C}$ until the pressure of the helium rises to 120 kPa . Determine (a) the final temperature of the helium, $(b)$ the final volume of the nitrogen, $(c)$ the heat transferred to the nitrogen, and $(d)$ the entropy generation during this process.


FIGURE P7-163
7-164 A $0.8-\mathrm{m}^{3}$ rigid tank contains carbon dioxide $\left(\mathrm{CO}_{2}\right)$ gas at 250 K and 100 kPa . A $500-\mathrm{W}$ electric resistance heater placed in the tank is now turned on and kept on for 40 min after which the pressure of $\mathrm{CO}_{2}$ is measured to be 175 kPa . Assuming the surroundings to be at 300 K and using constant specific heats, determine $(a)$ the final temperature of $\mathrm{CO}_{2}$, (b) the net amount of heat transfer from the tank, and (c) the entropy generation during this process.


FIGURE P7-164
7-165 Helium gas is throttled steadily from 500 kPa and $70^{\circ} \mathrm{C}$. Heat is lost from the helium in the amount of $2.5 \mathrm{~kJ} / \mathrm{kg}$ to the surroundings at $25^{\circ} \mathrm{C}$ and 100 kPa . If the entropy of the helium increases by $0.25 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$ in the valve, determine (a) the exit pressure and temperature and (b) the
entropy generation during this process. Answers: (a) 442 kPa , $69.5^{\circ} \mathrm{C}$, (b) $0.258 \mathrm{~kJ} / \mathrm{kg} \cdot \mathrm{K}$
7-166 Refrigerant-134a enters a compressor as a saturated vapor at 200 kPa at a rate of $0.03 \mathrm{~m}^{3} / \mathrm{s}$ and leaves at 700 kPa . The power input to the compressor is 10 kW . If the surroundings at $20^{\circ} \mathrm{C}$ experience an entropy increase of $0.008 \mathrm{~kW} / \mathrm{K}$, determine (a) the rate of heat loss from the compressor, (b) the exit temperature of the refrigerant, and (c) the rate of entropy generation.
7-167 Air at 500 kPa and 400 K enters an adiabatic nozzle at a velocity of $30 \mathrm{~m} / \mathrm{s}$ and leaves at 300 kPa and 350 K . Using variable specific heats, determine (a) the isentropic efficiency, (b) the exit velocity, and (c) the entropy generation.


FIGURE P7-167
7-168 Show that the difference between the reversible steady-flow work and reversible moving boundary work is equal to the flow energy.
7-169 An insulated tank containing $0.4 \mathrm{~m}^{3}$ of saturated water vapor at 500 kPa is connected to an initially evacuated, insulated piston-cylinder device. The mass of the piston is such that a pressure of 150 kPa is required to raise it. Now the valve is opened slightly, and part of the steam flows to the cylinder, raising the piston. This process continues until the pressure in the tank drops to 150 kPa . Assuming the steam that remains in the tank to have undergone a reversible adiabatic process, determine the final temperature (a) in the rigid tank and $(b)$ in the cylinder.


FIGURE P7-169
7-170 One ton of liquid water at $80^{\circ} \mathrm{C}$ is brought into a well-insulated and well-sealed $4-\mathrm{m} \times 5-\mathrm{m} \times 7-\mathrm{m}$ room initially at $22^{\circ} \mathrm{C}$ and 100 kPa . Assuming constant specific heats for both air and water at room temperature, determine (a) the final equilibrium temperature in the room and $(b)$ the total entropy change during this process, in $\mathrm{kJ} / \mathrm{K}$.


FIGURE P7-170

7-171E A piston-cylinder device initially contains $15 \mathrm{ft}^{3}$ of helium gas at 25 psia and $70^{\circ} \mathrm{F}$. Helium is now compressed in a polytropic process $\left(P V^{n}=\right.$ constant) to 70 psia and $300^{\circ} \mathrm{F}$. Determine ( $a$ ) the entropy change of helium, $(b)$ the entropy change of the surroundings, and (c) whether this process is reversible, irreversible, or impossible. Assume the surroundings are at $70^{\circ} \mathrm{F}$. Answers: (a) $-0.016 \mathrm{Btu} / \mathrm{R}$, (b) $0.019 \mathrm{Btu} / \mathrm{R}$, (c) irreversible

7-172 Air is compressed steadily by a compressor from 100 kPa and $17^{\circ} \mathrm{C}$ to 700 kPa at a rate of $5 \mathrm{~kg} / \mathrm{min}$. Determine the minimum power input required if the process is $(a)$ adiabatic and $(b)$ isothermal. Assume air to be an ideal gas with variable specific heats, and neglect the changes in kinetic and potential energies. Answers: (a) 18.0 kW , (b) 13.5 kW
7-173 Air enters a two-stage compressor at 100 kPa and $27^{\circ} \mathrm{C}$ and is compressed to 900 kPa . The pressure ratio across each stage is the same, and the air is cooled to the initial temperature between the two stages. Assuming the compression process to be isentropic, determine the power input to the compressor for a mass flow rate of $0.02 \mathrm{~kg} / \mathrm{s}$. What would your answer be if only one stage of compression were used? Answers: $4.44 \mathrm{~kW}, 5.26 \mathrm{~kW}$


FIGURE P7-173

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7-174 Consider a three-stage isentropic compressor with two intercoolers that cool the gas to the initial temperature between the stages. Determine the two intermediate pressures $\left(P_{x}\right.$ and $\left.P_{y}\right)$ in terms of inlet and exit pressures $\left(P_{1}\right.$ and $\left.P_{2}\right)$ that will minimize the work input to the compressor. Answers: $P_{x}=\left(P_{1}^{2} P_{2}\right)^{1 / 3}, P_{y}=\left(P_{1} P_{2}^{2}\right)^{1 / 3}$

7-175 Steam at 6 MPa and $500^{\circ} \mathrm{C}$ enters a two-stage adiabatic turbine at a rate of $15 \mathrm{~kg} / \mathrm{s}$. Ten percent of the steam is extracted at the end of the first stage at a pressure of 1.2 MPa for other use. The remainder of the steam is further expanded in the second stage and leaves the turbine at 20 kPa . Determine the power output of the turbine, assuming (a) the process is reversible and (b) the turbine has an isentropic efficiency of 88 percent. Answers: (a) $16,291 \mathrm{~kW}$, (b) $14,336 \mathrm{~kW}$


FIGURE P7-175

7-176 Steam enters a two-stage adiabatic turbine at 8 MPa and $550^{\circ} \mathrm{C}$. It expands in the first stage to a pressure of 2 MPa . Then steam is reheated at constant pressure to $550^{\circ} \mathrm{C}$ before it is expanded in a second stage to a pressure of 200 kPa . The power output of the turbine is 80 MW . Assuming an isentropic efficiency of 84 percent for each stage of the turbine, determine the required mass flow rate of steam. Also, show the process on a $T$-s diagram with respect to saturation lines. Answer: $85.8 \mathrm{~kg} / \mathrm{s}$
7-177 Refrigerant-134a at 140 kPa and $-10^{\circ} \mathrm{C}$ is compressed by an adiabatic $0.7-\mathrm{kW}$ compressor to an exit state of 700 kPa and $50^{\circ} \mathrm{C}$. Neglecting the changes in kinetic and potential energies, determine (a) the isentropic efficiency of the compressor, (b) the volume flow rate of the refrigerant at the compressor inlet, in L/min, and (c) the maximum volume flow rate at the inlet conditions that this adiabatic $0.7-\mathrm{kW}$ compressor can handle without violating the second law.

7-178E Helium gas enters a nozzle whose isentropic efficiency is 94 percent with a low velocity, and it exits at 14 psia, $180^{\circ} \mathrm{F}$, and $1000 \mathrm{ft} / \mathrm{s}$. Determine the pressure and temperature at the nozzle inlet.

7-179 An adiabatic air compressor is to be powered by a direct-coupled adiabatic steam turbine that is also driving a generator. Steam enters the turbine at
12.5 MPa and $500^{\circ} \mathrm{C}$ at a rate of $25 \mathrm{~kg} / \mathrm{s}$ and exits at 10 kPa and a quality of 0.92 . Air enters the compressor at 98 kPa and 295 K at a rate of $10 \mathrm{~kg} / \mathrm{s}$ and exits at 1 MPa and 620 K . Determine (a) the net power delivered to the generator by the turbine and $(b)$ the rate of entropy generation within the turbine and the compressor during this process.


FIGURE P7-179

7-180 EGS Reconsider Prob. 7-179. Using EES (or other) software, determine the isentropic efficiencies for the compressor and turbine. Then use EES to study how varying the compressor efficiency over the range 0.6 to 0.8 and the turbine efficiency over the range 0.7 to 0.95 affect the net work for the cycle and the entropy generated for the process. Plot the net work as a function of the compressor efficiency for turbine efficiencies of $0.7,0.8$, and 0.9 , and discuss your results.

7-181 Consider two bodies of identical mass $m$ and specific heat $c$ used as thermal reservoirs (source and sink) for a heat engine. The first body is initially at an absolute temperature $T_{1}$ while the second one is at a lower absolute temperature $T_{2}$. Heat is transferred from the first body to the heat engine, which rejects the waste heat to the second body. The process continues until the final temperatures of the two bodies $T_{f}$ become equal. Show that $T_{f}=\sqrt{T_{1} T_{2}}$ when the heat engine produces the maximum possible work.


FIGURE P7-181

7-182 The explosion of a hot water tank in a school in Spencer, Oklahoma, in 1982 killed 7 people while injuring 33 others. Although the number of such explosions has decreased dramatically since the development of the ASME Pressure Vessel Code, which requires the tanks to be designed to withstand four times the normal operating pressures, they still occur as a result of the failure of the pressure relief valves and thermostats. When a tank filled with a highpressure and high-temperature liquid ruptures, the sudden drop of the pressure of the liquid to the atmospheric level causes part of the liquid to flash into vapor, and thus to experience a huge rise in its volume. The resulting pressure wave that propagates rapidly can cause considerable damage.

Considering that the pressurized liquid in the tank eventually reaches equilibrium with its surroundings shortly after the explosion, the work that a pressurized liquid would do if allowed to expand reversibly and adiabatically to the pressure of the surroundings can be viewed as the explosive energy of the pressurized liquid. Because of the very short time period of the explosion and the apparent calm afterward, the explosion process can be considered to be adiabatic with no changes in kinetic and potential energies and no mixing with the air.
Consider a 80-L hot-water tank that has a working pressure of 0.5 MPa . As a result of some malfunction, the pressure in the tank rises to 2 MPa , at which point the tank explodes. Taking the atmospheric pressure to be 100 kPa and assuming the liquid in the tank to be saturated at the time of explosion, determine the total explosion energy of the tank in terms of the TNT equivalence. (The explosion energy of TNT is about $3250 \mathrm{~kJ} / \mathrm{kg}$, and 5 kg of TNT can cause total destruction of unreinforced structures within about a $7-\mathrm{m}$ radius.) Answer: 1.972 kg TNT


FIGURE P7-182
7-183 Using the arguments in the Prob. 7-182, determine the total explosion energy of a $0.35-\mathrm{L}$ canned drink that explodes at a pressure of 1.2 MPa . To how many kg of TNT is this explosion energy equivalent?
7-184 Demonstrate the validity of the Clausius inequality using a reversible and an irreversible heat engine operating between the same two thermal energy reservoirs at constant temperatures of $T_{L}$ and $T_{H}$.


FIGURE P7-184

7-185 The inner and outer surfaces of a $2-\mathrm{m} \times 2-\mathrm{m}$ window glass in winter are $10^{\circ} \mathrm{C}$ and $3^{\circ} \mathrm{C}$, respectively. If the rate of heat loss through the window is $3.2 \mathrm{~kJ} / \mathrm{s}$, determine the amount of heat loss, in kilojoules, through the glass over a period of 5 h . Also, determine the rate of entropy generation during this process within the glass.

7-186 Two rigid tanks are connected by a valve. Tank A is insulated and contains $0.2 \mathrm{~m}^{3}$ of steam at 400 kPa and 80 percent quality. Tank B is uninsulated and contains 3 kg of steam at 200 kPa and $250^{\circ} \mathrm{C}$. The valve is now opened, and steam flows from tank A to tank B until the pressure in tank A drops to 300 kPa . During this process 600 kJ of heat is transferred from tank B to the surroundings at $0^{\circ} \mathrm{C}$. Assuming the steam remaining inside tank A to have undergone a reversible adiabatic process, determine (a) the final temperature in each tank and (b) the entropy generated during this process. Answers: (a) $133.5^{\circ} \mathrm{C}, 113.2^{\circ} \mathrm{C}$; (b) $0.916 \mathrm{~kJ} / \mathrm{K}$


FIGURE P7-186

7-187 Heat is transferred steadily to boiling water in the pan through its flat bottom at a rate of 500 W . If the temperatures of the inner and outer surfaces of the bottom of the tank
are $104^{\circ} \mathrm{C}$ and $105^{\circ} \mathrm{C}$, respectively, determine the rate of entropy generation within bottom of the pan, in W/K.


FIGURE P7-187
7-188 A 1200-W electric resistance heating element whose diameter is 0.5 cm is immersed in 40 kg of water initially at $20^{\circ} \mathrm{C}$. Assuming the water container is well-insulated, determine how long it will take for this heater to raise the water temperature to $50^{\circ} \mathrm{C}$. Also, determine the entropy generated during this process, in $\mathrm{kJ} / \mathrm{K}$.
7-189 A hot-water pipe at $80^{\circ} \mathrm{C}$ is losing heat to the surrounding air at $5^{\circ} \mathrm{C}$ at a rate of 2200 W . Determine the rate of entropy generation in the surrounding air, in W/K.
7-190 In large steam power plants, the feedwater is frequently heated in closed feedwater heaters, which are basically heat exchangers, by steam extracted from the turbine at some stage. Steam enters the feedwater heater at 1 MPa and $200^{\circ} \mathrm{C}$ and leaves as saturated liquid at the same pressure. Feedwater enters the heater at 2.5 MPa and $50^{\circ} \mathrm{C}$ and leaves $10^{\circ} \mathrm{C}$ below the exit temperature of the steam. Neglecting any heat losses from the outer surfaces of the heater, determine (a) the ratio of the mass flow rates of the extracted steam and the feedwater heater and $(b)$ the total entropy change for this process per unit mass of the feedwater.
7-191 € $\in S$ Reconsider Prob. 7-190. Using EES (or other) software, investigate the effect of the state of the steam at the inlet of the feedwater heater. Assume the entropy of the extraction steam is constant at the value for 1 $\mathrm{MPa}, 200^{\circ} \mathrm{C}$ and decrease the extraction steam pressure from 1 MPa to 100 kPa . Plot both the ratio of the mass flow rates of the extracted steam and the feedwater heater and the total entropy change for this process per unit mass of the feedwater as functions of the extraction pressure.
7-192E A 3-ft ${ }^{3}$ rigid tank initially contains refrigerant-134a at 100 psia and 100 percent quality. The tank is connected by a valve to a supply line that carries refrigerant-134a at 140 psia and $80^{\circ} \mathrm{F}$. The valve is now opened, allowing the refrigerant to enter the tank, and is closed when it is observed that the tank contains only saturated liquid at 120 psia . Determine (a) the mass of the refrigerant that entered the tank, (b) the
amount of heat transfer with the surroundings at $110^{\circ} \mathrm{F}$, and (c) the entropy generated during this process.

7-193 During a heat transfer process, the entropy change of incompressible substances, such as liquid water, can be determined from $\Delta S=m c_{\text {avg }} \ln \left(T_{2} / T_{1}\right)$. Show that for thermal energy reservoirs, such as large lakes, this relation reduces to $\Delta S=Q / T$.
7-194 The inner and outer glasses of a $2-\mathrm{m} \times 2-\mathrm{m}$ doublepane window are at $18^{\circ} \mathrm{C}$ and $6^{\circ} \mathrm{C}$, respectively. If the glasses are very nearly isothermal and the rate of heat transfer through the window is 110 W , determine the rates of entropy transfer through both sides of the window and the rate of entropy generation within the window, in W/K.


FIGURE P7-194

7-195 A well-insulated $4-\mathrm{m} \times 4-\mathrm{m} \times 5-\mathrm{m}$ room initially at $10^{\circ} \mathrm{C}$ is heated by the radiator of a steam heating system. The radiator has a volume of 15 L and is filled with superheated vapor at 200 kPa and $200^{\circ} \mathrm{C}$. At this moment both the inlet and the exit valves to the radiator are closed. A $120-\mathrm{W}$ fan is used to distribute the air in the room. The pressure of the steam is observed to drop to 100 kPa after 30 min as a result of heat transfer to the room. Assuming constant specific heats for air at room temperature, determine (a) the average temperature of air in $30 \mathrm{~min},(b)$ the entropy change of the steam, (c) the entropy change of the air in the room, and (d) the entropy generated during this process, in $\mathrm{kJ} / \mathrm{K}$. Assume the air pressure in the room remains constant at 100 kPa at all times.

7-196 A passive solar house that is losing heat to the outdoors at $3^{\circ} \mathrm{C}$ at an average rate of $50,000 \mathrm{~kJ} / \mathrm{h}$ is maintained at $22^{\circ} \mathrm{C}$ at all times during a winter night for 10 h . The house is to be heated by 50 glass containers, each containing 20 L of water that is heated to $80^{\circ} \mathrm{C}$ during the day by absorbing solar energy. A thermostat controlled 15 kW backup electric resistance heater turns on whenever necessary to keep the house at $22^{\circ} \mathrm{C}$. Determine how long the electric heating system was on that night and the amount of entropy generated during the night.
7-197E A $15-\mathrm{ft}^{3}$ steel container that has a mass of 75 lbm when empty is filled with liquid water. Initially, both the steel tank and the water are at $120^{\circ} \mathrm{F}$. Now heat is transferred, and
the entire system cools to the surrounding air temperature of $70^{\circ} \mathrm{F}$. Determine the total entropy generated during this process.
7-198 Air enters the evaporator section of a window air conditioner at 100 kPa and $27^{\circ} \mathrm{C}$ with a volume flow rate of 6 $\mathrm{m}^{3} / \mathrm{min}$. The refrigerant-134a at 120 kPa with a quality of 0.3 enters the evaporator at a rate of $2 \mathrm{~kg} / \mathrm{min}$ and leaves as saturated vapor at the same pressure. Determine the exit temperature of the air and the rate of entropy generation for this process, assuming (a) the outer surfaces of the air conditioner are insulated and (b) heat is transferred to the evaporator of the air conditioner from the surrounding medium at $32^{\circ} \mathrm{C}$ at a rate of $30 \mathrm{~kJ} / \mathrm{min}$. Answers: (a) $-15.9^{\circ} \mathrm{C}, 0.00193 \mathrm{~kW} / \mathrm{K}$, (b) $-11.6^{\circ} \mathrm{C}, 0.00223 \mathrm{~kW} / \mathrm{K}$


FIGURE P7-198

7-199 A 4-m $\times 5-\mathrm{m} \times 7-\mathrm{m}$ well-sealed room is to be heated by 1500 kg of liquid water contained in a tank that is placed in the room. The room is losing heat to the outside air at $5^{\circ} \mathrm{C}$ at an average rate of $10,000 \mathrm{~kJ} / \mathrm{h}$. The room is initially at $20^{\circ} \mathrm{C}$ and 100 kPa and is maintained at a temperature of $20^{\circ} \mathrm{C}$ at all times. If the hot water is to meet the heating requirements of this room for a 24 -h period, determine (a) the minimum temperature of the water when it is first brought into the room and (b) the entropy generated during a $24-\mathrm{h}$ period. Assume constant specific heats for both air and water at room temperature.
7-200 Consider a well-insulated horizontal rigid cylinder that is divided into two compartments by a piston that is free to move but does not allow either gas to leak into the other side. Initially, one side of the piston contains $1 \mathrm{~m}^{3}$ of $\mathrm{N}_{2}$ gas at 500 kPa and $80^{\circ} \mathrm{C}$ while the other side contains $1 \mathrm{~m}^{3}$ of He gas at 500 kPa and $25^{\circ} \mathrm{C}$. Now thermal equilibrium is established in the cylinder as a result of heat transfer through the piston. Using constant specific heats at room temperature, determine $(a)$ the final equilibrium temperature in the cylinder and (b) the entropy generation during this process. What would your answer be if the piston were not free to move?
7-201 © $\in S$ Reconsider Prob. 7-200. Using EES (or other) software, compare the results for constant specific heats to those obtained using built-in variable specific heats built into EES functions.

7-202 Repeat Prob. 7-200 by assuming the piston is made of 5 kg of copper initially at the average temperature of the two gases on both sides.
7-203 An insulated $5-\mathrm{m}^{3}$ rigid tank contains air at 500 kPa and $57^{\circ} \mathrm{C}$. A valve connected to the tank is now opened, and air is allowed to escape until the pressure inside drops to 200 kPa . The air temperature during this process is maintained constant by an electric resistance heater placed in the tank. Determine (a) the electrical energy supplied during this process and (b) the total entropy change. Answers: (a) 1501 kJ , (b) $4.40 \mathrm{~kJ} / \mathrm{K}$
7-204 In order to cool 1-ton of water at $20^{\circ} \mathrm{C}$ in an insulated tank, a person pours 80 kg of ice at $-5^{\circ} \mathrm{C}$ into the water. Determine (a) the final equilibrium temperature in the tank and $(b)$ the entropy generation during this process. The melting temperature and the heat of fusion of ice at atmospheric pressure are $0^{\circ} \mathrm{C}$ and $333.7 \mathrm{~kJ} / \mathrm{kg}$.
7-205 An insulated piston-cylinder device initially contains $0.02 \mathrm{~m}^{3}$ of saturated liquid-vapor mixture of water with a quality of 0.1 at $100^{\circ} \mathrm{C}$. Now some ice at $-18^{\circ} \mathrm{C}$ is dropped into the cylinder. If the cylinder contains saturated liquid at $100^{\circ} \mathrm{C}$ when thermal equilibrium is established, determine (a) the amount of ice added and (b) the entropy generation during this process. The melting temperature and the heat of fusion of ice at atmospheric pressure are $0^{\circ} \mathrm{C}$ and $333.7 \mathrm{~kJ} / \mathrm{kg}$.


FIGURE P7-205
7-206 Consider a 5-L evacuated rigid bottle that is surrounded by the atmosphere at 100 kPa and $17^{\circ} \mathrm{C}$. A valve at the neck of the bottle is now opened and the atmospheric air is allowed to flow into the bottle. The air trapped in the bottle eventually reaches thermal equilibrium with the atmosphere as a result of heat transfer through the wall of the bottle. The valve remains open during the process so that the trapped air also reaches mechanical equilibrium with the atmosphere. Determine the net heat transfer through the wall of the bottle and the entropy generation during this filling process.
Answers: $0.5 \mathrm{~kJ}, 0.0017 \mathrm{~kJ} / \mathrm{K}$
7-207 (a) Water flows through a shower head steadily at a rate of $10 \mathrm{~L} / \mathrm{min}$. An electric resistance heater placed in the

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water pipe heats the water from 16 to $43^{\circ} \mathrm{C}$. Taking the density of water to be $1 \mathrm{~kg} / \mathrm{L}$, determine the electric power input to the heater, in kW , and the rate of entropy generation during this process, in $\mathrm{kW} / \mathrm{K}$.
(b) In an effort to conserve energy, it is proposed to pass the drained warm water at a temperature of $39^{\circ} \mathrm{C}$ through a heat exchanger to preheat the incoming cold water. If the heat exchanger has an effectiveness of 0.50 (that is, it recovers only half of the energy that can possibly be transferred from the drained water to incoming cold water), determine the electric power input required in this case and the reduction in the rate of entropy generation in the resistance heating section.


FIGURE P7-207

7-208 © $\in S$ Using EES (or other) software, determine the work input to a multistage compressor for a given set of inlet and exit pressures for any number of stages. Assume that the pressure ratio across each stage is identical and the compression process is polytropic. List and plot the compressor work against the number of stages for $P_{1}=100 \mathrm{kPa}$, $T_{1}=17^{\circ} \mathrm{C}, P_{2}=800 \mathrm{kPa}$, and $n=1.35$ for air. Based on this chart, can you justify using compressors with more than three stages?

7-209 A piston-cylinder device contains air that undergoes a reversible thermodynamic cycle. Initially, air is at 400 kPa and 300 K with a volume of $0.3 \mathrm{~m}^{3}$ Air is first expanded isothermally to 150 kPa , then compressed adiabatically to the initial pressure, and finally compressed at the constant pressure to the initial state. Accounting for the variation of specific heats with temperature, determine the work and heat transfer for each process.
7-210 Consider the turbocharger of an internal combustion engine. The exhaust gases enter the turbine at $450^{\circ} \mathrm{C}$ at a rate of $0.02 \mathrm{~kg} / \mathrm{s}$ and leave at $400^{\circ} \mathrm{C}$. Air enters the compressor at $70^{\circ} \mathrm{C}$ and 95 kPa at a rate of $0.018 \mathrm{~kg} / \mathrm{s}$ and leaves at 135
kPa . The mechanical efficiency between the turbine and the compressor is 95 percent ( 5 percent of turbine work is lost during its transmission to the compressor). Using air properties for the exhaust gases, determine (a) the air temperature at the compressor exit and (b) the isentropic efficiency of the compressor. Answers: (a) $126.1^{\circ} \mathrm{C}$, (b) 0.642


FIGURE P7-210
7-211 Air is compressed steadily by a compressor from 100 kPa and $20^{\circ} \mathrm{C}$ to 1200 kPa and $300^{\circ} \mathrm{C}$ at a rate of 0.4 $\mathrm{kg} / \mathrm{s}$. The compressor is intentionally cooled by utilizing fins on the surface of the compressor and heat is lost from the compressor at a rate of 15 kW to the surroundings at $20^{\circ} \mathrm{C}$. Using constant specific heats at room temperature, determine (a) the power input to the compressor, (b) the isothermal efficiency, and (c) the entropy generation during this process.

7-212 A $0.25-\mathrm{m}^{3}$ insulated piston-cylinder device initially contains 0.7 kg of air at $20^{\circ} \mathrm{C}$. At this state, the piston is free to move. Now air at 500 kPa and $70^{\circ} \mathrm{C}$ is allowed to enter the cylinder from a supply line until the volume increases by 50 percent. Using constant specific heats at room temperature, determine ( $a$ ) the final temperature, $(b)$ the amount of mass that has entered, $(c)$ the work done, and $(d)$ the entropy generation.


FIGURE P7-212

7-213 When the transportation of natural gas in a pipeline is not feasible for economic reasons, it is first liquefied using nonconventional refrigeration techniques and then transported in super-insulated tanks. In a natural gas liquefaction plant, the liquefied natural gas (LNG) enters a cryogenic turbine at 40 bar and $-160^{\circ} \mathrm{C}$ at a rate of $55 \mathrm{~kg} / \mathrm{s}$ and leaves at 3 bar. If

350 kW power is produced by the turbine, determine the efficiency of the turbine. Take the density of LNG to be 423.8 $\mathrm{kg} / \mathrm{m}^{3}$. Answer: 72.9 percent


FIGURE P7-213

## Fundamentals of Engineering (FE) Exam Problems

7-214 Steam is condensed at a constant temperature of $30^{\circ} \mathrm{C}$ as it flows through the condensor of a power plant by rejecting heat at a rate of 55 MW . The rate of entropy change of steam as it flows through the condenser is
(a) - $1.83 \mathrm{MW} / \mathrm{K}$
(b) $-0.18 \mathrm{MW} / \mathrm{K}$
(c) $0 \mathrm{MW} / \mathrm{K}$
(d) $0.56 \mathrm{MW} / \mathrm{K}$
(e) $1.22 \mathrm{MW} / \mathrm{K}$

7-215 Steam is compressed from 6 MPa and $300^{\circ} \mathrm{C}$ to 10 MPa isentropically. The final temperature of the steam is
(a) $290^{\circ} \mathrm{C}$
(b) $300^{\circ} \mathrm{C}$
(c) $311^{\circ} \mathrm{C}$
(d) $371^{\circ} \mathrm{C}$
(e) $422^{\circ} \mathrm{C}$

7-216 An apple with an average mass of 0.15 kg and average specific heat of $3.65 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$ is cooled from $20^{\circ} \mathrm{C}$ to $5^{\circ} \mathrm{C}$. The entropy change of the apple is
(a) $-0.0288 \mathrm{~kJ} / \mathrm{K}$
(b) $-0.192 \mathrm{~kJ} / \mathrm{K}$
(c) $-0.526 \mathrm{~kJ} / \mathrm{K}$
(d) $0 \mathrm{~kJ} / \mathrm{K}$
(e) $0.657 \mathrm{~kJ} / \mathrm{K}$

7-217 A piston-cylinder device contains 5 kg of saturated water vapor at 3 MPa . Now heat is rejected from the cylinder at constant pressure until the water vapor completely condenses so that the cylinder contains saturated liquid at 3 MPa at the end of the process. The entropy change of the system during this process is
(a) $0 \mathrm{~kJ} / \mathrm{K}$
(b) $-3.5 \mathrm{~kJ} / \mathrm{K}$
(c) $-12.5 \mathrm{~kJ} / \mathrm{K}$
(d) $-17.7 \mathrm{~kJ} / \mathrm{K}$
(e) $-19.5 \mathrm{~kJ} / \mathrm{K}$

7-218 Helium gas is compressed from 1 atm and $25^{\circ} \mathrm{C}$ to a pressure of 10 atm adiabatically. The lowest temperature of helium after compression is
(a) $25^{\circ} \mathrm{C}$
(b) $63^{\circ} \mathrm{C}$
(c) $250^{\circ} \mathrm{C}$
(d) $384^{\circ} \mathrm{C}$
(e) $476^{\circ} \mathrm{C}$

7-219 Steam expands in an adiabatic turbine from 8 MPa and $500^{\circ} \mathrm{C}$ to 0.1 MPa at a rate of $3 \mathrm{~kg} / \mathrm{s}$. If steam leaves the turbine as saturated vapor, the power output of the turbine is
(a) 2174 kW
(b) 698 kW
(c) 2881 kW
(d) 1674 kW
(e) 3240 kW

7-220 Argon gas expands in an adiabatic turbine from 3 MPa and $750^{\circ} \mathrm{C}$ to 0.2 MPa at a rate of $5 \mathrm{~kg} / \mathrm{s}$. The maximum power output of the turbine is
(a) 1.06 MW
(b) 1.29 MW
(c) 1.43 MW
(d) 1.76 MW
(e) 2.08 MW

7-221 A unit mass of a substance undergoes an irreversible process from state 1 to state 2 while gaining heat from the surroundings at temperature $T$ in the amount of $q$. If the entropy of the substance is $s_{1}$ at state 1 , and $s_{2}$ at state 2 , the entropy change of the substance $\Delta s$ during this process is
(a) $\Delta s<s_{2}-s_{1}$
(b) $\Delta s>s_{2}-s_{1}$
(c) $\Delta s=s_{2}-s_{1}$
(d) $\Delta s=s_{2}-s_{1}+q / T$
(e) $\Delta s>s_{2}-s_{1}+q / T$

7-222 A unit mass of an ideal gas at temperature $T$ undergoes a reversible isothermal process from pressure $P_{1}$ to pressure $P_{2}$ while losing heat to the surroundings at temperature $T$ in the amount of $q$. If the gas constant of the gas is $R$, the entropy change of the gas $\Delta s$ during this process is
(a) $\Delta s=R \ln \left(P_{2} / P_{1}\right)$
(b) $\Delta s=R \ln \left(P_{2} / P_{1}\right)-q / T$
(c) $\Delta s=R \ln \left(P_{1} / P_{2}\right)$
(d) $\Delta s=R \ln \left(P_{1} / P_{2}\right)-q / T$
(e) $\Delta s=0$

7-223 Air is compressed from room conditions to a specified pressure in a reversible manner by two compressors: one isothermal and the other adiabatic. If the entropy change of air $\Delta s_{\text {isot }}$ during the reversible isothermal compression, and $\Delta s_{\text {adia }}$ during the reversible adiabatic compression, the correct statement regarding entropy change of air per unit mass is
(a) $\Delta s_{\text {isot }}=\Delta s_{\text {adia }}=0$
(b) $\Delta s_{\text {isot }}=\Delta s_{\text {adia }}>0$
(c) $\Delta s_{\text {adia }}>0$
(d) $\Delta s_{\text {isot }}<0$
(e) $\Delta s_{\text {isot }}=0$

7-224 Helium gas is compressed from $15^{\circ} \mathrm{C}$ and 5.40 $\mathrm{m}^{3} / \mathrm{kg}$ to $0.775 \mathrm{~m}^{3} / \mathrm{kg}$ in a reversible and adiabatic manner. The temperature of helium after compression is
(a) $105^{\circ} \mathrm{C}$
(b) $55^{\circ} \mathrm{C}$
(c) $1734^{\circ} \mathrm{C}$
(d) $1051^{\circ} \mathrm{C}$
(e) $778^{\circ} \mathrm{C}$

7-225 Heat is lost through a plane wall steadily at a rate of 600 W . If the inner and outer surface temperatures of the wall are $20^{\circ} \mathrm{C}$ and $5^{\circ} \mathrm{C}$, respectively, the rate of entropy generation within the wall is
(a) $0.11 \mathrm{~W} / \mathrm{K}$
(b) $4.21 \mathrm{~W} / \mathrm{K}$
(c) $2.10 \mathrm{~W} / \mathrm{K}$
(d) 42.1 W/K
(e) $90.0 \mathrm{~W} / \mathrm{K}$

7-226 Air is compressed steadily and adiabatically from $17^{\circ} \mathrm{C}$ and 90 kPa to $200^{\circ} \mathrm{C}$ and 400 kPa . Assuming constant specific heats for air at room temperature, the isentropic efficiency of the compressor is
(a) 0.76
(b) 0.94
(c) 0.86
(d) 0.84
(e) 1.00

7-227 Argon gas expands in an adiabatic turbine steadily from $500^{\circ} \mathrm{C}$ and 800 kPa to 80 kPa at a rate of $2.5 \mathrm{~kg} / \mathrm{s}$. For
isentropic efficiency of 80 percent, the power produced by the turbine is
(a) 194 kW
(b) 291 kW
(c) 484 kW
(d) 363 kW
(e) 605 kW

7-228 Water enters a pump steadily at 100 kPa at a rate of $35 \mathrm{~L} / \mathrm{s}$ and leaves at 800 kPa . The flow velocities at the inlet and the exit are the same, but the pump exit where the discharge pressure is measured is 6.1 m above the inlet section. The minimum power input to the pump is
(a) 34 kW
(b) 22 kW
(c) 27 kW
(d) 52 kW
(e) 44 kW

7-229 Air at $15^{\circ} \mathrm{C}$ is compressed steadily and isothermally from 100 kPa to 700 kPa at a rate of $0.12 \mathrm{~kg} / \mathrm{s}$. The minimum power input to the compressor is
(a) 1.0 kW
(b) 11.2 kW
(c) 25.8 kW
(d) 19.3 kW
(e) 161 kW

7-230 Air is to be compressed steadily and isentropically from 1 atm to 25 atm by a two-stage compressor. To minimize the total compression work, the intermediate pressure between the two stages must be
(a) 3 atm
(b) 5 atm
(c) 8 atm
(d) 10 atm
(e) 13 atm

7-231 Helium gas enters an adiabatic nozzle steadily at $500^{\circ} \mathrm{C}$ and 600 kPa with a low velocity, and exits at a pressure of 90 kPa . The highest possible velocity of helium gas at the nozzle exit is
(a) $1475 \mathrm{~m} / \mathrm{s}$
(b) $1662 \mathrm{~m} / \mathrm{s}$
(c) $1839 \mathrm{~m} / \mathrm{s}$
(d) $2066 \mathrm{~m} / \mathrm{s}$
(e) $3040 \mathrm{~m} / \mathrm{s}$

7-232 Combustion gases with a specific heat ratio of 1.3 enter an adiabatic nozzle steadily at $800^{\circ} \mathrm{C}$ and 800 kPa with a low velocity, and exit at a pressure of 85 kPa . The lowest possible temperature of combustion gases at the nozzle exit is
(a) $43^{\circ} \mathrm{C}$
(b) $237^{\circ} \mathrm{C}$
(c) $367^{\circ} \mathrm{C}$
(d) $477^{\circ} \mathrm{C}$
(e) $640^{\circ} \mathrm{C}$

7-233 Steam enters an adiabatic turbine steadily at $400^{\circ} \mathrm{C}$ and 3 MPa , and leaves at 50 kPa . The highest possible percentage of mass of steam that condenses at the turbine exit and leaves the turbine as a liquid is
(a) $5 \%$
(b) $10 \%$
(c) $15 \%$
(d) $20 \%$
(e) $0 \%$

7-234 Liquid water enters an adiabatic piping system at $15^{\circ} \mathrm{C}$ at a rate of $8 \mathrm{~kg} / \mathrm{s}$. If the water temperature rises by $0.2^{\circ} \mathrm{C}$ during flow due to friction, the rate of entropy generation in the pipe is
(a) $23 \mathrm{~W} / \mathrm{K}$
(b) $55 \mathrm{~W} / \mathrm{K}$
(c) $68 \mathrm{~W} / \mathrm{K}$
(d) $220 \mathrm{~W} / \mathrm{K}$
(e) $443 \mathrm{~W} / \mathrm{K}$

7-235 Liquid water is to be compressed by a pump whose isentropic efficiency is 75 percent from 0.2 MPa to 5 MPa at a rate of $0.15 \mathrm{~m}^{3} / \mathrm{min}$. The required power input to this pump is
(a) 4.8 kW
(b) 6.4 kW
(c) 9.0 kW
(d) 16.0 kW
(e) 12 kW

7-236 Steam enters an adiabatic turbine at 8 MPa and $500^{\circ} \mathrm{C}$ at a rate of $18 \mathrm{~kg} / \mathrm{s}$, and exits at 0.2 MPa and $300^{\circ} \mathrm{C}$. The rate of entropy generation in the turbine is
(a) $0 \mathrm{~kW} / \mathrm{K}$
(b) $7.2 \mathrm{~kW} / \mathrm{K}$
(c) $21 \mathrm{~kW} / \mathrm{K}$
(d) $15 \mathrm{~kW} / \mathrm{K}$
(e) $17 \mathrm{~kW} / \mathrm{K}$

7-237 Helium gas is compressed steadily from 90 kPa and $25^{\circ} \mathrm{C}$ to 600 kPa at a rate of $2 \mathrm{~kg} / \mathrm{min}$ by an adiabatic compressor. If the compressor consumes 70 kW of power while operating, the isentropic efficiency of this compressor is
(a) $56.7 \%$
(b) $83.7 \%$
(c) $75.4 \%$
(d) $92.1 \%$
(e) $100.0 \%$

## Design and Essay Problems

7-238 It is well-known that the temperature of a gas rises while it is compressed as a result of the energy input in the form of compression work. At high compression ratios, the air temperature may rise above the autoignition temperature of some hydrocarbons, including some lubricating oil. Therefore, the presence of some lubricating oil vapor in highpressure air raises the possibility of an explosion, creating a fire hazard. The concentration of the oil within the compressor is usually too low to create a real danger. However, the oil that collects on the inner walls of exhaust piping of the compressor may cause an explosion. Such explosions have largely been eliminated by using the proper lubricating oils, carefully designing the equipment, intercooling between compressor stages, and keeping the system clean.

A compressor is to be designed for an industrial application in Los Angeles. If the compressor exit temperature is not to exceed $250^{\circ} \mathrm{C}$ for safety consideration, determine the maximum allowable compression ratio that is safe for all possible weather conditions for that area.

7-239 Identify the major sources of entropy generation in your house and propose ways of reducing them.

7-240 Obtain the following information about a power plant that is closest to your town: the net power output; the type and amount of fuel; the power consumed by the pumps, fans, and other auxiliary equipment; stack gas losses; temperatures at several locations; and the rate of heat rejection at the condenser. Using these and other relevant data, determine the rate of entropy generation in that power plant.

7-241 Compressors powered by natural gas engines are increasing in popularity. Several major manufacturing facilities have already replaced the electric motors that drive their compressors by gas driven engines in order to reduce their energy bills since the cost of natural gas is much lower than the cost of electricity. Consider a facility that has a $130-\mathrm{kW}$ compressor that runs $4400 \mathrm{~h} / \mathrm{yr}$ at an average load factor of 0.6 . Making reasonable assumptions and using unit costs for natural gas and electricity at your location, determine the potential cost savings per year by switching to gas driven engines.


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[^1]:    Courtesy of Ebara International Corporation, Cryodynamics Division, Sparks, Nevada.

[^2]:    *This section can be skipped without a loss in continuity.

[^3]:    *Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with a CD-EES icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed DVD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

